Andrzej BIEŃ, Krzysztof DUDA

Akademia Górniczo-Hutnicza w Krakowie, Katedra Metrologii

An induction motor speed measurement method based on supplying current analysis

Streszczenie. *Przedstawiony tekst opisuje nową technikę pomiaru prędkości obrotowej – poślizgu silnika indukcyjnego. Prezentowana metoda opiera się o analizę czasowo-częstotliwościową prądu zasilania silnika w jednej fazie. Wyznaczana jest interpolowana DFT w analizowanych oknach czasowych, tak by uzyskać konieczną rozdzielczość częstotliwościową spektrogramów prądu. Poślizg i prędkość obrotowa wyznaczana jest na podstawie fluktuacji amplitudy podstawowej harmonicznej. Opisano wyniki badań laboratoryjnych metody. (**Pomiar prędkości obrotowej silnika indukcyjnego w oparciu o analizę widmową prądu zasilania**)*

***Abstract**. We present a new technique for estimating speed and slip of the induction motor. The method is based on the time-frequency analysis of the current supplying the motor. Interpolated DFT, computed in the sliding time window, is used for obtaining current spectrograms with high frequency resolution. The speed and the slip are estimated from the fluctuation of amplitude of the main current harmonic. The proposed method is validated by laboratory experiment.*

Słowa kluczowe: silnik indukcyjny, poślizg, prędkość obrotowa, interpolowane DFT, analiza czasowo-częstotliwościowa.
Keywords: induction motor, slip, rotational speed, interpolated DFT, time-frequency analysis.

Introduction

The need of the speed measurement of induction motors working in the inaccessible and important places is often necessary. The possibility of on-line digital processing of voltage and current measurement signals caused rapid development of the induction motor speed measuring methods. The methods for the induction motor speed measurement described in scientific literature and patents are based on the two following procedures:

- spectral analysis around the current induction motor main frequency (electricity) or detection of its envelope [1], [2], [3], [4] and patents: SU 1037401 A, DE 100 56 199 A 1, United States Patent 4,358,734
- spectral analysis in the frequency domain connected with heterogeneity of rotor construction- cage bars [5], [6], [7], [8] and patent WO 2006/131878 A1.

The first method of measurement can be implemented in a very easy way even without digital signal processing for the noticeable eccentricity motors [9]. However, its application makes the dynamical speed changes observation impossible and the measurement lasts relatively long, the envelope oscillation frequencies are low (about 1 Hz or smaller). The second method gives the possibility of dynamical speed changes observation. It is more frequent in the modern measurement system solution. It requires the acquaintance of the engine construction parameters and motor current main frequency. Additionally the current signal is often sampled synchronously with the main frequency.

The current signal may also be analyzed with the use of wavelets and neural networks, however we obtained satisfactory results with, computationally simpler than methods based on Fourier analysis. The envelop of AC induction motor supplying current is practically sinusoidal and its dynamic changes in the properly running motor are slow in relation to the basic frequency, thus the signal is well suited for Fourier analysis; on the other hand in the diagnosis wavelet analysis may be used.

The contribution of this paper is a connection of both procedures. High resolution spectral analysis for estimating the envelope of the current main frequency is used but there is no need to know the construction parameters of the induction motor.

The proposed method

A. The model of the induction motor.

It is assumed that a real induction motor is not built symmetrically about an axis. Slight asymmetries result from the motor structure and its supply method. Making these assumptions, to analyze current of nonsymmetrical induction motor, the mathematical dependences describing the state of synchronous motor which performs asynchronously are used. Using the model of bi-axial machine and classical Park's transformation we obtain dependences on the current induction motor for the basic frequency in mechanical dynamic position such as the mechanical rotor speed changes. Phase current im is described by the following relation [10]:

(1)
$$i_m = U_m Y_a(s)\cos(\omega_0 t + \gamma + \alpha(s) + \varphi_0) - U_m Y_b(s)\cos[(1-2s)\omega_0 t + \beta(s) + 2\varphi_0 - \gamma]$$

(2)
$$Y_a(s) = \frac{1}{2}\left|\frac{1}{X_d(s)} + \frac{1}{X_q(s)}\right| \quad Y_b(s) = \frac{1}{2}\left|\frac{1}{X_d(s)} - \frac{1}{X_q(s)}\right|$$

(3)
$$\alpha(s) = \mathrm{arctg}\left(\frac{\mathrm{Re}\left(\frac{1}{X_d(s)} + \frac{1}{X_q(s)}\right)}{\mathrm{Im}\left(\frac{1}{X_d(s)} + \frac{1}{X_q(s)}\right)}\right) \quad \beta(s) = \mathrm{arctg}\left(\frac{\mathrm{Re}\left(\frac{1}{X_d(s)} - \frac{1}{X_q(s)}\right)}{\mathrm{Im}\left(\frac{1}{X_d(s)} - \frac{1}{X_q(s)}\right)}\right)$$

where: s – slip, φ_0 and γ - electrical and mechanical phase shift, U_m - supply voltage amplitude, ω_0 - supply voltage frequency in radians, t – time. Parameters X_d and X_q in Park's model are functions of the slip s.

For the constant slip s the second part of (1) becomes the amplitude modulation signal or simply the envelope of the main current frequency. Neglecting constant values we get:

(4)
$$\cos[(1-2s)\omega_0 t] = \cos(\omega_0 t)\cos(2s\omega_0 t) + \sin(\omega_0 t)\sin(2s\omega_0 t)$$

It is seen from (4) that the frequency of the current main component envelope ω_{sp} is:

(5) $$\omega_{sp} = 2s\omega_0$$

and from (5) the slip is:

(6) $$s = \omega_{sp}/(2\omega_0)$$

Rotational speed ω is given by:

(7) $$\omega = (1-s)\omega_0$$

If the supply voltage is deformed then the envelop frequency ω_{sp} may also be estimated from the current harmonics, that is from spectral components with frequencies $h \cdot \omega_0$, h=2,3,4...

In the case, when a number of ωsp frequencies is estimated from a different current harmonics the mean or median of all results is accepted as the result of measurement.

B. Time-Frequency analysis.

The estimation and tracking of the frequency, amplitude and phase of the sinusoidal signal is an issue widely encountered in many fields of engineering. Classical tracking methods are based on the Kalman filter which is an optimal LQG (Linear Quadratic Gaussian) state estimator [11]. A number of tracking methods may be interpreted as special cases of the general Kalman filter. For example, adaptive RLS filter is a Kalman filter with filter coefficients being state variables [12]; and Software PLL (Phase Locked Loop) is also a Kalman filter [13].

Spectrum estimation methods may be distinguished as DFT based and modern spectrum analysis [14], [15]. Modern spectrum analysis is based on linear algebra, and is claimed in literature, for superior frequency resolution, especially for short data recordings, but the computational burden is quite significant as to compare to DFT methods.

DFT analysis is biased by a spectral-leakage and a picket-fence effect [16], [17]. Spectral leakage is reduced by the use of the proper window and the picket-fence effect is reduced by the interpolated DFT (IpDFT) algorithms [18]. Compare to parametric methods, DFT analysis does not require signal's model, and computational burden is low due to FFT algorithms.

In this paper we use the three-point IpDFT algorithm with the Rife-Vincent class I order 2 window. This window is defined as [20]:

(8)
$$w_n = \begin{cases} 1 - 4/3\cos(\omega_h n) + 1/3\cos(2\omega_h n), & 0 \le n < N \\ 0, & \text{otherwise} \end{cases}$$

where $\omega_h = 2\pi/N$ is the base frequency of the cosine periodic window (8). The frequency of the signal estimated by IpDFT equals [20]:

(9) $$\omega = (k \pm \delta)\frac{2\pi}{N}, \quad 0 < \delta \le 0.5$$

where correction δ is the distance to the DFT bin with the highest amplitude. If the second DFT bin with the highest amplitude has an index $k+1$ then we have the summation in (9); if the second bin with the highest amplitude has an index k-1 then we have subtraction in (9). For the three-point interpolation with the window (8) the correction δ is given by [19], [20]:

(10) $$\delta = 3\frac{|V_{k+1}| - |V_{k-1}|}{2|V_k| + |V_{k-1}| + |V_{k+1}|}$$

where V_k is the k-th DFT bin of the measurement signal x_n analyzed with the window w_n defined by (8), that is $V_k = \mathrm{DFT}\{x_n w_n\}$. The amplitude of the signal is given by [20]:

(11) $$|V(\omega_0)| = \frac{2\pi}{\sin(\delta\pi)} \frac{|V_{k-1}| + 2|V_k| + |V_{k+1}|}{S_{k-1} + 2S_k + S_{k+1}}$$

where:

S_{k-1}=|-(1/6)/(δ+3)+(4/6)/(δ+2)-1/(δ+1)+(4/6)/δ-(1/6)/(δ-1)|,
S_k =|(1/6)/(δ+2)-(4/6)/(δ+1)+1/δ-(4/6)/(δ-1)+(1/6)/(δ-2)|,
S_{k+1}=|-(1/6)/(δ+1)+(4/6)/δ-1/(δ-1)+(4/6)/(δ-2)-(1/6)/(δ-3)|.

The frequency (9) and the amplitude (11) are computed in the window sliding along the analyzed signal, similarly to time-depended Fourier transform computations [18], with the difference that we only track parameters of the main frequency current signal.

C. Measurement system.

Measurement system is depicted in Fig. 1. The current supplying induction motor is measured by Data Acquisition Board (DAQ) connected by an analog current transformer. The Digital Signal Processing (DSP) part can be implemented on the DAQ for real-time measurement or it can be an independent software for off-line analysis.

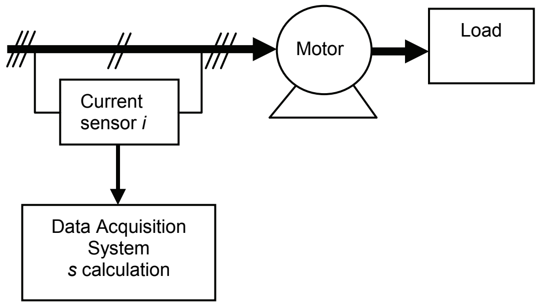

Fig.1 Measurement system to work in real condition

Laboratory experiments

The method proposed in section II was validated in the laboratory. The setup of the experiment is shown in Fig. 2. Sensors u, i were used for scale conversion and galvanic separation of DAQ. We used 12 bits AD converters and sampling frequency was set to 6.5 kHz.

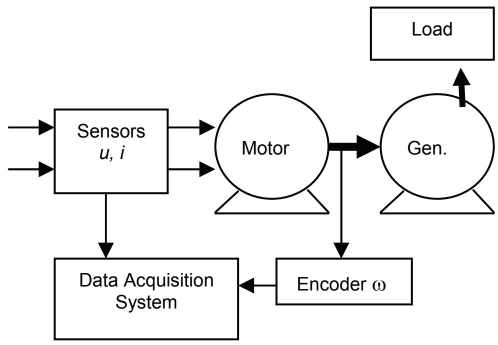

Fig.2. The setup of the laboratory experiment

The induction motor, with nominal power PN= 4kW was used, charged by DC generator. Reference rotational speed measurement was performed by counting pulses.

Time-frequency analysis was performed with IpDFT algorithm described in section IIB. The window was moved by one sample each iteration. The length of the DFT was set to 1024 samples, thus IpDFT algorithm was used to estimate the frequency bin in the neighborhood of the DFT bin with index k=8. Fig. 3 presents exemplary results of the envelope of the main frequency of the current. The frequency ωsp of this magnitude, depicted in Fig. 4, is next used to estimate the slip (6) and the rational speed (7). Fig. 4 shows the DFT spectrum of the envelope presented in Fig. 3. The given value of the frequency ω_{sp}=21,9 rad, marked by 'x', was estimated by another IpDFT algorithm. The rational speed estimated for the above example is given in Table 1, line 6.

Table 1 gives the results of rotational speed measurement of 4 pole induction motor at different static charges. It is seen from Table 1 that the results for the proposed method (5-th column) are in good agreement with the reference method (3-rd column). Last column shows percentage errors between the reference method and the proposed method. Determined frequencies of the current envelope given in Table I (4-th column) were rounded to 0.1 rad.

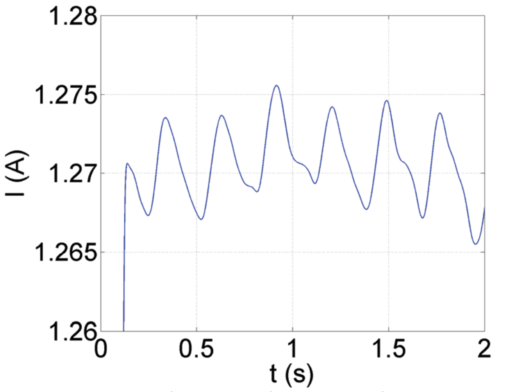

Fig.3. The envelope of the main frequency of the current signal

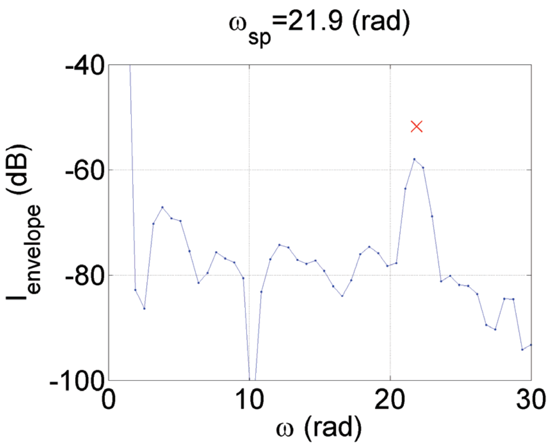

Fig.4. The DFT spectrum of the current signal envelope

Table 1 Results of rotational speed measurement

	Percent of nominal power P_N=4 kW (%)	Measured speed ω (rpm)	Solved frequency ω_{sp} (rad)	Calculated (7) speed ω (rpm)	Error (%)
1.	11,25	1496	0,6	1498,5	0,17
2.	25,25	1491	3,8	1492,5	0,10
3.	38,00	1485	6,3	1485	0
4.	75,50	1468	13,2	1468,5	0,03
5.	100,50	1452	18,8	1455	0,21
6.	113,75	1447	21,9	1447,5	0,03

Conclusion

The paper presents a new method for induction motor speed and slip measurement on the basis of its supplying current time-frequency analysis. This is a method based on Fourier transformation, easy to use for the current signals. The method was validated experimentally. The results with easy to find frequency of the current envelop are available for power over 10% of nominal power, for that case the local spectrum maximum is over 10 dB higher than the background. In conducted laboratory experiments the highest error of speed measurement is 0,21 % with the reference to counting pulses speed measurement. Determination of rational speed for its dynamic changes is limited to observation times longer than 0,5 s.

Beside easy to define number of pole pairs, the presented method does not require using induction motor construction parameters in calculations, what is its unquestionable advantage as compared to those described in quoted literature.

REFERENCES

[1] Dixon J. W., Rivarola J. N., *Induction Motor Speed Estimator and Synchronous Motor Position Estimator Base on Fixed Carrier Frequency Signal*, IEEE Transactions on Industrial Electronics, vol. 43, no. 4, August 1996, pp 505-509

[2] Beliczynski B., Grzesiak L., *Induction motor speed estimation: limitations of certain neural schemes based on time instances of periodic signals,* IECON '01 The 27th Annual Conference of the IEEE, vol. 1, 2001, pp. 653-657

[3] Mehrotra P., Quaicoe J.E., Venkatesan R., *Development of an artificial neural network based induction motor speed estimator*, Power Electronics Specialists Conference, 1996. PESC '96 Record., 27th Annual IEEE, vol. 1, pp. 682-688

[4] Guohai Liu, Zijian Hu, *Estimation of induction motor speed based on artificial neural networks inversion system method*, Control Conference, 2008. CCC 2008. 27th Chinese, pp. 649-653

[5] Zhao Tong, D.S. *Zinger, Induction motor speed detection by applying Goertzel's algorithm to current harmonics*, Applied Power Electronics Conference and Exposition, 2000. APEC 2000, vol. 1, pp. 140-143

[6] Nutt K., Tarbouchi M., McGaughey D., Chikhani A., *Current harmonic detection for induction motor speed sensorless control using fast orthogonal search,* American Control Conference, 2002, vol. 4, pp. 3088-3092

[7] Aiello M., Cataliotti A., Nuccio S., *An induction motor speed measurement method based on current harmonic analysis with the chirp-z transform*, IEEE Trans. Instrum. Meas., vol. 54, no. 5, Oct. 2005, pp. 1811-1819

[8] Shi Dongfeng, Unsworth P.J., Gao R.X., *Sensorless speed measurement of induction motor using Hilbert transform and interpolated fast Fourier transform,* IEEE Trans. Instrum. Meas., vol 55, no 1, Feb. 2006, pp. 290 – 299

[9] Rusek J., *Effect of non-steady air gap on induction machine performance*, 43 Intern. Wiss. Koll. Ilmenau, 1998, pp 446-449

[10] Bień A., Czajkowski J., Morończyk A., *Determining the RMS Current Components During the Synchronous Machine Starting to Answer a Diagnostic Purpose,* 9th International Symposium on Electrical Instruments in Industry (IMECO TC-4) Glasgow, Scotland, 1997, pp. 161-163

[11] Grewal M. S., Andrews A. P., *Kalman Filtering Theory and Practice Using MATLAB,* Third Edition John Wiley & Sons, 2008.

[12] Sayed A.H., Kailath T., *A State-Space Approach to Adaptive RLS Filtering*, IEEE Signal Processing Magazine, 11, 1994, pp. 18-16

[13] Patapoutian A., *On Phase-Locked Loops and Kalman Filters*, IEEE Transactions on Communications, vol. 47, no. 5, May 1999, pp. 670-672

[14] Kay S. M., *Modern Spectrum Analysis*, Englewood Cliffs, NJ: Prentice-Hall, 1987.

[15] Marple L., *Digital Spectrum Analysis with Applications*, Englewood Cliffs, NJ: Prentice-Hall, 1987.

[16] Harris F. J., *On the use of windows for harmonic analysis with the discrete Fourier transform*, Proc. IEEE, vol. 66, 1978, pp. 51–83

[17] Oppenheim A.V., Schafer R.W., Buck J.R., *Discrete-Time Signal Processing,* 2nd Edition, Prentice-Hall, 1999.

[18] Agrež D., *Weighted Multipoint Interpolated DFT to Improve Amplitude Estimation of Multifrequency Signal*, IEEE Trans. Instrum. Meas., vol. 51, 2002, pp. 287-292

Autorzy: *dr hab. inż. Andrzej Bień prof. n. AGH, Akademia Górniczo-Hutnicza w Krakowie, Katedra Metrologii, al. Mickiewicza 30, 30-059 Kraków, E-mail: abien@agh.edu.pl. dr inż. Krzysztof Duda, Akademia Górniczo-Hutnicza w Krakowie, Katedra Metrologii, al. Mickiewicza 30, 30-059 Kraków, E-mail: kduda@agh.edu.pl .*