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Calculation of Optimized Parameters for the Rectangular Microstrip Antennas by means of Taguchi Method

***Abstract**. Based on the orthogonal arrays (OAs) concept, Taguchi's method is a procedure effectively reducing the number of test iterations required in an optimization process. Even though the method has a quite wide spectrum of applications in general, applications of Taguchi method in electromagnetic theory are rare. In this study, the method is applied for rectangular microstrip patch antenna design. The obtained results show that the antenna design parameters are extracted in a successful manner.*

***Streszczenie.** Bazując na ortogonalnej matrycy zredukowano skutecznie liczbę iteracji niezbędnych do optymalizacji projektu anteny. Metodę zastosowano do projektu prostokątnej mikropaskowej anteny. (**Obliczenia optymalnych parametrów mikroanteny prostokątnej przy wykorzystaniu metody Taguchi**)*

Keywords: Taguchi method, optimization, rectangular microstrip patch antenna, resonant frequency.
Słowa kluczowe: antenna, częstotliwość rzonansowa, metoda Tageuchi.

1. Introduction

For the solution of most of the inverse engineering problems, trial-and-error based approaches are usually applied. The class of optimization algorithms, which are known as "Metaheuristics", are nothing but procedures performing the trial-and-error operations in a more intelligent and systematic manner.

Especially for test engineering applications, in which numerous test cases shall be executed for coverage of whole possible input combinations, Taguchi method is one of the most efficient and hence popular approaches for lowering the number of cases to be executed while satisfying a reasonable coverage percent. Developed by Dr. Genichi Taguchi, who is a quality engineer, the method is based on the concept of orthogonal arrays. That is the main reason for satisfying maximum coverage with a minimum number of trials.

Even though the method has so far widely been used for different applications of chemical engineering, power electronics, and many areas in industry, there have been only a limited number of applications of Taguchi method in electromagnetic theory. Previously, Weng *et al.* applied the method to problems in electromagnetics, such as the linear antenna array synthesis [1-3], and the coplanar waveguide slot antenna [4,5] design problem.

In this study, we try to demonstrate the applicability of the method to the design of rectangular microstrip patch antennas, which is a very important structure in electromagnetics and microwave applications. For this purpose, we first try to summarize the concept of orthogonal arrays in Section 2. Then, we try to give a brief description of Taguchi method. In Section 3, along with discussions of how to apply the method for our problem, we present the results obtained for different problem setups. Finally, we will try to make our concluding remarks in Section 4.

2. Material and Method

In this section, the main ideas underlying the Taguchi method together with the concept orthogonal array, which constitutes a major and important aspect of the search operation, are presented. Certainly, due to space considerations, only brief descriptions of all concepts and ideas are given in this paper. Interested readers might proceed to [6] for a more detailed description with explicit examples.

2.1. Orthogonal Arrays

Let S be a set consisting of s symbols (or levels). In that case, an $N \times k$ matrix A with entries from S is said to be an orthogonal array with s levels and strength t ($0 \leq t \leq k$), if in every $N \times t$ sub-array of A, each t-tuple based on S appears exactly the same times as a row [6]. The notation for such an orthogonal array is OA(N,k,s,t).

In order to have a better understanding about this definition, let us consider the OA(27,10,3,2) with entries selected from S={1,2,3}. Hence, the number of levels is 3. In case that any 2 columns are selected (i.e. t=2), nine possible combinations can be observed as a row: (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3). It can be verified that each combination has the same number of occurrences as a row: three times. This is the main idea of orthogonality: ensuring a balanced and fair selection of parameters in all possible combinations [7].

Table 1. The orthogonal array OA(27,10,3,2)

Experiments	Elements									
	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	2	1	2	2	2	3	3	1	2	3
3	3	1	3	3	3	2	2	1	3	2
4	1	2	1	2	2	2	3	3	1	2
5	2	2	2	3	3	1	2	3	2	1
6	3	2	3	1	1	3	1	3	3	3
7	1	3	1	3	3	3	2	2	1	3
8	2	3	2	1	1	2	1	2	2	2
9	3	3	3	2	2	1	3	2	3	1
10	1	1	2	1	2	2	2	3	3	1
11	2	1	3	2	3	1	1	3	1	3
12	3	1	1	3	1	3	3	3	2	2
13	1	2	2	2	3	3	1	2	3	2
14	2	2	3	3	1	2	3	2	1	1
15	3	2	1	1	2	1	2	2	2	3
16	1	3	2	3	1	1	3	1	3	3
17	2	3	3	1	2	3	2	1	1	2
18	3	3	1	2	3	2	1	1	2	1
19	1	1	3	1	3	3	3	2	2	1
20	2	1	1	2	1	2	2	2	3	3
21	3	1	2	3	2	1	1	2	1	2
22	1	2	3	2	1	1	2	1	2	2
23	2	2	1	3	2	3	1	1	3	1
24	3	2	2	1	3	2	3	1	1	3
25	1	3	3	3	2	2	1	3	2	3
26	2	3	1	1	3	1	3	3	3	2
27	3	3	2	2	1	3	2	3	1	1

2.2. Taguchi Method

Taguchi method starts with the problem initialization, which consists of the selection of an appropriate OA following the definition of the fitness function. The number k in the quadruple (N,k,s,t) is nothing but the dimension of the problem. In other words, for example, if the number of parameters to be determined in the problem is equal to 5, then the number k should be set to 5. This would yield an orthogonal array with k columns, each of which represents a design parameter.

Usually, during the search, 3 levels are found to be sufficient for each parameter, which means that the number s is equal to 3. In most cases, an OA with strength 2 (i.e. t=2) is sufficient and feasible, since this yields a small number of rows in the array. After determining k, s and t, appropriate N is determined.

Next, the input parameters are selected in order to conduct the experiments, where an experiment corresponds to evaluation of the fitness function at any point inside the multidimensional search space. For each parameter (i.e. for each dimension of the search space), values for each level shall be determined. In the first iteration, generally the mid-level (i.e. 2nd level for 3-level problems) is chosen as the mid-point of the search space at that dimension. Mathematically stating:

$$a(n)_1^{(2)} = (\min + \max)/2 \tag{1}$$

where the subscript denotes the iteration number, and the superscript in parenthesis denotes the level number.

Values of the 1st and the 3rd levels are evaluated by subtracting/adding a level difference (LD) form/to the value of the 2nd level:

$$a(n)_1^{(1)} = a(n)_1^{(2)} - LD(n)_1 \tag{2}$$

$$a(n)_1^{(3)} = a(n)_1^{(2)} + LD(n)_1 \tag{3}$$

where the level difference in the first iteration (LD_1) is determined as follows:

$$LD(n)_1 = (\max - \min)/(s+1) \tag{4}$$

where $\max$ and $\min$ are respectively the upper and the lower bounds of the search space in that particular dimension. This procedure guarantees performing the search operation in a uniformly distributed manner.

Then, for all points in the search space, the values of the fitness function (FF) are evaluated. Then, all these values are converted to the signal-to-noise (S/N) ratio (η) using:

$$\eta = -20\log(FF) \quad \text{dB} \tag{5}$$

Hence, a small fitness function results in a large S/N ratio. After the S/N values are evaluated for all experiment points, a response table is constructed for that iteration by averaging the S/N ratios for each parameter n and each level m using:

$$\overline{\eta}(\min) = \frac{s}{N} \sum_{i,OA(i,n)=m} \eta_i \tag{6}$$

From this response table, the ideal level for each parameter is determined. Then the search range is reduced in the next iteration via reduction in the level difference:

$$LD(n)_{i+1} = RR \cdot LD(n)_i \tag{7}$$

where RR is the reduction rate, which is typically chosen between 0.5 and 1. In this study, this value is chosen as 0.8. The algorithm continues computations of fitness functions and relevant S/N ratios up to the ith iteration for which:

$$\frac{LD(n)_i}{LD(n)_1} < CR \tag{8}$$

where CR is the converged value, which is usually set between 0.001 and 0.01. In this study, 0.001 is chosen as the value of CR. In case the condition in Eq. (8) is satisfied, the algorithm terminates, and the combination of the ideal level values for each dimension constitutes the solution proposal.

3. Problem Definition and Results

A rectangular microstrip antenna is presented in Fig. 1 together with the definitions of length L, width W, and height h.

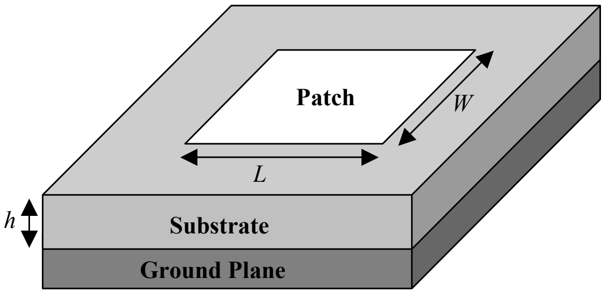

Fig.1. Rectangular microstrip patch antenna

The structure is quite popular due to its cheapness and production ease as well as conformity to mobile platforms without affecting their aerodynamics. On the other hand, the bandwidths of the microstrip patch antennas are quite narrow; hence, it is quite important to predict their resonant frequencies much before the design process. Thus, considerable amount of effort has been spent in order to obtain closed form expressions for the resonant frequency of the rectangular microstrip patch antenna. One of the most referred formulations of this sort is that of Kara [8]. In [8], Kara gave the formula for the resonant frequency of the rectangular microstrip antenna as:

$$f_r = \frac{c_0}{2(L + \Delta W)\sqrt{\varepsilon_e(W)}} \tag{9}$$

where c_0 is the velocity of the electromagnetic waves in free space, and $\varepsilon_e(W)$ is the effective dielectric constant given by:

$$\varepsilon_e(W) = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2\left(1 + \frac{10}{\mu}\right)^{1/2}} \tag{10}$$

where ε_r is the relative permittivity of the substrate, and $\mu = W/h$. ΔW is the line extension given by:

$$\Delta W = 0.412\,h\,\frac{(\varepsilon_e(W) + 0.300)(W/h + 0.264)}{(\varepsilon_e(W) - 0.258)(W/h + 0.813)} \tag{11}$$

Later in [9], Kara refined his resonant frequency formula. In this formulation, the effective dielectric constant $\varepsilon_e(W)$ is given by:

$$\varepsilon_e(W) = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2\left(1 + \frac{10}{\mu}\right)^{ab}} \tag{12}$$

in which:

$$a = 1 + \frac{1}{49}\ln\left[\frac{\mu^4 + (\mu/52)^2}{\mu^4 + 0.432}\right] + \frac{1}{18.7}\ln\left[1 + (\mu/18.1)^3\right] \tag{13}$$

$$b = 0.564\left(\frac{\varepsilon_r - 0.9}{\varepsilon_r + 0.3}\right)^{0.053} \tag{14}$$

and this time, the line extension ΔW is given by:

$$\Delta W = 0.824\,h\,\frac{(\varepsilon_e(W) + 0.300)(W/h + 0.264)}{(\varepsilon_e(W) - 0.258)(W/h + 0.813)} \tag{15}$$

The rectangular microstrip patch antenna design problem is considered as minimization of the function:

$$FF = \left| f_{r\_desired} - f_{r\_achieved} \right| \tag{16}$$

where $f_{r\_achieved}$ is the achieved resonant frequency during the heuristic search, in which the parameter setup (ε_r, L, W, h) yielding the desired resonant frequency ($f_{r\_desired}$) is tried to be found.

Previously, similar approach to the rectangular microstrip patch antenna design problem is presented in [10] and [11], in which the solution is carried out by means of Genetic Algorithm and Particle Swarm Optimization, respectively. The success of the optimization procedure can be measured via the following error measure denoted by *err*:

$$err = \frac{\left| f_{r\_desired} - f_{r\_achieved} \right|}{f_{r\_desired}} \tag{17}$$

which corresponds to nothing but the percentage error in the achieved resonant frequency with respect to the desired value.

3.1. Problem Setup I and the Corresponding Results

As seen throughout the formulations given in Eq. (7) to Eq. (13), the resonant frequency is dependent to the following four factors:

- ε_r: relative dielectric permittivity of the substrate,
- h: the height of the substrate,
- L: the length of the patch,
- W: the width of the patch.

Since there is no absolute freedom in the choice of the dielectric permittivity of the substrate, in the design procedure, ε_r is usually considered as an input. In other words, the design problem is to obtain the geometric parameters for a given substrate and a desired resonant frequency. Moreover, in most cases, the substrate height h is also considered as an input as in [10] and [11]. With these conditions, the problem reduces to a 2-dimensional problem in which the two parameters of the patch (i.e. L and W) are determined. This will be referred to as the Problem Setup 1 from now on. In this setup, 6 problems are selected from the literature, and all of them are solved by means of the Taguchi method. In all cases, the obtained L and W values are reasonable and feasible for manufacturing; and the desired resonant frequencies are achieved with a percentage error (*err*) smaller than 10^{-6} as seen in Table 2.

Table 2. Results for the Problem Setup 1 (in which the desired resonant frequency, substrate permittivity and height are given as inputs; the length and the width of the patch are determined)

Prob. No	Inputs			Outputs		
	$f_{r\ desired}$ (GHz)	ε_r	h (mm)	L (mm)	W (mm)	$f_{r\ achieved}$ (GHz)
1	6.200	2.55	2.0	14.389	2.500	6.200
2	8.450	2.22	0.17	11.820	1.817	8.450
3	7.740	2.22	0.17	12.909	19.996	7.740
4	5.060	2.33	1.57	18.618	21.144	5.060
5	5.600	2.55	1.63	16.014	7.291	5.600
6	4.805	2.33	1.57	19.655	17.278	4.805

The same problems were previously solved in [10] and [11] by means of Genetic Algorithm and Particle Swarm Optimization, respectively. The important point is that, as seen from Table 3, the number of experiments (i.e. fitness function evaluations) for the problem solution was 4000 for the Genetic Algorithm; meanwhile this number was 1000 for Particle Swarm Optimization. On the other hand, owing to the concept of orthogonal arrays, almost perfect solutions are obtained after only 477 fitness function evaluations in the Taguchi method. This demonstrates the power of the method compared to other metaheuristics.

Table 3. Number of fitness function evaluations required for the solution for the Problem Setup 1

	Solution via		
	Genetic Algorithm [10]	Particle Swarm Optimization [11]	Taguchi Method (This Study)
Number of Experiments (i.e. Number of Fitness Function Evaluations)	20×200=4000 (since 20 individuals and 200 iterations)	20×50=1000 (since 20 particles and 50 iterations)	3^2×53=477 (since 3 levels, 2 parameters and 53 iterations)

3.2. Problem Setup II and the Corresponding Results

As a next step, the substrate height h is also considered as a parameter to be determined during the optimization process, unlike most other studies in the literature. Under these circumstances, the problem becomes to a 3-dimensional problem in which the substrate height h in addition to the two parameters of the patch (i.e. L and W) are determined. This will be referred to as the Problem Setup 2 from now on. In this setup, 15 problems are selected from the literature, and all of them are solved by means of the Taguchi method. In all cases, the obtained h, L and W values are reasonable and feasible for manufacturing. Except for the Problems 2, 6, 12 and 13, the desired resonant frequencies are achieved with a percentage error (*err*) smaller than 10^{-6} as seen in Table 5. For problems 2, 6, 12 and 13, the error values are in the order of 10^{-4}. For each problem in this setup, the solution is obtained after 3^3×63=1701 fitness function evaluations (since there are 3 levels, 3 design parameters; and 63 iterations are required to achieve the algorithm termination criterion).

4. Conclusion

Thanks to the systematic distribution in the search mechanism by means of the orthogonal arrays, the Taguchi method is a powerful tool for the solution of multidimensional optimization problems even in cases where the fitness function is multimodal (i.e. having numerous local optima). Application of this well-known method in electromagnetics is relatively new. In this study, the method is applied for determining the parameters of the rectangular microstrip patch antenna with a desired resonant frequency.

Table 2. Results for the Problem Setup 2 (in which the desired resonant frequency and the substrate permittivity are given as inputs; the length and the width of the patch together with the substrate height are determined)

Prob. No	Inputs		Outputs			
	$f_{r\ desired}$ (GHz)	ε_r	L (mm)	W (mm)	h (mm)	$f_{r\ achieved}$ (GHz)
1	7.740	2.22	5.454	6.972	5.503	7.740
2	3.970	2.22	10.740	3.904	10.098	3.969
3	9.140	2.33	5.500	9.074	5.143	9.140
4	4.805	2.33	18.198	15.980	9.480	4.805
5	8.270	2.33	6.376	7.068	6.183	8.270
6	7.134	2.55	12.420	13.722	9.715	7.138
7	6.200	2.55	13.353	12.678	5.938	6.200
8	5.100	2.55	17.627	13.449	8.230	5.100
9	6.070	2.55	14.598	11.529	6.737	6.070
10	5.820	2.55	16.431	12.802	10.109	5.820
11	7.134	2.55	12.420	13.722	9.715	7.134
12	6.560	2.55	13.550	10.042	5.677	6.564
13	5.270	2.55	6.670	2.541	7.500	5.272
14	6.380	2.55	13.946	12.323	7.895	6.380
15	5.990	2.55	13.849	12.924	5.978	5.990

Table 2. Errors in the achieved resonant frequencies for the Problem Setup 2

Prob No.	*err*	Prob No.	*err*	Prob No.	*err*
1	$<10^{-6}$	6	5.6×10^{-4}	11	$<10^{-6}$
2	2.5×10^{-4}	7	$<10^{-6}$	12	6.1×10^{-4}
3	$<10^{-6}$	8	$<10^{-6}$	13	3.8×10^{-4}
4	$<10^{-6}$	9	$<10^{-6}$	14	$<10^{-6}$
5	$<10^{-6}$	10	$<10^{-6}$	15	$<10^{-6}$

The results of the Problem Setup 1 in Section 3.1 showed that the method finds almost perfect solutions within a quite smaller number of computations compared to Genetic Algorithm and Particle Swarm Optimization.

The results of the Problem Setup 2 in Section 3.2 demonstrated the power of the method once more, in the case where the problem definition became more complicated. For all cases, again almost perfect results were obtained in a timely manner by means of the Taguchi method.

Our ongoing research is about the application of the method for the design of more complicated microstrip patch antenna problems with different patch shapes and superstrates having multiple resonant frequencies.

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