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Sliding Mode Control with Grey Prediction for an Electro-Hydraulic Velocity Servo System

***Abstract**. This paper presents an improved reaching law, and a sliding mode speed controller, based on the improved reaching law and grey prediction compensator, is presented for an Electro-Hydraulic Velocity Servo System. The proposed control strategy can forecast and reject the disturbances and parameter variations. This system has a fast response and a good disturbance rejection capability. The numerical simulation is presented to verify the effectiveness of the proposed control scheme. It is shown from the experimental results that the proposed controller offers several advantages such as fast response, good disturbance rejection capability, good velocity tracking capability and so forth. It is also revealed from simulation results that the proposed control strategy is valid for the Electro-Hydraulic Velocity Servo System.*

***Streszczenie.** W artykule zaprezentowano ulepszony system sterowania ślizgowego bazującego na ulepszonej regule osiągania i szarej kompensacji przewidywań. System charakteryzuje się odpornością na zakłócenia i szybkością działania. System został zastosowany w elektro-hydraulicznym serwomechanizmie. (**Sterowanie ślizgowe z wykorzystaniem szarego systemu prognozowania w elektrohudraulicznym serwomechanizmie**).*

Keywords: Electro-hydraulics Velocity Servo System, Sliding mode control (SMC),Reaching Law, Grey Prediction, Numerical simulation,
Słowa kluczowe: serwomechanizm elektrohydrauliczny, sterowanie ślizgowe, system szarego prognozowania.

Introduction

The electro-hydraulic servo control systems are frequently used in the velocity servo system[1]. However, the actual electro-hydraulic servo system has many uncertainties, resulting from the flow-pressure relationship, oil leakage, and etc. Furthermore, an electro-hydraulic servo system suffers a variety of disturbances such as the null shift of a servo valve and load torque variations [2]. Consequently, the conventional control approaches may not guarantee satisfactory control performance for the system.

Recently, sliding mode control has been widely used in electro-hydraulic servo control [3], due to its simple algorithm, good robustness against parameter uncertainties and external disturbance , high reliability, fast response, and so on[4].

However, the most disadvantage of SMC is chattering, caused by the time lag switch, spatial lag switch, the system's inertia, uncertainties and other factors [5]. Because of the chattering, the unmodeled dynamics in the high-frequency is easy to be triggered. In order to reduce the chattering ,Quasi-sliding mode control, Sliding mode control based on trending law, Fuzzy sliding mode control, Dynamic sliding mode control, Adaptive sliding mode control ,Integral sliding mode control and so forth have been proposed[3].

In this respect, an improved trending law is proposed to reduce the chattering, then the sliding mode controller based on the proposed trending law with Grey prediction has been designed, and it is applied into a servo-valve controlled motor transmission system.

System Descriptions

Mathematical Model[6]

Hydraulic motor speed servo system, consisting of servo valve, hydraulic motor and load and so forth, is frequently used in the velocity servo system. The structure of the valve-controlled hydraulic motor is shown in figure 1.

In order to establish mathematical model of the hydraulic system, the following assumptions are made:

1).Connecting pipeline is short and thick, and friction loss in pipeline, the impact of fluid mass, and pipeline's dynamics are ignored.

2).The flowing state of the internal and external leakage is laminar flow.

3).The pressure of liquid chamber in the hydraulic motor is equal. Bulk modulus and oil temperature are constant.

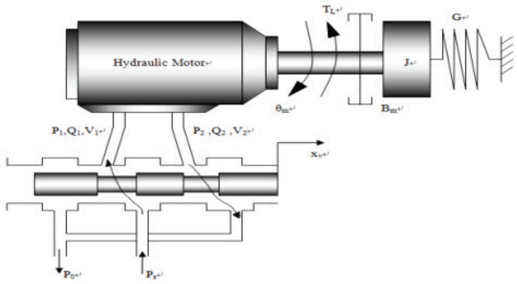

Fig.1.schematic diagram of valve-controlled hydraulic motor

Under the above assumptions, the dynamic equation of the valve-controlled hydraulic motor can be obtained.

1).Linearized flow equation of the valve

$$q_L = k_q x_v - k_c p_L \quad (1)$$

2).Continuity equation

$$q_L = D_m \dot{\theta}_m + C_{tm} p_L + (V_m / 4\beta_e)\dot{p}_L \quad (2)$$

3).Moment balance equation

$$T_s = D_m p_L = J\ddot{\theta}_m + B_m \dot{\theta}_m + G\theta_m + T_L \quad (3)$$

where q_L is load flow of the valve, x_v is spool displacement, k_c is flow-pressure coefficient of valve, p_L is load pressure, k_q is flow enhancement of valve on steady working-point, D_m is theoretical displacement of hydraulic motor, θ_m is angular displacement of hydraulic motor, C_{tm} is total leakage coefficient of hydraulic motor, $C_{tm} = C_{im} + 0.5C_{em}$. C_{im} is internal leakage and by pass leakage coefficient, C_{em} is external leakage coefficient of motor, V_m is total volume of connected pipes, Hydraulic motor and valve chamber, β_e is effective bulk modulus of working oil, J is the total inertia (the inertia between spool of Hydraulic motor and load (excluding oil) transfer into the motor shaft), B_m is viscous damping coefficient of load and motor, G is torsion spring stiffness of load, T_L is external load torque loaded on the motor shaft.

After Laplace transformation for (1), (2), (3),

(4) $$\begin{cases} q_L(s) = k_q x_v(s) - k_c p_L(s) \\ q_L(s) = D_m s\theta_m(s) + (C_{tm} p_L + (V_m/4\beta_e)s)p_L(s) \\ T_s(s) = Js^2\theta_m(s) + B_m s\theta_m(s) + G\theta_m(s) + T_L(s) \end{cases}$$

From (4), we can obtain the transfer function:

(5) $$\frac{\theta_m}{x_v} = \frac{\omega_h^2 k_q / D_m}{s(s^2 + 2\xi_h \omega_h s + \omega_h^2)}$$

Where: ω_h is natural frequency of hydraulic system, ξ_h is hydraulic damping ratio,

The structure of velocity servo system

The structure of hydraulic motor speed servo system is shown in figure2.From the figure we know that the system is consisted of electro-hydraulic servo valve, servo amplifier, hydraulic motor, sensors and so forth.

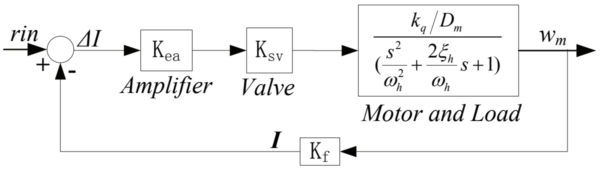

Fig.2. Structure of hydraulic motor speed servo system

In order to simplify the servo system, supposed that the system is only inertia loads, while the elastic load, viscous load were 0, and the dynamic characteristics of servo valve, amplifiers and other electrical components are ignored. The state equation can be written as:

$$\begin{cases} \dot{x} = Ax + Bw_m \\ y = Cx \end{cases}$$

(6)

where: $x = [x_1 \ \ x_2]^T, \dot{x} = [\dot{x}_1 \ \ \dot{x}_2]^T, A = \begin{bmatrix} 0 & 1 \\ -\omega_h^2 & -2\xi_h\omega_h \end{bmatrix}$,

$B = [0 \ \ k_q\omega_h^2 / D_m]^T, C = [1 \ \ 0]^T$.

The sliding mode controller

Reaching Law

Domestic experts proposed the reaching law approach to reduce or inhibit the chattering of SMC in the premise of ensuring the condition of sliding existence $S\dot{S} < 0$ has been met. And given four different methods of reaching law, such as constant reaching law, exponential approach law, power reaching law and general reaching law, where the exponential approach is applied widely [5], the discrete form of exponential reaching law's switching zone is zonal. So it can't be close to the origin ultimately but a chattering near the origin during the process of moving [5] . In order to solve the chattering phenomenon of exponential reaching law near the origin [7], the paper[8] proposed a variable rate reaching law for continuous system with its discrete form as follows:

(7) $$S(k+1) - S(k) = -\varepsilon T \|x\|_1 \operatorname{sgn}(S(k))$$

where: $\|x\|_1$ -- 1 norm of x.

Reaching speeds of variable rate reaching law is $\varepsilon\|x\|_1$, and is proportional to $\|x\|_1$. The switching zone passes through the origin with two rays, which can make S = 0 in the middle of the two rays, and can be stabilized at the origin. However, when the system just entered to switching zone $\|x\|_1$ will get a large value, and have a big chattering in SMC. In order to overcome the problem of the variable rate reaching law and exponential approach law, a new reaching law has been proposed based on paper [9] with its discrete form as follows:

$$S(k+1) - S(k) = -qTS(k) - \frac{|S(k)|}{2} T \tan sig(\|x\|) \operatorname{sgn}(S(k))$$

(8)

Grey Sliding mode controller

In this paper, in order to enhance the ability of anti-interference and reduce the chattering of VSC, the novel reaching law and grey prediction methods have been used to design a controller [10-11].

Assume G(s) is the transfer function of servo system, and its state space equation is expressed as follow:

Assuming G(s) is the transfer function of servo system, and its state space equation as follow:

(9) $$x(k+1) = Ax(k) + B(u(k) + D(x,k))$$

where: $x(k) = [x_1(k) \ \ x_2(k)]^T$, $x_1(k)$ is The actual velocity. $x_2(k)$ is the changing rate of velocity. $D(x,k)$ is the parameter uncertainty and external disturbances.

$$D(x,k) = \sum_{i=1}^{k} V_i x_i(k) + d(k)$$

(10)

Supposed $r(k)$, $\dot{r}(k)$ is the velocity order and its changing rate. $R_k = [r(k), \dot{r}(k)]$, predicted by linear extrapolation R_{k+1} is:

(11) $$r(k+1) = 2r(k) - r(k-1)$$

(12) $$\dot{r}(k+1) = 2\dot{r}(k) - \dot{r}(k-1)$$

Defined:

(13) $$e = r(k) - x(k)$$

Then:

(14) $$\dot{e} = \dot{r}(k) - \dot{x}(k)$$

Define switching function is:

(15) $$S(k) = C_e E = C_e(R_k - x(k))$$

(16) ∴ $$\begin{aligned} S(k+1) &= C_e E = C_e(R_{k+1} - x(k+1)) \\ &= C_e(R_{k+1} - Ax(k) - Bu(k)) \end{aligned}$$

The exponential reaching law's discrete form is:

(17) $$\begin{aligned} &S(k+1) \\ &= (1 - qT)S(k) - \frac{|S(k)|}{2} Tt\operatorname{an} sig(\|x\|) \operatorname{sgn}(S(k)) \end{aligned}$$

In order to weaken the chattering of the control, we use the saturation function sat (S) to replace the exponential reaching law's sign function sgn (S):

(18) $$sat(S(k)) \begin{cases} 1 & S(k) > \Delta \\ kS(k) & |S(k)| \le \Delta \\ -1 & S(k) < \Delta \end{cases}, k = \frac{1}{\Delta}$$

By (16), (17),(18)

(19) $$\begin{aligned} &S(k+1) \\ &= (1 - qT)S(k) - \frac{|S(k)|}{2} T \tan sig(\|x\|) sat(S(k)) \\ &= C_e(R_{k+1} - Ax(k) - Bu(k)) \end{aligned}$$

(20) $$\begin{aligned} u(k) &= CB^{-1}[C_e(R_{k+1} - R_k) - C_e(A - I)x(k) \\ &+ qTS(k) + \frac{|S(k)|}{2} T \tan sig(\|x\|) sat(S(k))] \end{aligned}$$

where: $C_e = [c, 1]$.

Stability Analysis

THEOREMS 1: the designed controller should ensure the exist condition of sliding mode.

PROOF: Consider the following Lyapunov function:

(21) $$V(k)=\frac{1}{2}[S(k)]^2$$

(22) $$V(k+1)=\frac{1}{2}[S(k+1)]^2$$

$$V(k+1)-V(k)$$

(23) $$=\frac{1}{2}\{[S(k+1)]^2-[S(k)]^2\}$$

$$=\frac{1}{2}\{[S(k+1)-S(k)][S(k+1)+S(k)]\}$$

$$\because [S(k+1)-S(k)]\operatorname{sgn}(S(k))$$

(24) $$=[-qTS(k)-\frac{|S(k)|}{2}T\tan sig(\|x\|)\operatorname{sgn}(S(k))]\operatorname{sgn}(s(k))$$

$$=-(q+0.5\tan sig(\|x\|))T|S(k)|<0$$

$$\because [S(k+1)+S(k)]\operatorname{sgn}(S(k))$$

(25) $$=[(2-qT)S(k)-\frac{|S(k)|}{2}T\tan sig(\|x\|)\operatorname{sgn}(S(k))]\operatorname{sgn}(s(k))$$

$$=(2-qT-0.5T\tan sig(\|x\|))|S(k)|$$

$$=(2-qT-0.5T)|S(k)|>0$$

For the former case, the following equation can be obtained:

$$V(k+1)-V(k)$$

(26) $$=\frac{1}{2}\{[(S(k+1)-S(k))\operatorname{sgn}(S(k))][(S(k+1)+S(k))\operatorname{sgn}(S(k)]\}$$

$$\therefore V(k+1)-V(k)<0$$

$$\therefore V(k+1)<V(k)$$

Therefore, the tracking error of system asymptotically tends to zero when $t\to\infty$, that is stable tracking can be achieved.

Grey Prediction

The grey theory proposed by Deng [12] has been successfully employed in control systems and motor control[13–15]. Grey prediction methodology requires only several output data to develop a grey model and to forecast a future value, without complex calculation.

The SMC controlling law is a function of state and related to system parameter and disturbance. In order to avoid the influence of system parameter and disturbance for the controlling rule, the grey prediction theory is used to identify system parameter and disturbance, and make a compensation for sliding model controller. Sliding model control with grey prediction theory includes grey prediction and compensation [11,16,17].

Grey prediction

Let $x^{(0)}$ be the original data sequence:

(27) $$x^{(0)}=(x^{(0)}(1)\ x^{(0)}(2)\ x^{(0)}(3)\ \ \cdots x^{(0)}(n))$$

Where: n—order of system

By accumulated generating operation, the first-order AGO (1-AGO) sequence is ［18］:

(28) $$x^{(1)}(k_1)\underline{\Delta}\sum_{m=1}^{k}x^{(0)}(m)$$

Supposed $x(k)=[x_1(k)\ x_2(k)]^T$, by 1-AGO:

(29) $$x_i^{(1)}(k_1)\underline{\Delta}\sum_{i=1}^{2}\sum_{m=1}^{k}x_i^{(0)}(m)$$

By (9), We can obtain :

(30) $$D(x,k)=B^{-1}[x(k+1)-Ax(k)-Bu(k)]$$

By (31), we will get the original data sequence $D^{(0)}$ as follows:

(31) $$D^{(0)}(k)=(D^{(0)}(1),D^{(0)}(2),D^{(0)}(3)\ \ \cdots D^{(0)}(N))$$

$D^{(1)}(k_1)$ can be got by 1-AGO:

(32) $$D^{(1)}(k_1)=\sum_{m=1}^{k}D^{(0)}(m)$$

where: $N=n+3$. $k_1=1,2,3,\cdots,N-2$. $k=1,2,3,\cdots,N$.

According to the grey system theory, we can establish GM (0, N) model, and estimate roughly on the uncertainty.

(33) $$\hat{V}=(\hat{V}_1\ \ \hat{V}_2\ \ \cdots\ \hat{V}_n\ \ \hat{d}\ )^T$$

Defined:

(34) $$B_{GREY}=\begin{bmatrix} x_1^{(1)}(2) & \dots & x_n^{(1)}(2) & 1 \\ x_1^{(1)}(3) & \cdots & x_n^{(1)}(3) & 2 \\ \vdots & \vdots & \vdots & \vdots \\ x_1^{(1)}(N) & \cdots & x_n^{(1)}(N) & N-2 \end{bmatrix}$$

If B_{GREY} is invertible, by the least square method:

(35) $$\hat{V}^T=(B_{GREY}^T B_{GREY})^{-1}B_{GREY}^T D^{(1)}$$

According to the grey theory, the identification of uncertainty is realized after N steps.

4.2 Compensation for controller

The uncertainty is predicted by the theory cited in 2.1. So sliding mode controller can be compensated, and the rule of compensator is:

(36) $$u_{GREY}(k)=-(\sum_{i=1}^{n}\hat{V}_i x_i+\hat{d})$$

The controlling law after compensation is:

(37) $$\begin{aligned} u &= u_{SMC}+u_{GREY}(k) \\ &= CB^{-1}[C_e(R_{k+1}-R_k)-C_e(A-I)x(k) \\ &\qquad +qTS(k)+\varepsilon T\tan sig(\|x\|)sat(S(k))] \\ &\quad -(\sum_{i=1}^{n}\hat{V}_i x_i+\hat{d}) \end{aligned}$$

where u_{SMC} is the controlling law of SMC

Simulation

In order to verify the effectiveness of the controller, MATLAB is used to make simulation for the electro-hydraulic velocity servo system. According to the hydraulic technology and the hydraulic servo-control, the open loop transfer function of system is obtained by corresponding parameters (be omitted):

(38) $$G(s)=\frac{1100.82}{s^2+54.34s+1747.24}$$

The parameters for simulation are:

$V_1=1.75$, $V_2=-1.75$, $d(k)=0.25$, $C_e=[20\ \ 1]$, $\Delta=0.05$, $q=60$, $\varepsilon=8$,

Simulation for SMC with grey prediction

The simulation results are shown in Figures from 3 to 10. Figure 3 is a general sliding mode velocity tracking; Figure4 shows sliding mode velocity tracking with grey prediction. Figure 5 and 6 are the phase trajectories of variable structure control; in figure 6 grey predictions and compensation are added. Figure 7 and 8 are the input of SMC and GSMC, and finally the sliding mode faces SMC and GSMC are shown in figure9 and10, respectively. The simulation results are analyzed and the conclusion is reached that the system after adding the grey prediction and compensation has a good velocity tracking, and the external disturbance and parameter perturbation can be predicted and compensated effectively. The chattering phenomenon was inhibited obviously; the functions of the controller can be realized.

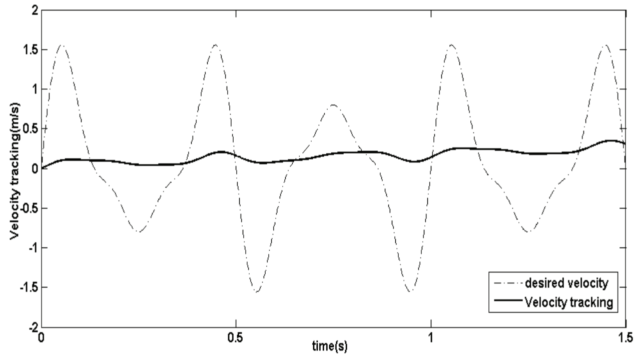

Fig.3. Velocity tracking of general SMC

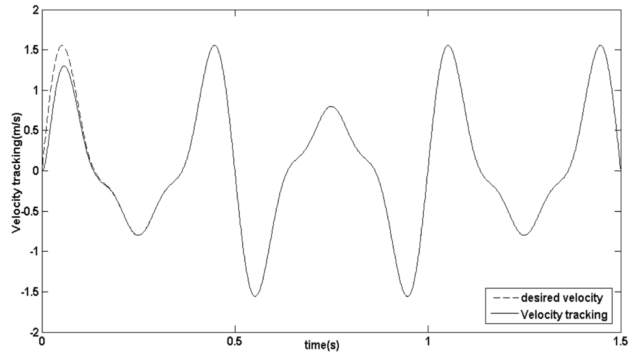

Fig.4. Velocity tracking of grey SMC.

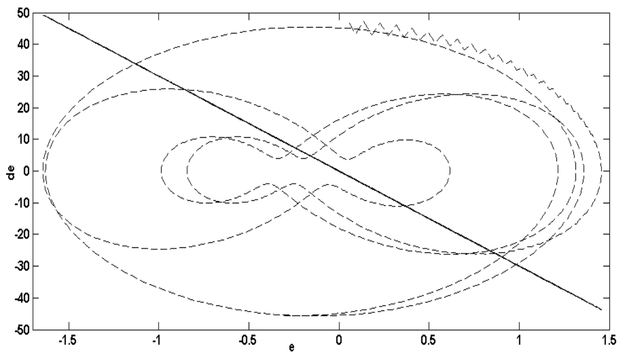

Fig.5. Phase trajectory of general SMC

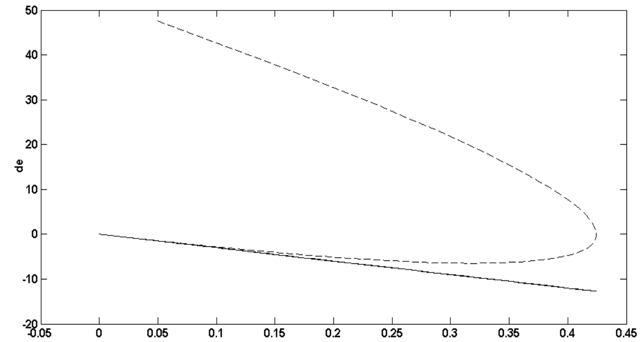

Fig.6. Phase trajectory of grey GSMC

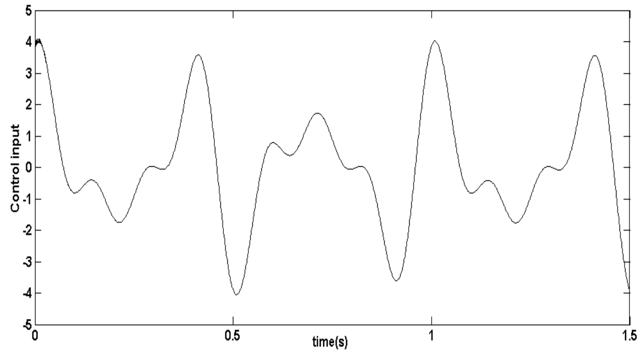

Fig.7. Input of general SMC

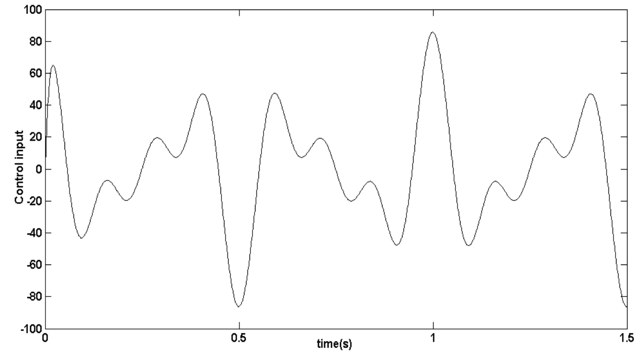

Fig.8. Input of grey SMC.

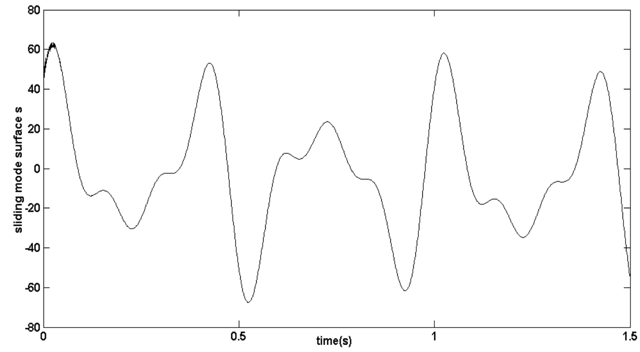

Fig.9. Sliding mode face of general SMC

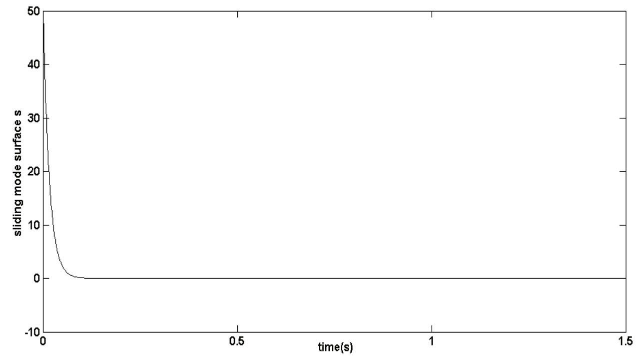

Fig.10. Sliding mode face of grey SMC

Conclusion

In the work reported here, we investigated the feasibility of variable structure methods for the velocity control of Electro- hydraulic servo systems. First, the description of the electro- hydraulic servo systems was introduced and its mathematical model was established. Then, an improved reaching law and the sliding mode control based on it with grey prediction were proposed, and the theoretical basis and stability analyses of the proposed control were described in detail. Moreover, simulation and experimentation were carried out testing the effectiveness of the proposed control systems.

Compared with the general sliding mode control scheme, the main advantages of the adopted nonlinear control design approach applied to the Electro- hydraulic servo systems are as follows:

1).It has a good performance in velocity tracking, and the proposed control strategy is valid for the Electro-Hydraulic Velocity Servo System.

2).The external disturbance and parameter perturbation of the servo system were predicted and compensated effectively.

3).The problem of chattering was inhibited obviously, and the ability of anti-interference and anti-parameters perturbation, stability and control quality of the system were all improved.

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