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Frequency symbolic models of linear parametric circuits

Abstract. *The methods of constructing symbolic models of linear parametric circuits in frequency domain are considered. The models introduce the systems of linear algebraic equations . The circuit is considered to contain only one parametric element and in the model it is replaced with additional source of signal or frequency model of parametric element or controlled source. Frequency symbolic models of single-circuit parametric amplifier are formed according to the given methods.*

Streszczenie. *Podano metodę tworzenia symbolicznego opisu obwodu liniowego zmiennego w czasie. Rozpatrzono obwód zawierający jeden kondensator zmienny w czasie. Kondensator jest zastępowany przez dodatkowe źródło starowane zależne od czasu i częstotliwości. Korzystając z opracowanej metody pokazano przykład modelowania wzmacniacza zawierającego kondensator zmienny w czasie.* ***(Symboliczne modele częstotliwościowe obwodów zmiennych w czasie)***

Keywords: Linear parametric circuits, symbolic analysis, frequency symbolic models.
Słowa kluczowe: Obwód zmienny w czasie, analiza symboliczna, model symboliczny w dziedzinie częstotliwości.

Introduction

Frequency symbolic method of analysis of linear parametric circuits with periodically variable parameters which allows for a known input signal $X(s)$ using the parametric transfer function $W(s,t)$ to determine the output signal $Y(s,t)$ in frequency domain is considered in [1] :

(1) $$Y(s,t) = \hat{W}(s,t) \cdot X(s),$$

where s, t - complex variable and time variable, respectively. The approximation $\hat{W}(s,t)$ of parametric transfer function $W(s,t)$ is determined in the form of truncated Fourier series which contains k harmonic components.

In this paper on the basis of frequency symbolic method the frequency symbolic models of parametric circuit are constructed and they represent the system of linear algebraic equations (SLAE) and they allow to conduct the further circuit analysis only in frequency domain. Three methods of construction of such models are considered in the paper .

Method 1. The method of additional independent source.

According to expression (1) we determine the current that goes through parametric element of the circuit $I_p(s,t)$:

(2) $$I_p(s,t) = \Lambda(s,t) \cdot I(s) ,$$

where $I(s)$ - input current, $\Lambda(s,t)$ - parametric transfer function of signal from input source of current to the current of parametric element. Then it can be seen from the theorem of substitution [2] that voltages and currents in parametric circuit will not change if its branch with parametric element is replaced with branch with source of current $I_p(s,t)$ that is determined by expression (2). After such replacement given parametric circuit becomes a circuit with constant parameters and two sources of current – input source $I(s)$ and source of current $I_p(s,t)$ respectively. Such treatment of parametric circuit gives the possibility to construct its frequency model in the form of SLAE constructed according to the rules of nodal solution in the same way as for the circuit with constant parameters:

(3) $$Y(s_i) \cdot U(s_i,t) = I(s,t) ,$$

where conductivity matrix $Y(s_i)$ contains the parameters of elements of given parametric circuit except for parametric element; the vector of sources of current $I(s,t)$ has non-zero values only in the elements that correspond to nodes of connecting sources $I(s)$ and $I_p(s,t)$; $U(s_i,t)$ - vector of unknown nodal voltage; s - complex variable of input signal and parametric transfer function from vector $I(s,t)$; s_i - complex variable which proves that signal $I_p(s,t)$ contains harmonic components with frequencies $(\omega \pm i\Omega)$ and $i = 0,1,2,\ldots,k$, that is why according to the principle of superposition the SLAE (3) should be done $(2k+1)$ times, substituting different values of complex frequency $j(\omega \pm i\Omega)$ every time; and the results should be added. But frequency symbolic method is symbolic and it is enough to solve SLAE (3) one time with symbolic frequency s_i , according to some rules in the given solution the symbol s_i should be replaced with symbols of complex frequencies $s_{-k}, \ldots , s_{-1}, s, s_{+1}, \ldots , s_{+k}$ and nodal voltages should be formed like the sums of nodal voltages determined for every harmonic of the signals. So, according to model (3) in complex expressions of circuit functions (input resistance of the circuit, transfer of current from input to output) and nodal voltages there are two symbol complex variables s and s_i but only during the last section of calculations before determining time dependences in expressions for nodal voltages, s_i will be replaced with symbolic complex variables from series $s_{-k},\ldots,s_{-1},s,s_{+1},\ldots,s_{+k}$.

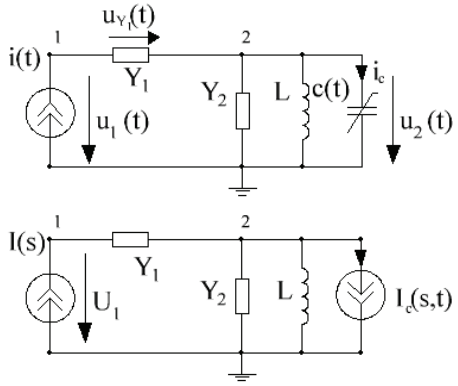
As model (3) of parametric circuit is algebraic, it can be analysed with the help of programs of analysis of linear circuits with constant parameters. It is also clear that the change of parameters of input signal $I(s)$ must be accounted by corresponding change of current $I_p(s,t)$.

Example 1. According to the method of additional independent source it is necessary to construct frequency symbolic model of parametric amplifier (Fig. 1,a) with additional source of current $I_p(s,t)$, and to determine instantaneous value of voltage U_1 in separate points of time and compare them with instantaneous values of voltages determined with the help of MicroCap 7 program in the same points of time .

Equation that connects input current and current of parametric capacity in time domain is the following:

$$i''_C\cdot(L^2c(t)Y_2c'(t)-Lc(t)^2)+i'_C\cdot(L^2Y_2{}^2c'(t)-Y_2c(t)L-$$
$$-Y_2L^2c(t)c''(t)+2\cdot Y_2L^2c'(t)^2-Lc(t)c'(t))+i_C\cdot(2LY_2c'(t)-$$
$$-Y_2{}^2L^2c''(t)-c(t))=i'\cdot(2L^2Y_2c'(t)^2-Lc(t)c'(t)-$$
$$-Y_2L^2c(t)c''(t))+i''\cdot(Y_2L^2c(t)c'(t)-Lc(t)^2) \quad (4)$$

Equation (4) results in equation that is made relative to function *Λ(s,t)* of transferring input current *I(s)* into current of parametric capacity $I_C(s,t)$ in frequency domain:



(a)

(b)

Fig.1. Single-circuit parametric amplifier– (a), equivalent circuit of amplifier made according to the method of additional independent sources, – (b), $c(t)=c_0(1+m\cdot\cos(\Omega\cdot t));c_0=0.1F;m=0.1;\Omega=20\,\mathrm{rad/s}$; $I(s)=A_m\cdot\exp(s\cdot t+\varphi);s=j\cdot\omega;\omega=10\,\mathrm{rad/s};A_m=1A;\varphi=\pi/4$; $Y_1=0.25S;Y_2=0.1S;L=0.1H$.

$$(L^2c(t)Y_2c'(t)-Lc(t)^2)\cdot\Lambda''(s,t)+(s\cdot 2(L^2c(t)Y_2c'(t)-$$
$$-Lc(t)^2)+(L^2Y_2{}^2c'(t)-Y_2c(t)L-Y_2L^2c(t)c''(t)+2Y_2L^2c'(t)^2-$$
$$-Lc(t)c'(t)))\cdot\Lambda'(s,t)+(s^2\cdot(L^2c(t)Y_2c'(t)-Lc(t)^2)+$$
$$+s\cdot(L^2Y_2{}^2c'(t)-Y_2c(t)L-Y_2L^2c(t)c''(t)+2\cdot Y_2L^2c'(t)^2-$$
$$-Lc(t)c'(t))+(2LY_2c'(t)-Y_2{}^2L^2c''(t)-c(t)))\cdot\Lambda(s,t)=$$
$$=s\cdot(2L^2Y_2c'(t)^2-Lc(t)c'(t)-Y_2L^2c(t)c''(t))+$$
$$+s^2\cdot(Y_2L^2c(t)c'(t)-Lc(t)^2), \quad (5)$$

the solution of which with the help of frequency symbolic method when $k=1$ is the following:

$$\Lambda(s,t)=\Lambda_{-1}(s,t)\cdot\exp(-j\Omega t)+\Lambda_0(s)+\Lambda_{+1}(s,t)\cdot\exp(j\Omega t), \quad (6)$$

where $\Lambda_{-1}(s,t)=\lambda_{-1}(s)/d(s),\Lambda_0(s)=\lambda_0(s)/d(s),\Lambda_{+1}(s,t)=\lambda_{+1}(s)/d(s)$,

$$\lambda_{-1}(s)=(0.49\cdot10^{-10}+0.24\cdot10^{-13}i)s^5+(0.50\cdot10^{-8}+0.99\cdot10^{-11}i)s^4+(0.49\cdot10^{-7}+$$
$$+0.99\cdot10^{-7}i)s^3+(0.25\cdot10^{-5}-0.19\cdot10^{-6}i)s^2+(0.78\cdot10^{-5}+0.29\cdot10^{-4}i)s,$$
$$\lambda_0(s)=-0.71\cdot10^{-7}s+0.99\cdot10^{-4}s^4+0.95\cdot10^{-3}s^3+0.19\cdot10^{-5}s^5+0.9\cdot10^{-4}s^2+0.99\cdot10^{-9}s^6,$$
$$\lambda_{+1}(s)=(0.49\cdot10^{-10}-0.24\cdot10^{-13}i)s^5+(0.50\cdot10^{-8}-0.99\cdot10^{-11}i)s^4+(0.49\cdot10^{-7}-$$
$$-0.99\cdot10^{-7}i)s^3+(0.25\cdot10^{-5}+0.19\cdot10^{-6}i)s^2+(0.78\cdot10^{-5}-0.29\cdot10^{-4}i)s,$$
$$d_\lambda(s)=0.18\cdot10^{-3}s+0.10\cdot10^{-5}s^4+0.21\cdot10^{-5}s^3+0.91\cdot10^{-2}+0.19\cdot10^{-3}s^2+$$
$$+0.99\cdot10^{-9}s^6+0.29\cdot10^{-8}s^5.$$

Together with current $I(s)$ expression (6) that results from (1) determine the current of parametric capacity $I_C(s,t)$ in frequency domain as

$$I_C(s,t)=\Lambda(s,t)\cdot I(s)\,. \quad (7)$$

Regarding the determined current of parametric element $I_C(s,t)$ as independent source of current, we construct the equivalent circuit of amplifier (Fig.1.b) from which we construct the searched frequency symbolic model of amplifier with additional independent source with the help of nodal solution in the form of SLAE:

$$\begin{bmatrix} Y_1 & -Y_1 \\ -Y_1 & Y_1+Y_2+1/Ls_i \end{bmatrix}\cdot\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}=\begin{bmatrix} I(s) \\ -\Lambda(s,t)\cdot I(s) \end{bmatrix}. \quad (8)$$

Despite having the parameter of independent source $I_C(s,t)$ that changes with time model (8) describes parametric amplifier (Fig. 1.a) in frequency domain and can be the basis for further analysis, statistical investigation and optimization of given circuit only in this frequency domain without the help of differential equation. So, for example, symbolic nodal voltage U_1 is determined from model (8) in the following form:

$$U_1=\left[((Y_1+Y_2+1/Ls_i)-Y_1\Lambda(s,t))/Y_1(Y_2+1/Ls_i)\right]\cdot I(s)\,. \quad (9)$$

Substituting (9) by expression for $\Lambda(s,t)$ from (6) we receive complex voltage U_1 of amplifier during approximation of function $\Lambda(s,t)$ by one harmonic:

$$U_1=\frac{\left[((Y_1+Y_2+1/Ls_i)-Y_1[\Lambda_{-1}(s,t)+\Lambda_0(s)+\Lambda_{+1}(s,t)])\right]}{Y_1(Y_2+1/Ls_i)}\cdot I(s)\,. \quad (10)$$

Further calculation of expression (10) should be done considering some peculiarities that result from principle of superposition of signals and concern algebraic operations of multiplication and summation of expressions in one of which there is variable s and exponential multiplier that is determined by inferior index and in the second – variable s_i.

The main rules of such operations are the following.

The rule of multiplication:

$$\mathrm{A}_r(s,t)\cdot B(s_i)=\mathrm{A}_r(s,t)\cdot B(s_r)\,,$$
$$r=-k,-(k-1),\ldots,-1,0,+1,\ldots,+(k-1),+k,\ \ s_0=s \quad (11)$$

Rule of summation:

$$[\mathrm{A}_{-k}(s,t)+\ldots+\mathrm{A}_{-1}(s,t)+\mathrm{A}_0(s)+\mathrm{A}_{+1}(s,t)+\ldots$$
$$+\mathrm{A}_{+k}(s,t)]+B(s_i)=[\mathrm{A}_{-k}(s,t)+\ldots+\mathrm{A}_{-1}(s,t)+$$
$$+\mathrm{A}_0(s)+B(s)+\mathrm{A}_{+1}(s,t)+\ldots+\mathrm{A}_{+k}(s,t)] \quad (12)$$

So, considering (11)-(12), we receive the following in (10):

$$U_1=\left[\frac{[(Y_1+Y_2+1/Ls_0)-Y_1\Lambda_0(s,t)]}{Y_1(Y_2+1/Ls_0)}-\frac{Y_1\Lambda_{-1}(s,t)}{Y_1(Y_2+1/Ls_{-1})}-\right.$$
$$\left.-\frac{Y_1\Lambda_{+1}(s,t)}{Y_1(Y_2+1/Ls_{+1})}\right]\cdot I(s). \quad (13)$$

Table 1.

Time, s	Output voltage U1, V		
	Micro Cap 7	Method 1	
		$k=1$	$k=3$
400	-0.349	0.486	-0.349
401	-5.859	-6.064	-5.869
402	10.112	10.936	10.112
403	-10.350	-9.851	-10.350

The results of calculation of expression (13) and similar to it but with more harmonic components k in parametric function $\Lambda(s,t)$ for different meanings of time t are presented in Table 1.

Method 2. The method of frequency symbolic model of parametric element.

We determine the current that goes through parametric element and voltage on it according to expression (1).The ratio of these values

$$\frac{I_p(s,t)}{U_p(s,t)}=\frac{[\Lambda(s,t)\cdot I(s)]}{[Z(s,t)\cdot I(s)]}=\frac{\Lambda(s,t)}{Z(s,t)}=S(s,t) \quad (14)$$

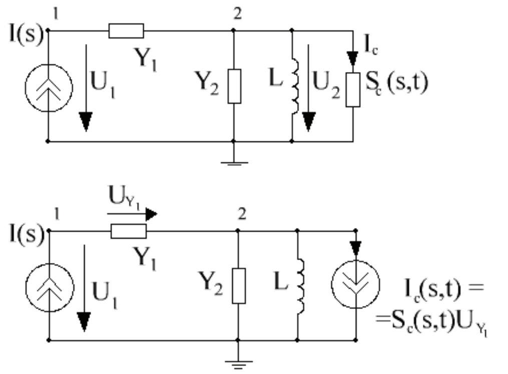
due to circuit linearity does not depend on its voltages and currents and that is why it determines some parameter (conductivity) $S(s,t)$ that will be called frequency symbolic model in particular conductivity of parametric capacity.

Unlike linear circuits with constant parameters the frequency model of parametric element that is in the circuit can not be determined separately from the circuit itself. Moreover the frequency model of parametric element does nor exist separately from the circuit. Here we can see the peculiarity of frequency analysis of parametric circuits and infeasibility of applying Laplace transformation to them.

The parameter of frequency symbolic conductivity (14) like constant parameters of other elements of the circuit can be inscribed into Y-matrix with the help of the nodal solution and as a result of it in the form of SLAE frequency symbolic model of parametric circuit as a whole will be constructed:

(15) $$Y(s,s_i)\cdot U(s_i,t)=I(s)$$

where variables s,s_i have the same content as for model (3).



(a)
(b)

Fig.2. Equivalent circuit of parametric amplifier from Fig.1,a made according to the method of frequency symbolic model of parametric element – (a); controlled source – (b).

Example 2. Using expression (14) we construct the equivalent circuit of amplifier presented at fig.2.a, which in turn determines the frequency model of amplifier:

(16) $$\begin{bmatrix} Y_1 & -Y_1 \\ -Y_1 & Y_1+Y_2+S_C(s,t)+1/Ls_i \end{bmatrix}\cdot\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}=\begin{bmatrix} I(s) \\ 0 \end{bmatrix}.$$

The received frequency symbolic model of the circuit (16) can be used to determine necessary secondary parameters and nodal voltages with the help of which other voltages and currents of the circuit can be determined.

Method 3. The method of controlled source.

The method of controlled source is the elaboration of the method of frequency symbolic model of parametric element. The last one has such a peculiarity that frequency symbolic model of the element is fractional expression. Fractional symbolic expressions can lead to inconvenience of doing algebraic operations with them. Therefore the method of controlled source eliminates such inconvenience by simplification or full elimination of denominators in expressions that are analogous to expression (14). This happens in the following manner. The parameter of the parametric element is not determined by its relation of current to voltage on it, but by the relation of its current to any other voltage of the circuit. Such a relation determines the controlled source where controlling branch – chosen voltage, the controlled one – current of the parametric element. According to traditional rules the parameter of conductivity $S(s,t)$ of such controlled source

(17) $$S(s,t)=I_p(s,t)/U_i(s,t)=\Lambda(s,t)/Z_i(s,t)$$

together with conductivity of the other elements of the circuits inscribed like for the circuits with constant parameters into matrix of circuit conductivity as a whole and this forms the algebraic model of all parametric circuit in the form of (15).

Example 3. Using expression (17) as $S_C(s,t)= =I_p(s,t)/U_{Y_1}(s,t)=\Lambda(s,t)\cdot Y_1$ we construct the equivalent circuit of amplifier presented at fig.2.b, which in turn determines the frequency model of amplifier:

(18) $$\begin{bmatrix} Y_1 & -Y_1 \\ -Y_1+S_C(s,t) & Y_2+Y_2+1/Ls_i-S_C(s,t) \end{bmatrix}\cdot\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}=\begin{bmatrix} I(s) \\ 0 \end{bmatrix}.$$

Obtained frequency symbolic model (18), as frequency symbolic model (16), contains all the information about the parametric amplifier in frequency domain.

Conclusions

1. The analyzed frequency models fully describe parametric circuit in frequency domain and as shown by experiments are adequate.
2. The accuracy of results received by frequency symbolic models of the circuit increases when the number of harmonic components in approximations of parametric transfer functions increases.
3. Frequency models of parametric elements are determined by the circuit in which these parameters are present. It is most probably that frequency models of parametric elements do not exist beyond the circuit.
4. The parameters of elements that are going to be changed in the process of circuit investigation must be left in symbol both in frequency model and in parametric transfer functions that are present in these frequency models. In this case the necessary change of such parameters is quite quick because of substitution of new values in frequency model of the circuit and parametric functions without any additional calculations.
5. The developed frequency models of the circuits contain two complex variables as they describe the circuit on a frequency of input signal and frequencies that appear in the circuit of harmonic components of signals.
6. The developed frequency models allow to carry out the analysis, statistical investigation, optimization and designing of parametric circuits in frequency domain by analogy with circuits with constant parameters.
7. The developed frequency models are the systems of linear algebraic equations. That is why during their analysis the software for symbolic analysis of linear circuits with constant parameters can be widely used. We consider this fact to be the most practical result of the given paper.

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