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Diffraction of incoherent light for thin, glass fibre diameter characterization

***Abstract**. The aim of this paper is to provide some conceptual basis of a measurement method for non-invasive in situ diameter characterization of a thin, glass fibre. The method involves small-angle scattering of incoherent light. A few selected results achieved with the aid of mathematical models of light scattering indicate that a practical implementation of the method is possible.*

Streszczenie.** W niniejszej pracy przedyskutowano koncepcję metody nieinwazyjnego pomiaru in situ średnicy włókna szklanego, wykorzystującej promieniowanie niespójne jako narzędzie poznawcze. W drodze modelowania matematycznego i symulacji numerycznych zbadano właściwości pola rozproszonego pod małym kątem oraz zaproponowano metodę jednoznacznego rozwiązania problemu odwrotnego w pomiarze średnicy.**(Wykorzystanie dyfrakcji promieniowania niespójnego w pomiarze średnicy włókna szklanego).

Keywords: Diffraction, incoherent light, non-invasive measurement, diameter, glass fibre.
Słowa kluczowe: Dyfrakcja, promieniowanie niespójne, pomiar nieinwazyjny, średnica, włókno szklane.

Introduction

In recent years there has been steady progress in the reinforcement fibres industry. Ultra-thin fibres with diameter of 4 to 40 μm made of glass are the essential component of composites used in many applications including aerospace, automotive, marine, construction industries to name but a few. Reinforcement fibres are manufactured in continuous forming process where a vertical jet is generated by flow of molten glass through a nozzle [1]. The jet of glass is cooled, solidified, and finally wound under a mechanical tension on a take-up reel (Fig.1).

In order to control some physical properties of a fibre being formed (diameter in particular), laboratory verification is usually carried out [1-4]. Such *ex situ* characterization, kept in isolation from the forming process, does not allow to control fibre's properties in real-time, which affects both the quality of the product as well as the efficiency of this process.

An empirical investigation intended to acquire *in situ*, quantitative information in real-time may be realized by means of light scattering. The way in which scientific knowledge is in this case obtained is causal, i.e. quantitative properties of the particle under test (reason) are deduced on the basis of some characteristics of the registered and processed scattered radiation (effect). Such methodology, among others, is the basis of diffractometry where light scattered under a small angle is analysed. When the particle dimension is much larger than the wavelength of light and the particle is highly absorptive, the Fraunhofer approximation can be applied to describe the far-field intensity. The approximation is very attractive for the purpose of non-invasive measurement as, in contrast to Lorenz-Mie theory, any optical properties are irrelevant. Moreover, since Fraunhofer diffraction by an aperture is mathematically equivalent to the Fourier transform of the aperture shape, the measurement system may be configured in such way so as to ensure the weak impact of particle movement on the registered pattern.

Laser diffractometry has been extensively studied and adopted for the purpose of sizing of single particles like a metallic wire [5]. Straight application of this method for the characterization of a transparent fibre is difficult, because simple interpretation of waves propagation in terms of the Huygens-Fresnel principle, valid for non-transparent objects, is insufficient to describe the light scattered on a weakly-absorbing particle. This fact illustrates the drawing comparison, showing the field scattered in the vicinity of a metallic wire (Fig. 2A) and the field in the vicinity and inside a glass fibre (Fig. 2B). The mutual interaction between the incident field and a transparent particle is complex and encompasses the concepts of diffraction, reflection, refraction, and absorption. Their combination give rise to various phenomena including resonance scattering as in the case of an optical microcavity [6, 7]. An overall description of the scattering problem requires solving Maxwell equations within, e.g. Lorenz-Mie theory framework [7]. Unfortunately, this approach turns out to be rather complicated analytically, in particular when inversion of measurement data is considered. This is so because the scattered field is non-linearly dependent on the particle properties [8]. Furthermore, mathematical models of light scattering are usually ill-conditioned even if identifiable in theory (as in the case of models based on the Fredholm integral equation [9, 10]), leading to numerical instabilities and inconsistent results. So far, the only method for the inversion of the laser diffraction diagrams of small transparent fibres has been developed by Onofri *et al.* [11]. The method relies on Lorenz-Mie theory calculations and the definition of a correlation estimator. However, detailed knowledge of the refractive index and its changes in the course of forming process must be provided.

The central idea of this paper is to influence some spectral features of the incident radiation in order to expose those phenomena, which would be described by means of simplified physical and mathematical concepts correctly. This idea is intended to be a basis for a simple method for the diameter of a transparent fibre characterization.

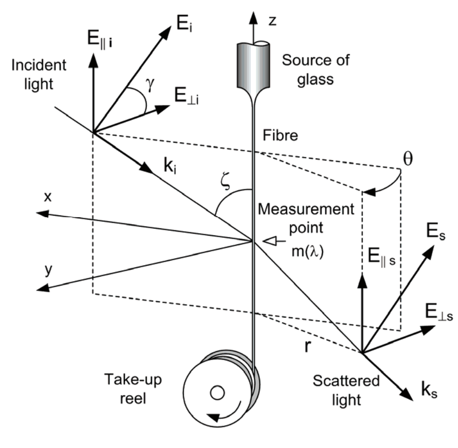

Fig.1. Glass fibre drawing process and the scattering geometry

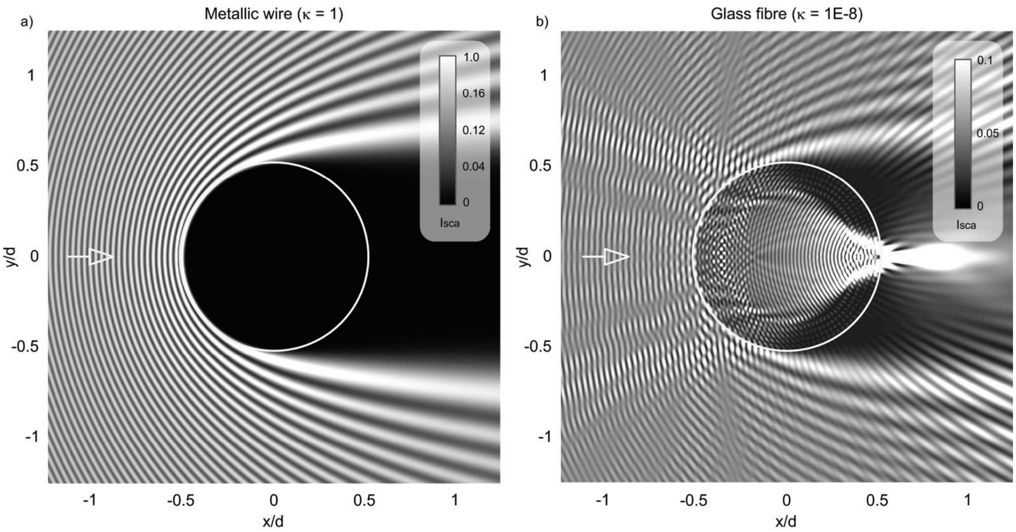

Fig.2. Normalized internal and near-field external intensity distributions over the scattering plane for: (A) metallic wire, (B) glass fibre. (d = 10 μm, λ = 0.6328 μm, m = 1.5505 + 0i)

Small-angle scattering of incoherent light

To treat the subject of a quasi-monochromatic, temporally incoherent beam scattering in a suitable way, it would be necessary to use a statistical approach. However, for the purposes of this work a heuristic rather than rigorous approach would be more practical. Consequently, the scattering effect in terms of intensity will be treated as the result of incoherent (i.e. phase independent) superposition of independently scattered, monochromatic waves from the spectrum of the incident radiation:

$$I_s(\theta) = \int_{\lambda_1}^{\lambda_2} E_s \cdot E_s^* d\lambda, \tag{1}$$

where E_s is the complex amplitude of the electric field evaluated for an independently scattered wave of length λ from the source's emission bandwidth $<\lambda_1, \lambda_2>$. A mathematical formalism which allows calculations of the electric field E_s outside a regular particle is typically based on the separation-of-variables solution of Maxwell's equations (i.e. Lorenz-Mie theory) for the case of a plane wave [7, 12] and also a Gaussian-beam [13, 14] incidence. Here the scattering problem is considered in the circular cylindrical coordinate system (r, θ, z) (Fig. 1), where fibre's shape is approximated by an infinitely-long, homogeneous cylinder of diameter d and complex, wavelength-dependent refractive index $m(\lambda)$. The fibre is illuminated by a perfectly collimated beam making angle ζ with its axis. It is convenient to split the incident and scattered electric fields into two orthogonal components – parallel ($E_{\|s}$) and perpendicular ($E_{\perp s}$) to the fibre axis. Following Bohren and Huffman [7], the fields are related through:

$$\begin{pmatrix} E_{\|s} \\ E_{\perp s} \end{pmatrix} = \exp(i3\pi/4)\left(\frac{2}{\pi k r \sin\zeta}\right)^{1/2} \times \exp\left[ik\left(r\sin\zeta - z\cos\zeta\right)\right]\begin{pmatrix} T_1 & T_4 \\ T_3 & T_2 \end{pmatrix}\begin{pmatrix} E_{\|i} \\ E_{\perp i} \end{pmatrix}, \tag{2}$$

where $E_{\|i}$, $E_{\perp i}$ are the complex amplitudes of the incident electric field components and $k = 2\pi/\lambda$ is a propagation constant. The terms T_1-T_4 of the amplitude scattering matrix depend on the scattering angle θ and have the form:

$$\begin{aligned} T_1(\theta) &= b_{0\mathrm{I}} + 2\sum_{n=1}^{\infty} b_{n\mathrm{I}} \cos(n\theta), \\ T_2(\theta) &= a_{0\mathrm{II}} + 2\sum_{n=1}^{\infty} a_{n\mathrm{II}} \cos(n\theta), \\ T_3(\theta) &= -2i\sum_{n=1}^{\infty} a_{n\mathrm{I}} \sin(n\theta), \\ T_4(\theta) &= -T_3(\theta). \end{aligned} \tag{3}$$

The scattered field expansion coefficients, a_n and b_n, are obtained from matching the boundary conditions at the surface of the fibre, and depend on its shape, size, and index of refraction. The solutions are known for the case of a homogeneous [7] and also a multi-layered cylinder [15].

The incoherent superposition according to equation (1) must be carried out by taking into account different amplitudes of each independently scattered wave according to the spectral profile of the source. For the sake of this discussion, Gaussian approximated spectral profile, which is valid for single-colour light-emitting diodes [16], is assumed:

$$I(\lambda) = I_0 \exp\left(4\log(0.5)\left(\frac{\lambda - \lambda_0}{fwhm}\right)^2\right), \tag{4}$$

where I_0 is the peak intensity (usually normalized to 1), λ_0 – the peak wavelength, and $fwhm$ – the spectral linewidth. To perform calculations according to equation (1), the spectral profile needs to be sampled at discrete points within the range of limiting wavelengths, i.e. from λ_1 to λ_2.

It was assumed earlier, that the intrinsic optical properties of the fibre characterize a wavelength-dependent index of refraction. This dependence results from the chromatic dispersion of glass. For most transparent media, in the visible spectral region where normal dispersion arises, the real part of refractive index (n) increases considerably, while absorption remains negligibly weak [17]. For the fairly well specification of $n(\lambda)$, Sellmeier formula may be used [17]:

$$n^2(\lambda) = 1 + \sum_i \frac{A_i \lambda^2}{\lambda^2 - \lambda_i^2}, \tag{5}$$

where A_i, λ_i^2 are coefficients empirically determined by a glass manufacturer.

For the purpose of numerical research, let us assume that the fibre is made of N-PSK3 glass with dispersion coefficients determined by Schott [18]. The imaginary part of the glass refractive index (absorption coefficient) is 1E-8. The collimated beam of incident light makes angle ζ with the fibre axis and is polarized parallel to the axis (Fig. 1). Calculations will be performed for 2201 wavelengths spaced every 0.0001 nm relative to the peak wavelength λ_0 specified later on. The spectral linewidth of the source is selected from the range of 1-80 nm. The scattered intensity calculated from $E_{\|s} \cdot E_{\|s}^*$ is recorded in the far field, i.e. where $kr\sin\zeta >> 1$ [7].

An illustrative result of performed calculations is given in figure 3, which demonstrates how the measurement data, i.e. the angular position of the first dark fringe (θ_{1-}), depends on the diameter of the fibre and the spectral linewidth. The same experiment was conducted for two different setups: (A) λ_0 = 0.430 μm, ζ = 30° and (B) λ_0 = 0.5145 μm, ζ = 90°. The calculations of the intensity as a function of θ were made at intervals of 0.01° within the range of 0-15°. The significant nonlinearities present in the graphs are caused by the part of light propagating through the weakly absorbing fibre. In a marginal case no dark fringe is formed, which is manifested in the form of a steep extreme on the graph. Comparison of parts (A) and (B) of figure 3 reveals, however, that these nonlinearities may be diminished significantly so that the relationship between θ_{1-} and d becomes monotonic. This may be particularly achieved by tightening the angle of incidence and shifting the peak wavelength towards the UV region what, in fact, causes that more fringes of the scattered field appear within the angular range of θ subjected to observation.

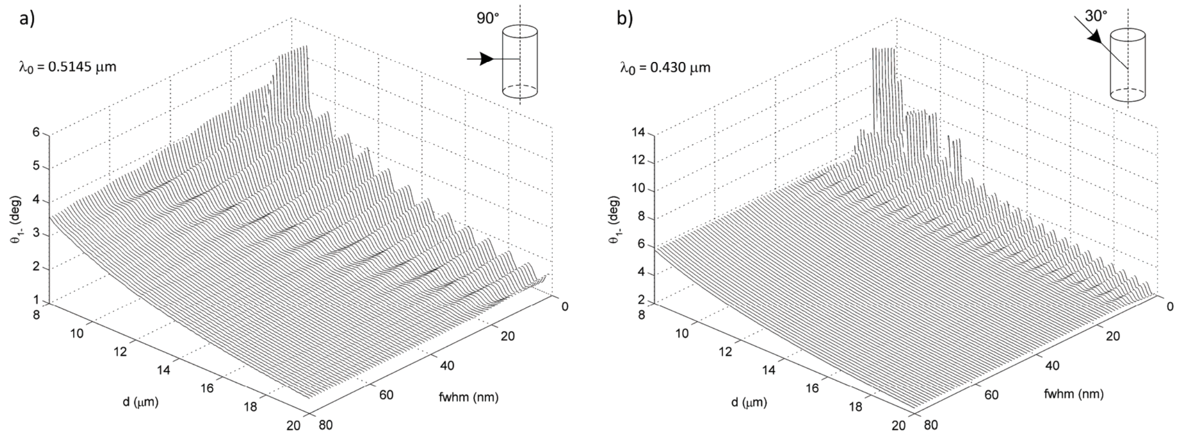

Fig.3. Influence of the spectral linewidth *fwhm* on the angular location θ_{1-} of the 1st dark fringe in the scattered field vs. the glass fibre diameter d. (A) normal incidence, λ_0 = 0.5145 μm, (B) diagonal incidence, λ_0 = 0.430 μm

The regular behavior of the first dark fringe suggests that the light scattered under a small angle may be regarded as if it was diffracted. This is because high-frequency ripples present in the scattering pattern for monochromatic light are averaged over θ on an incoherent superposition according to equation (1). A very special case is when the superposing waves coincide in phase so that the averaging effect becomes weaker. However, no significant influence on the scattered *far field* has been for such case observed. At this point it is worth noting that incoherent superposition of monochromatic waves corresponds to averaging over θ or, equivalently, to low-pass filtration of the scattering pattern in the frequency domain.

Diffraction-like effect of scattering raises the question whether and if so, to what extent the refractive index profile of the fibre influences the scattering pattern. For the sake of this matter, several simulations in the presence of a solid and an air core embedded into the fibre's structure were carried out. In these experiments, the relative size of the core and fibre, d_{core}/d, was being increased, and the information of the angular position, θ_{1-}, of the first dark fringe was being collected. Selected results are shown in figure 4. The left side of this figure corresponds to the case of a solid-core fibre, with the refractive index greater (+0.1) than that of the fibre, while the opposite side refers to the case of an air-core fibre. On both graphs, there are areas indicating the low susceptibility or even insensitivity of θ_{1-} to the refraction disturbance. This is a distinctive feature of the method which involves incoherent light scattering for a glass fibre diameter characterization, since the real part of the refractive index undergoes various changes of both isotropic as well as anisotropic nature (due to gas bubbles, inclusions, stress-induced birefringence, density fluctuations, etc.) [1, 19].

Synthetic data inversion

The inverse problem investigated in this paper is to convert measurement data obtained from the scattered field, into unambiguous information about the diameter of a fibre under test. To address this problem, first and foremost a simple causal model (i.e. forward model) will be introduced on the basis of the theory of Fraunhofer diffraction. This proposal results from the conclusions drawn above, according to which diffraction-like effect may be, under certain conditions, observed when incoherent light is scattered on a transparent fibre.

A classical description of diffraction in terms of the Fraunhofer approximation is to formulate the Kirchoff-Fresnel integral for 1D rectangular obstacle. As a consequence of a simplified description of vector electric field with a single scalar function, polarization effects are ignored. Therefore, the amplitude scattering matrix in equation (2) for normally incident plane wave (ζ = 90°) takes the form [7]:

$$(6)\qquad \begin{pmatrix} T_1 & T_4 \\ T_3 & T_2 \end{pmatrix} = \frac{\pi d}{\lambda}\left(\frac{1+\cos\theta}{2}\right)\sin(u)/u\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

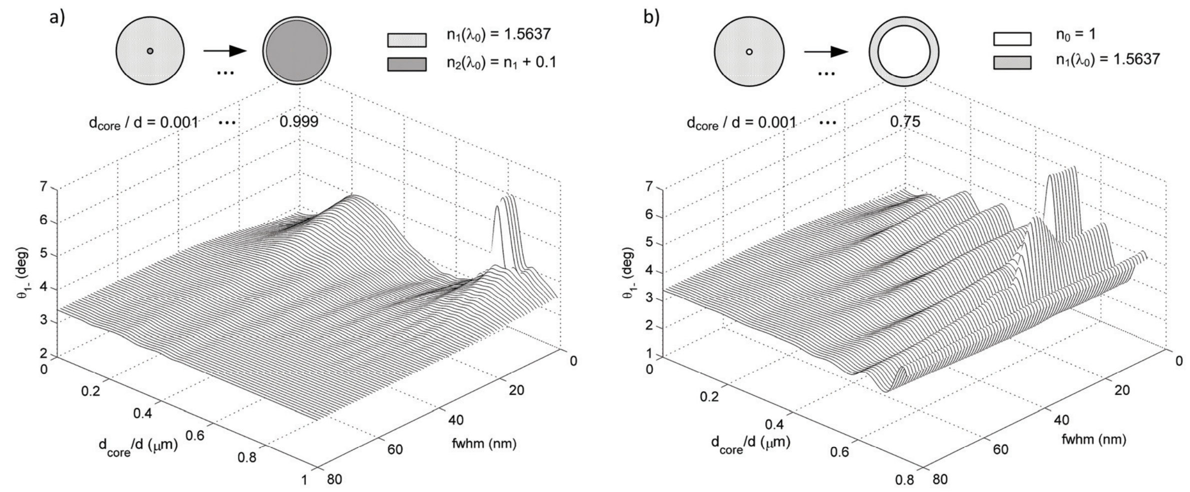

Fig.4. Influence of the size of inhomogeneity (solid core, A) and discontinuity (air core, B) included into a glass fibre on the angular location θ_{1-} of the 1st dark fringe in the scattered field. Normally incident beam of λ_0 = 0.430 μm and of the spectral linewidth *fwhm* is assumed

The coefficient u in (6) is defined as:

$$u = (\pi d/\lambda)\sin\theta. \quad (7)$$

Under the formula (7), the angle of the first minimum of scattering intensity (θ_{1-}) is simply related to the fibre size by:

$$\sin\theta_{1-} = \lambda/d. \quad (8)$$

Let us consider validity of the inverse model (8) in terms of systematic error. Synthetic measurement data, i.e. θ_{1-}, will be acquired from the scattering intensity patterns calculated with the use of Lorenz-Mie formalism for small-angle scattering of incoherent light, as introduced above. The experimental setup and simulation conditions correspond to those for which figure 3(A) was plotted, except for λ_0 = 0.430 μm. The results of the systematic error calculations are shown in figure 5. For comparative purposes, the experiment was carried out for two fibres with extremely different absorption characteristics, i.e. for a glass fibre and a metallic wire. Different linewidths of the source were chosen as well.

The nature of the systematic error observed in this case may be explained by taking into account two issues. Firstly, the approximate nature of the forward model causes that the measurement data (θ_{1-}) departs from the exact solution. However, as observed in figure 5, this component of error is reduced significantly for the case of incoherent illumination, even if the fibre shows transparency. In the border case where *fwhm* = 65 nm, the optical properties of a transparent fibre do not affect the error plot significantly, since data points for a glass fibre and a metallic wire coincide. At this point it is also worth noting that the diameter is over-estimated. Secondly, the systematic error is noisily dependent on θ_{1-} readings, carried out with a limited resolution (0.01°).This is particularly apparent for large fibres, for which most of the scattering pattern is concentrated near the central lobe and θ_{1-} undergoes small movements due to changes in the fibre size.

Conclusions

The theoretical considerations and numerical results covered by this paper indicate that a practical implementation of non-invasive *in situ* characterization of a thin, transparent fibre is possible. The idea proposed is to use a quasi-monochromatic, incoherent light as a measurement tool. Some benefits were outlined, however, numerous issues need to be settled by experimental work. A key issue is forming a well-collimated, high-intensity beam to obtain the scattering pattern of a good quality. Activities in this field will aim to use light-emitting diodes in a prototype of compact, integrated illuminator. So far, the problem of heat dissipation as well as thermal stabilization of LEDs has been discussed by Mroczka *et al.* [20, 21].

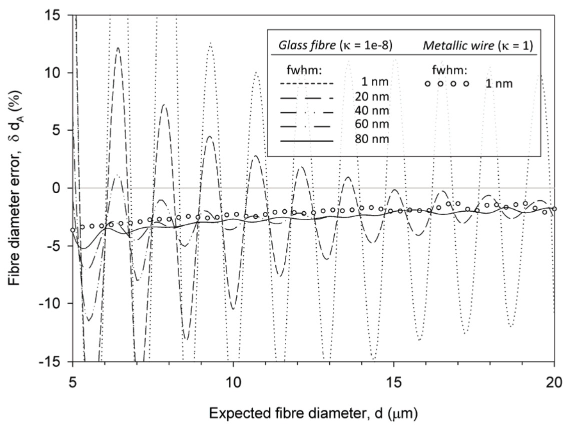

Fig.5. Fibre diameter error vs. expected diameter for a glass fibre and a metallic wire. Normally incident beam of λ_0 = 0.430 μm and of the spectral linewidth *fwhm* is assumed

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REFERENCES

[1] Gupta, P. K., "Glass Fibers for Composite Materials," in *Fiber Reinforcements for Composite Materials*. vol. II, A. D. Bunsell, Ed., The Netherlands: Elsevier Science Publishers B.V. (1988), pp. 19-71.
[2] Girasole, T., *et al.*, "Fiber orientation and concentration analysis by light scattering: experimental setup and diagnosis," *Review of Scientific Instruments,* vol. 68 (1997), pp. 2805-2811.
[3] Girasole, T., *et al.*, "Cylindrical fibre orientation analysis by light scattering: Part 1: Numerical aspects," *Particle & Particle Systems Characterization,* vol. 14 (1997), pp. 163-174.

[4] Girasole, T., *et al.*, "Cylindrical fibre orientation analysis by light scattering: Part 2: Experimental aspects," *Particle & Particle Systems Characterization,* vol. 14 (1997), pp. 211-218.
[5] Lebrun, D., *et al.*, "Enhancement of wire diameter measurements: comparison between Fraunhofer diffraction and Lorenz-Mie theory," *Optical Engineering,* vol. 4 (1996), pp. 946-950.
[6] Chýlek, P., *et al.*, "Narrow resonance structure in the Mie scattering characteristics," *Applied Optics,* vol. 17 (1978), pp. 3019-3021.
[7] Bohren, C. F. and Huffman, D. R., *Absorption and Scattering of Light by Small Particles*. New York: John Wiley & Sons (1983).
[8] Devaney, A. J. and Sherman, G. C., "Nonuniqueness in Inverse Source and Scattering Problems," *IEEE Transactions on Antennas and Propagation,* vol. AP-30 (1982), pp. 1034-1037.
[9] Mroczka, J. and Szczuczyński, D., "Improved regularized solution of the inverse problem in turbidimetric measurements," *Applied Optics,* vol. 49 (2010), pp. 4591-4603.
[10] Mroczka, J. and Szczuczyński, D., "Inverse problems formulated in terms of first-kind Fredholm integral equations in indirect measurements," *Metrology and Measurement Systems,* vol. 16 (2009), pp. 333-357.
[11] Onofri, F., *et al.*, "High-resolution laser diffractometry for the on-line sizing of small transparent fibres," *Optics Communications,* vol. 234 (2004), pp. 183-191.
[12] Barber, P. W. and Hill, S. C., *Light Scattering by Particles: Computational Methods* vol. 2. Singapore: World Scientific Publishing (1990).
[13] Ren, K. F., *et al.*, "Scattering of a Gaussian beam by an infinite cylinder in the framework of Generalized Lorenz-Mie Theory: formulation and numerical results," *Journal of Optical Society of America A,* vol. 14 (1997), pp. 3014-3025.
[14] Mroczka, J. and Wysoczański, D., "Plane-wave and Gaussian-beam scattering on an infinite cylinder," *Optical Engineering,* vol. 39 (2000), pp. 763-770.
[15] Jiang, H., *et al.*, "Improved algorithm for electromagnetic scattering of plane waves by a radially stratified tilted cylinder and its application," *Optics Communications,* vol. 266 (2006), pp. 13-18.
[16] Schubert, E. F., *Light-Emitting Diodes*, 2nd ed. New York: Cambridge University Press (2006).
[17] Bach, H. and Neuroth, N., Eds., *The Properties of Optical Glass*. Berlin: Springer-Verlag (1998).
[18] Schott, "Optical Glass. Data Sheets.," in *Optics for devices*, S. AG, Ed., ed (2007).
[19] Corpus, J. M. and Gupta, P. K., "Diameter dependence of the refractive index of melt-drawn glass fibers," *Journal of the American Ceramic Society,* vol. 76 (1993), pp. 1390-1392.
[20] Mroczka, J. and Parol, M., "Methods of temperature stabilization of light-emitting diode radiation," *Review of Scientific Instruments,* vol. 65 (1994), pp. 803-806.
[21] Mroczka, J., "Temperature stabilisation of light-emitting diode radiation," *Journal of Physics E: Scientific Instruments,* vol. 21 (1988), pp. 306-309.

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