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Calculation of Characteristic Impedance of Eccentric Rectangular Coaxial Lines

***Abstract**. In this paper the characteristic impedance for coaxial lines with a rectangular inner and outer conductor, concentric, eccentric and with rotated structure, has been obtained by using the Equivalent Electrodes Method (EEM). Our results have been compared with those reported in the literature, obtained by other analytical and numerical methods, and those obtained by using the COMSOL Multiphysics software package and a very good agreement with those results has been found.*

***Streszczenie.** W artykule przedstawiono obliczenia impedancji charakterystycznej linii koncentrycznej z prostokątnym przewodnikiem usytuowanym centrycznie i ekscentrycznie. Obliczenia wykonano z wykorzystaniem metody ekwiwalentnych elektrod EEM. (**Obliczenia impedancji charakterystycznej linii koncentrycznej z ekscentrycznym przewodem prostokątnym**)*

Keywords: Rectangular coaxial line, Equivalent Electrodes Method, Characteristic impedance.
Słowa kluczowe: linia koncentryczna, impedancja charakterystyczna.

Introduction

As rectangular coaxial lines are commonly used in microwave technology to transmit energy, it is very important to know their characteristic parameters. The main parameters are: the capacitance per unit length, the inductance per unit length, and characteristic impedance. Exact analytical solutions for the determination of these parameters are not possible and so numerical methods and approximate analytical expressions are used to obtain them.

Since the early 1960s several authors have been interested in analysing rectangular coaxial lines. The first significant analysis was performed by Chen [1] using the Schwarz-Christoffel transformation and approximate analytical expressions for the capacitance and inductance per unit length and characteristic impedance of rectangular coaxial lines were presented. Various other approximate analytical solutions for the capacitance per unit length, especially corner capacitance, have been presented in [2]. In 1965, relaxation techniques for the solution of Laplace's equation in two dimensions were applied by [3] to derive the characteristic impedance. The conformal mapping method and Schwarz-Christoffel transformation were used in 1972 by [4], for the calculation of the dimensions of a line for a given characteristic impedance. The step current density approximation has been used to determine the characteristic impedance by [5]. Costamagna and Fanni [6] in 1992 computed the characteristic impedances of various coaxial lines with rectangular conductors by means of a numerical inversion of the Schwarz-Christoffel conformal transformation. A quasi-analytical method of the multipole theory (MT) method has been used for the analysis of a rectangular transmission line family and the results obtained for the characteristic impedance have been presented in [7]. The Galerkin method has been applied to solve a system of integral equations, by means of basis functions taking into account the right-edge behaviour of the currents, for calculating the characteristic impedance by [8]. Results from 2007, for the capacitance per unit length of rectangular coaxial lines, obtained by using the Finite Element Method software package COMSOL have been presented by [9].

In our paper a simple numerical procedure called the Equivalent Electrodes Method (EEM) has been proposed for the analysis of rectangular coaxial lines. Previously, EEM has been used to solve static and quasi-static electromagnetic fields and other potential fields of theoretical physics [10, 11]. It has also been successfully used for transmission line analysis [12-15] and very good agreement between these results has been found by comparison with results obtained by using other analytical and numerical methods. Therefore, EEM has been used in this paper for calculating the characteristic impedance of coaxial lines with rectangular inner and outer conductor, concentric, eccentric and with rotated structure. The results obtained by using EEM have been compared with those found in the literature and those obtained by using numerous models of these lines made in the software package COMSOL.

EEM Application

EEM has been applied to the calculation of the characteristic impedance of a double eccentric rectangular coaxial line (Fig. 1). This line contains two rectangular conductors. The inner conductor is displaced from the longitudinal axis of symmetry in both horizontal and vertical directions. Also, the inner conductor has a different angular position relative to the outer conductor. The electric potentials of the conductors are φ_1 and φ_2.

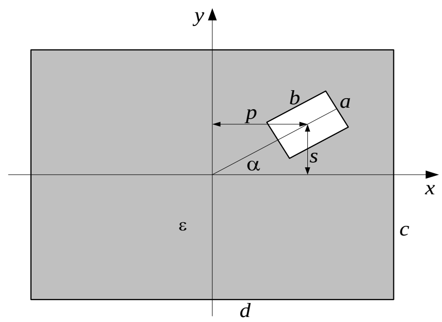

Fig.1. Double eccentric rectangular coaxial line with rotated inner conductor

In applying EEM, each conductor should be replaced by a finite system of equivalent electrodes (EE) placed on its surface. As the sides of the conductors are not the same size, each has to be divided by a different number of EE. In our example, the sides of the conductors with width a, b, c and d have been divided by N_1, N_2, N_3 and N_4 parts, respectively, and the widths of all of these parts should be same, Δx (Fig. 2). The systems of equivalent electrodes (EE) placed on the surface of the conductors are: q'_i, $i=1,2,\cdots,N_1$, for one vertical side of the inner electrode, q'_j, $j=1,2,\cdots,N_2$, for one horizontal side of the inner electrode, Q'_n, $n=1,2,\cdots,N_3$, for one vertical side of the outer electrode, and Q'_m, $m=1,2,\cdots,N_4$, for one horizontal side of the outer electrode. The total number of EE is therefore; $N_u=2N_1+2N_2+2N_3+2N_4$.

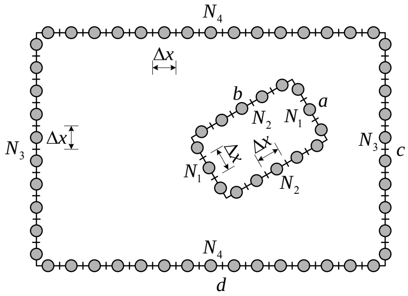

Fig.2. Arrangement of equivalent electrodes

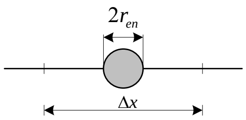

Fig.3. Thin flat strip conductor replaced with cylindrical EE

Each part of the original conductors, in our example thin flat strip conductors with a width Δx, has been replaced by a cylindrical EE with a circular cross-section as shown in Fig. 3. The equivalent radius of these EE can be calculated as: $r_{en}=\Delta x/4$ [12], where: $\Delta x=a/N_1=b/N_2=c/N_3=d/N_4$. The equivalent electrodes that replace these parts have the same radius, potential and charge as the part of the real conductor they represent.

Axes of EE have been placed in: $x=x_n$, $y=y_n$ for the inner conductor, and $x=x_m$, $y=y_m$ for the outer conductor, where:

(1) $$\begin{bmatrix} x_n \\ y_n \end{bmatrix} = \begin{bmatrix} p \\ s \end{bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \dfrac{b}{2} \\ -\dfrac{a}{2}+\dfrac{n}{N_1+1}a \end{bmatrix},$$

for $n=1,2,\cdots,N_1$,

(2) $$\begin{bmatrix} x_n \\ y_n \end{bmatrix} = \begin{bmatrix} p \\ s \end{bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \dfrac{b}{2}-\dfrac{n-N_1+1}{N_2+1}b \\ \dfrac{a}{2} \end{bmatrix},$$

for $n=N_1+1, N_1+2, \cdots, N_1+N_2$,

(3) $$\begin{bmatrix} x_n \\ y_n \end{bmatrix} = \begin{bmatrix} p \\ s \end{bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} -\dfrac{b}{2} \\ \dfrac{a}{2}-\dfrac{n-N_1-N_2+1}{N_1+1}a \end{bmatrix},$$

for $n=N_1+N_2+1, N_1+N_2+2, \cdots, 2N_1+N_2$,

(4) $$\begin{bmatrix} x_n \\ y_n \end{bmatrix} = \begin{bmatrix} p \\ s \end{bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} -\dfrac{b}{2}+\dfrac{n-2N_1-N_2+1}{N_2+1}b \\ -\dfrac{a}{2} \end{bmatrix},$$

for $n=2N_1+N_2+1, 2N_1+N_2+2, \cdots, 2N_1+2N_2$,

(5) $$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \dfrac{d}{2} \\ -\dfrac{c}{2}+\dfrac{m}{N_3+1}c \end{bmatrix},$$

for $m=1,2,\cdots,N_3$,

(6) $$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \dfrac{d}{2}-\dfrac{m-N_3+1}{N_4+1}d \\ \dfrac{c}{2} \end{bmatrix},$$

for $n=N_3+1, N_3+2, \cdots, N_3+N_4$,

(7) $$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} -\dfrac{d}{2} \\ \dfrac{c}{2}-\dfrac{m-N_3-N_4+1}{N_3+1}c \end{bmatrix},$$

for $m=N_3+N_4+1, N_3+N_4+2, \cdots, 2N_3+N_4$,

(8) $$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} -\dfrac{d}{2}+\dfrac{n-2N_3-N_4+1}{N_4+1}d \\ -\dfrac{c}{2} \end{bmatrix},$$

for $n=2N_3+N_4+1, 2N_3+N_4+2, \cdots, 2N_3+2N_4$.

Systems of EE create the electric potential:

(9) $$\varphi=\varphi_0-\sum_{i=1}^{N_I}\frac{q_i'}{4\pi\varepsilon}\ln\left[\left(x-x_i\right)^2+\left(y-y_i\right)^2\right]- -\sum_{j=1}^{N_O}\frac{Q_j'}{4\pi\varepsilon}\ln\left[\left(x-x_j\right)^2+\left(y-y_j\right)^2\right]$$

where: $N_I=2N_1+2N_2$ and $N_O=2N_3+2N_4$, which should satisfy the boundary conditions on the surfaces of the inner and outer electrodes of the coaxial line. By this procedure, a system of linear equations with the unknown charges of EE can be obtained.

As φ_1-$\varphi_2=U$ and $\sum_{i=1}^{N_I} q_i' + \sum_{j=1}^{N_O} Q_j' = 0$, the complete system of linear equations is:

(10) $$\begin{bmatrix} U \\ \vdots \\ U \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & \cdots & A_{1N_I} & B_{11} & \cdots & B_{1N_O} & 1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{N_I1} & \cdots & A_{N_IN_I} & B_{N_I1} & \cdots & B_{N_IN_O} & 1 \\ C_{11} & \cdots & C_{1N_I} & D_{11} & \cdots & D_{1N_O} & 1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{N_O1} & \cdots & C_{N_ON_I} & D_{N_O1} & \cdots & D_{N_ON_O} & 1 \\ 1 & 1 & \cdots & \cdots & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1' \\ \vdots \\ q_{N_I}' \\ Q_1' \\ \vdots \\ Q_{N_O}' \\ -\varphi_0' \end{bmatrix}$$

where:

(11) $$A_{nm}=-\frac{1}{4\pi\varepsilon}\ln\left[\left|r_1\right|^2+r_{en}^2\delta_{mn}\right],$$

for $x=x_n, y=y_n, n=1,2,\cdots,N_I, m=1,2,\cdots,N_I$,

(12) $$B_{nm}=-\frac{1}{4\pi\varepsilon}\ln\left[\left|r_2\right|^2\right],$$

for $x=x_n, y=y_n, n=1,2,\cdots,N_I, m=1,2,\cdots,N_O$,

(13) $$C_{nm}=-\frac{1}{4\pi\varepsilon}\ln\left[\left|r_1\right|^2\right],$$

for $x=x_m, y=y_m, n=1,2,\cdots,N_O, m=1,2,\cdots,N_I$,

(14) $$D_{nm}=-\frac{1}{4\pi\varepsilon}\ln\left[\left|r_2\right|^2+r_{en}^2\delta_{mn}\right],$$

for $x=x_m, y=y_m, n=1,2,\cdots,N_O, m=1,2,\cdots,N_O$,

(15) $r_1^2 = \left(x - x_n\right)^2 + \left(y - y_n\right)^2$, $r_2^2 = \left(x - x_m\right)^2 + \left(y - y_m\right)^2$

and δ_{nm} is the Kronecker symbol.

By solving this system, the unknown charges of EE can be determined.

The capacitance per unit length and the characteristic impedance can be easily calculated as:

$$C' = \frac{\sum_{i=1}^{N_t} q_i'}{U} , \; Z_C = \frac{\sqrt{\varepsilon_r}}{c\,C'} , \tag{16}$$

where: c is speed of electromagnetic waves in free space, and ε_r is the relative dielectric constant of the dielectric medium inside line.

Numerical Results

EEM has been applied to the analysis of the general shape of the double eccentric rectangular coaxial line with a rotated inner conductor (Fig. 1). The basic shape of rectangular coaxial lines has been analysed at beginning of this section. Then, lines with single or double eccentricity have been analysed. Lines with a rotated inner conductor, both concentric and eccentric, have been analysed at the end of this section.

Concentric Lines

The line from Fig. 1 has a concentric shape when p=s=0 and α=0.

When applying a numerical method for the calculation of a quantity, it is necessary to determine the convergence of the obtained solution. In the case of EEM, the obtained solution converges with an increasing number of equivalent electrodes. The convergence of the characteristic impedance for concentric lines has been presented in Table 1. Here, axial and transversal symmetry has been used, so the presented number of EE is four times greater than the case without symmetry. According to the results presented in Table 1, the total number of EE in other calculations has been set between 10000 and 16000 when symmetry has been used, or between 2500 and 4000 without symmetry.

This type of line has been analysed by many authors and the results obtained for the characteristic impedance have been presented in numerous papers. These results have been compared in Tables 2, 3 and 4 with the results obtained by using EEM. Table 5 presents the comparison of the results for the characteristic impedance of this type of line obtained by using EEM and COMSOL. A very good agreement of all these results is evident for various line dimensions. In particular, the best agreement of the presented results have been found between those obtained by using EEM and those given in [6].

By analysing all these results it can be seen that the dimensions of the line have a great impact on the characteristic impedance value by causing it to decrease as the ratios a/d and b/d increase.

Table 2 Z_c for different ratios a/d and b/d when d/c=1

a/d	b/d	Z_c MEE	Z_c [5]	Z_c [6]	Z_c [19]
0.05	0.2	121.81	121.63	121.75	121.14
0.1	0.2	109.10	108.53	109.05	108.96
0.2	0.2	91.155	91.20	91.12	92.44
0.1	0.4	79.939	80.32	79.90	80.92
0.2	0.4	67.3788	67.20	67.36	67.62
0.2	0.6	49.816	50.02	49.81	50.54

Table 3 Z_c for different ratios a/c when d-b=c-a, b/c=0.5

a/c	Z_c MEE	Z_c [1]	Z_c [6]	Z_c [7]
0.1	79.0583	76.896	79.034	79.124
0.2	66.318	65.717	66.295	66.341
0.3	55.5015	55.367	55.487	55.342
0.4	45.7736	45.759	45.767	45.789
0.5	36.8054	36.815	36.807	36.822
0.6	28.4531	28.468	28.461	28.476
0.7	20.6421	20.661	20.657	20.670
0.8	13.3193	13.342	13.341	13.357

Single Eccentric Lines

The line from Fig. 1 has a single eccentric shape when $p\neq0$ or $s\neq0$ and α=0.

The results for the characteristic impedance of lines with this shape have been presented in Tables 6 and 7. Table 6 presents the comparison of the results obtained by using EEM and those given in [5] and a very good agreement with those results has been obtained. The results presented in Table 7 have been obtained by using EEM for various eccentricities.

Table 4 Z_c for different ratios b/d when c/d=0.5, a/c =0.1

b/d	Z_c MEE	Z_c [1]	Z_c [2]	Z_c [3	Z_c [6]	Z_c [8]	Z_c [16]	Z_c [17]	Z_c [18]
0.9	27.5145	29.3	27.3	27.7	27.52	27.52	27.62	27.1	27.7
0.75	38.0314	38.7	38.1	38.3	38.03	38.03	38.10	37.8	38.0
0.45	58.4999	58.6	58.8	58.7	58.49	58.49	58.61	59.1	58.6
0.225	86.8448	86.7	87.2	85.9	86.82	86.82	87.27	87.0	86.5

Table 1 Convergence of the Z_c when a/d=0.2, b/d=0.4, c/d=0.5

N_u	Z_c MEE
96	39.9148
168	40.4090
336	40.7176
672	40.8972
1344	40.9967
2688	41.0500
5376	41.0779
8064	41.0875
10752	41.0924
13440	41.0953
16128	41.0973

Table 5 Z_c for different ratios b/d when c/d=0.5, a/d=0.2

b/d	Z_c MEE	Z_c COMSOL
0.1	75.1266	75.039614
0.2	59.0226	58.953132
0.3	48.5612	48.50593
0.4	41.0920	41.045987
0.5	35.3752	35.336003
0.6	30.6852	30.651742
0.7	26.4790	26.450546
0.8	22.1401	22.117489
0.9	16.3405	16.328868

From Table 7 it can be seen that increasing the eccentricity of the line decreases the value of the characteristic impedance. This decrease is smaller than the one that occurs when the dimensions of the line increase, except when the eccentricity is of the same order as the dimensions of line.

Table 6 Z_c for different ratios a/d, b/d and s/d when d/c=1 and p=0

a/d	b/d	s/d	Z_c MEE	Z_c [5]	Z_c [6]	Z_c [8]
0.1	0.2	0.35	69.8328	69.54	69.84	69.84
0.2	0.2	0.30	62.2342	62.04	62.24	62.24
0.1	0.4	0.35	47.5623	47.86	47.58	47.58
0.2	0.4	0.30	43.2709	43.27	43.32	43.32

Table 7 Z_c for different ratios b/d and p/d when c/d=0.5, a/d=0.2, s=0

b/d	Z_c MEE	Z_c MEE	Z_c MEE	Z_c MEE
	p/d=0.05	p/d=0.1	p/d=0.15	p/d=0.2
0.1	75.0425	74.7885	74.2494	73.1985
0.2	58.9183	58.6177	57.975	56.7034
0.3	48.4292	48.0362	47.1832	45.4423
0.4	40.9143	40.3566	39.1072	36.3774
0.5	35.1172	34.2537	32.181	26.8044
0.6	30.2714	28.7595	24.4317	-
0.7	25.7063	22.3359	-	-
0.8	20.1924	-	-	-

Double Eccentric Lines

The line from Fig. 1 has a double eccentric shape when $p\neq 0$ and $s\neq 0$ and $\alpha=0$.

The results for the characteristic impedance of lines with this shape have been presented in Tables 8, 9 and 10. Table 8 presents the comparison of the results obtained by using EEM and those given in [5] and a very good agreement with those results has been obtained. The results presented in Tables 9 and 10 have been obtained by using EEM, when eccentricity or dimensions of the line have been changed.

A double eccentric line has a smaller characteristic impedance than a single eccentric line, or a line without eccentricity.

Table 8 Z_c for different ratios a/d, b/d, p/d and s/d when d/c=1

a/d	b/d	p/d	s/d	Z_c MEE	Z_c [5]	Z_c [6]	Z_c [8]
0.05	0.2	2	2.75	96.6753	97.20	96.64	96.64
0.1	0.2	2	2.5	86.9425	86.78	86.92	86.92
0.1	0.4	1	2.5	64.9165	65.58	64.93	64.93
0.2	0.2	2	2	73.6543	73.54	73.64	73.64
0.2	0.4	1	2	56.6902	56.73	56.68	56.68
0.2	0.6	0	2	43.722	44.04	43.72	43.72

Table 9 Z_c for different ratios p/d and s/d when c/d=0.5, b/d =0.3, a/d=0.2

p/d	Z_c MEE	Z_c MEE	Z_c MEE
	s/d=0	s/d=0.05	s/d=0.1
0	48.5432	44.9082	32.3999
0.05	48.4292	44.8143	32.3566
0.1	48.0362	44.4900	32.2063
0.15	47.1832	43.7830	31.8743
0.2	45.4423	42.3266	31.1728
0.25	41.7792	39.2077	29.5897
0.3	33.1192	31.5779	25.2524

Rotated Structures

The line from Fig. 1 has a rotated shape when $\alpha\neq 0$.

The results for the characteristic impedance of the rotated structures, with or without eccentricity, obtained by using EEM and COMSOL have been presented in Table 11 and a good agreement between the results is evident. The variation of the characteristic impedance with a rotation angle, obtained by using EEM, has been presented in Figs. 4 and 5.

The characteristic impedance of concentric and eccentric lines decreases when increasing the rotation angle α from 0 to $\pi/2$. A symmetrical distribution of results has been obtained when the rotation angle has a value between $\pi/2$ and π (Figs. 4 and 5) and a similar distribution of results has been obtained for values greater than π. When eccentricity of the line is large, variation of the characteristic impedance with rotation angle is not so simple (Figs. 4 and 5). Thus, a combination of large eccentricity and rotation can lead to an undesirable increase of the characteristic impedance.

Table 10 Z_c for different ratios b/d and p/d when c/d=0.5, a/d=0.2, s/d=0.1

b/d	Z_c MEE	Z_c MEE	Z_c MEE	Z_c MEE
	p/d=0.05	p/d=0.1	p/d=0.15	p/d=0.2
0.1	55.8819	55.7649	55.515	55.0216
0.2	40.9939	40.8706	40.6042	40.0664
0.3	32.3566	32.2063	31.8743	31.1728
0.4	26.6594	26.4541	25.9801	24.8756
0.5	22.5624	22.2476	21.4476	19.0792
0.6	19.3832	18.8165	16.9866	-
0.7	16.6565	15.2644	-	-
0.8	13.7081	-	-	-

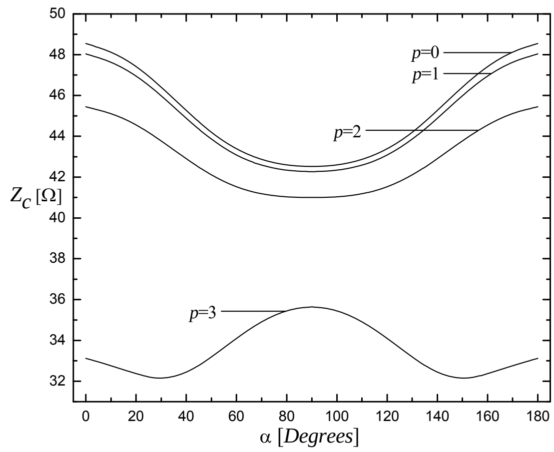

Fig. 4 – Variation of the characteristic impedance with rotation angle and eccentricity when c/d=0.5, a/d=0.2, b/d=0.3, s=0.

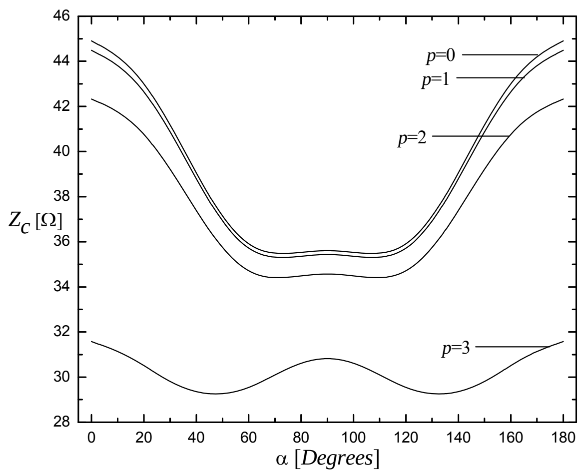

Fig. 5 – Variation of the characteristic impedance with rotation angle and eccentricity when c/d=0.5, a/d=0.2, b/d=0.3, s=0.5.

Table 11 Z_c for different rotation angle α and ratio p/d when c/d=0.5, a/d=0.2, b/d =0.3, s=0

α	Zc MEE	Zc COMSOL	Zc MEE	Zc COMSOL	Zc MEE	Zc COMSOL	Zc MEE	Zc COMSOL
	p/d=0	p/d=0	p/d=0.1	p/d=0.1	p/d=0.2	p/d=0.2	p/d=0.3	p/d=0.3
0	48.5432	48.50593	48.0362	47.998894	45.4423	45.409024	33.1192	33.1197811
π/12	47.9684	47.931376	47.4827	47.446429	44.9868	44.953978	32.5905	32.591744
π/6	46.4629	46.427551	46.0333	45.998132	43.81	43.778111	31.9188	31.916412
π/4	44.6458	44.612034	44.2856	44.254093	42.4255	42.394	32.6391	32.631137
π/3	43.2626	43.233384	42.9617	42.931642	41.4292	41.401904	34.1512	34.137033
5π/12	42.6254	42.599676	42.3611	42.33511	41.0378	41.013704	35.3413	35.327064
π/2	42.4839	42.458483	42.2317	42.207446	40.9782	40.9541723	35.7872	35.773517

Conclusion

In this paper, EEM is proposed for the calculation of the characteristic impedance of rectangular coaxial lines both with eccentricity and without it, and especially for lines with a rotated inner conductor. The results obtained have been compared with those found in the literature. Some of the results have been compared with those obtained by using the COMSOL software package. All the results have been found to be in very good agreement over a wide range of line dimensions, eccentricity and rotation angle.

The rectangular line is very important in microwave technology because matching impedance and transitional structure between round coaxial line and stripline, microstrip line or other planar line is very important and therefore, this type of line must have well defined characteristics. One of these parameters, the characteristic impedance, has been calculated in this paper with a very high accuracy with respect to other numerical methods previously used for such calculations. The results have been obtained for a wide range of dimensions and shapes of the line and thus provides a clear picture of how this parameter changes with dimensions and shapes. It can be concluded that eccentricity and rotation of the inner conductor significantly impacts on the characteristic impedance which takes a wide range of values. This allows this type of line to have continuously characteristic impedance which can be easily calculated by using EEM.

Comparing the results obtained by using EEM and those found in the literature, it can be seen that EEM is a very accurate method that has no limitations in terms of the dimensions and shapes of lines. Additionally, considering the entirely new results for lines with a rotated inner conductor, it can be concluded that EEM has significant advantages over other methods. The application of EEM is very straightforward and the programming is simple and fast without numerical integrations using only simple mathematical operations.

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