Grzegorz ŚWIRNIAK

Chair of Electronic and Photonic Metrology, Wrocław University of Technology

On the use of Airy fringes for indirect measurement of glass fibre diameter

***Abstract**. The paper is devoted to the concept of method for non-invasive diameter characterization of a homogeneous glass fibre using the scattered radiation of low temporal coherence recorded at a high angle. A few theoretical considerations and numerical results are covered to discuss properties of the scattered radiation. Finally, a method for data inversion employing Airy rainbow theory is discussed.*

Streszczenie.** W pracy przedstawiono koncepcję metody aktywnego pomiaru średnicy jednorodnego włókna szklanego, wykorzystującą niespójne promieniowanie rozproszone rejestrowane pod dużym kątem. W drodze badań modelowych i symulacyjnych przedyskutowano właściwości pola rozproszonego i zaproponowano rozwiązanie zadania odwrotnego pozwalające określić średnicę włókna na podstawie kątowego położenia wybranych prążków tęczy Airy'ego. **(Wykorzystanie prążków Airy'ego w pośrednim pomiarze średnicy włókna szklanego).

Keywords: Glass fibre, diameter, rainbow, Airy rainbow theory, low-coherent radiation, indirect measurement.
Słowa kluczowe: Włókno szklane, średnica, tęcza, teoria tęczy Airy'ego, promieniowanie niespójne, pomiar pośredni.

Introduction

The paradigm of mathematical and empirical understanding may begin from the observation and constantly emerging cognitive problems engendered on its basis seem to confirm the words of Hamlet: "There are more things in heaven and earth, Horatio Than are dreamt of in your philosophy." The observations of a rainbow and its mathematical interpretations made by Descartes, Young, and many other researchers, verified by means of instrumental analysis, are the foundations of modern scientific methods where light is used as a tool for cognition.

A rainbow is a splendid demonstration of light scattering by a weakly absorbing particle. Within the framework of causal inference, it is possible to obtain some knowledge of the physical characteristics of a particle (cause) on the basis of recorded and processed scattered radiation (effect). In such a non-invasive experiment, *in situ* quantitative information in real-time may be potentially acquired.

Rainbows caused in nature by sunlight may be reproduced in a laboratory with the use of laser radiation. A numerical example of the intensity pattern in the vicinity of the so-called primary rainbow derived for a glass fibre is shown in Fig. 1a (in nature secondary rainbow may be observed in favourable conditions as well). Coherent nature of light contributes to the multiplicity of physical phenomena occurring in the process of scattering and, in consequence, to the complexity of the observed pattern. The origin of the main rainbow maximum and the supernumerary bows explains diffraction theory announced in the nineteenth century by Sir George Biddell Airy. In his honour, these bows are referred to as Airy fringes or the Airy rainbow pattern [1, 2]. The Airy's theory does not justify the strong nonlinearities, i.e. ripple structure superimposed on the Airy fringes. These ripples result from an influence of additional scattering components not accounted for by Airy, including radiation reflected from the surface of the particle, radiation scattered many times in its structure, and their mutual interaction taking the form of resonant scattering as in the case of an optical microcavity [1]. An overall description of the light scattered at a high angle is possible within the framework of Lorenz-Mie theory (LMT), for instance [3, 4].

Causal inference on the basis of light scattered by a single, weakly absorbing particle encounters limitations related to the complexity of physical as well as mathematical interpretation of the field. The essence of this issue illustrates the far-zone intensity pattern in the vicinity of a primary rainbow from a glass fibre, drawn as a function of fibre diameter, Fig. 1b. Due to the ripple structure and resonance microforms (MDRs, see zoom) sensitive to small changes in diameter, it is difficult to determine exact locations of the Airy fringes.

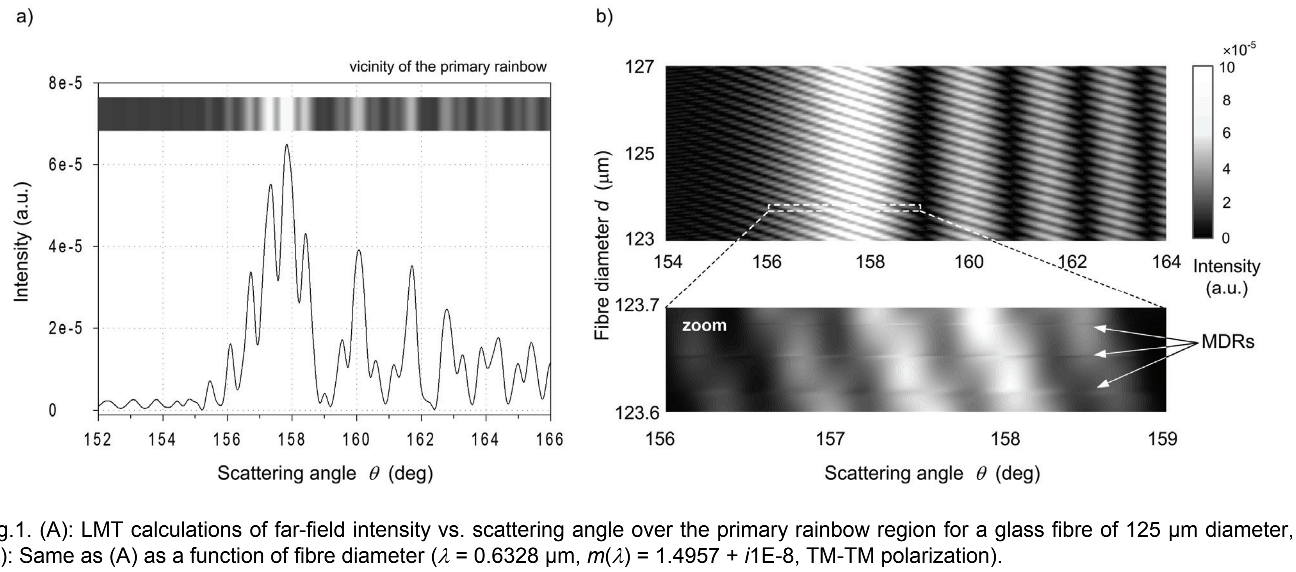

Fig.1. (A): LMT calculations of far-field intensity vs. scattering angle over the primary rainbow region for a glass fibre of 125 μm diameter, (B): Same as (A) as a function of fibre diameter (λ = 0.6328 μm, $m(\lambda)$ = 1.4957 + i1E-8, TM-TM polarization).

These ripples may even interchange Airy's minima and maxima [5]. Therefore, formulation of an unambiguous cause-effect link between the measurement data and physical characteristics of the particle (such as diameter) is problematic. Data inversion is further complicated by the fact, that complex models of light scattering, e.g. LMT, identifiable in theory, usually turn out to be numerically ill-conditioned in practice (as in the case of models based on the Fredholm integral equation [6, 7]), making impossible to interpret the results correctly.

It is worth mentioning, that the resonance effects are typical to the problem of scattering by a single particle and do not usually occur in experiments on multi-phase/ polydisperse systems composed of irregularly shaped, weakly absorbing particles [8-15]. In short, each particle in such a system contributes to the scattered field differently, so that the ripple structure and resonances are smoothed.

Within the framework of Lorenz-Mie theory, assuming plane-wave illumination, it is not possible to spatially diversify the components contributing to the scattered and recorded field. In particular, such separation cannot be achieved with respect to the Airy rainbow pattern. One way to address this issue is by mathematical analysis of the recorded pattern. In principle, spectral investigation is usually performed on the basis of a meaningful interpretation of the rainbow spectrum provided by van Beeck and Riethmuller [16]. A typical procedure applied by Roth *et al*. [17] involves low-pass filtration carried out to extract the Airy fringes. Another remarkable method described by Han *et al.* relies on spectral representation of the ripple structure [18]. Important to note is that the results of spectral analysis are useful only when the spectral components are easily distinguishable, which is met when the size of a particle is significantly larger than the wavelength of the scattered radiation. In practice, characterization of particles with a diameter of several hundred microns is feasible.

There are also solutions that allow to form the Airy fringes by a suitable arrangement of the experiment. The method described by Mèés *et al*. [19] relies on temporal separation of the scattered components what is achieved by illumination with an ultra-short pulse of laser light. Another way is to illuminate a particle with a shaped beam to excite only the scattered components accounted for by Airy rainbow theory [20]. Both of these methods, however, are impractical in most applications.

The idea promoted in this paper is to influence some spectral properties of the radiation incident on a transparent particle in order to dampen the effect of the ripples and resonances on the Airy fringes, so that simplified physical and mathematical interpretations of scattering become applicable. This idea is intended to be a basis for a method for non-invasive diameter characterization of a glass fibre.

Scattering of low-coherent radiation in the vicinity of a primary rainbow

To put the topics discussed here into a general context of possible applications, the subject of low-coherent beam scattering will be referred to the universal configuration shown in Fig. 2. The scattering problem is considered in the circular cylindrical coordinate system (r, θ, z). The incident plane wave propagates in the negative x-direction and illuminates the fibre normally. Only the transverse magnetic (TM) case of polarization is considered, i.e. the electric field is polarized parallel to the cylinder axis. The emission spectrum is approximated by a Gaussian function which is typical for single-colour light emitting diodes:

(1) $$I(\lambda) = \exp\left[4\log(0.5)\left((\lambda-\lambda_0)/fwhm\right)^2\right],$$

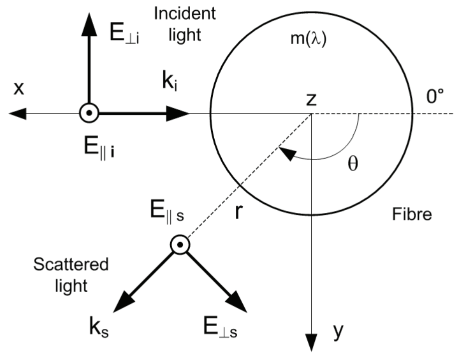

Fig.2. Geometry of a glass fibre of diameter *d* normally illuminated by a plane wave propagating in the -*x* direction.

where I_0 is the peak intensity (normalized to 1), λ_0 – the peak wavelength (0.6328 μm assumed), and *fwhm* – the spectral linewidth. The radiation is scattered by an infinitely-long, axisymmetric, homogeneous glass fibre of diameter *d* and complex, wavelength-dependent refractive index $m(\lambda) = n(\lambda) + i\kappa$. For most transparent media, in the visible spectral region where normal dispersion occurs, the real part of refractive index (n) increases noticeably, while absorption is negligibly weak [21]. In this study, the Sellemier dispersion formula is applied for the specification of $n(\lambda)$ [21]:

(2) $$n^2(\lambda) = 1 + \sum_i \frac{A_i\lambda^2}{\lambda^2 - \lambda_i^2},$$

where A_i, λ_i^2 are empirical constants specific for NPK52A glass and determined by a glass manufacturer (Schott) [22]. The extinction coefficient (κ) is assumed to be 1E-8. The scattered intensity is detected in the far field (kr >> 1). Since for normal incidence the polarization of the scattered wave is maintained [3], only the scattered component parallel to the cylinder axis ($E_{\|s}$) will be considered (TM polarization).

Rigorous description of quasi-monochromatic, low temporal coherence radiation scattering is possible on the basis of statistical optics. For the purposes of this work, however, a heuristic approach is more practical. Within this approach, the emission bandwidth of the source is divided into N discrete wavelengths. As the fibre is assumed to be linear, superposition applies and the total solution in terms of the scattered intensity is the incoherent sum of independently scattered N waves. Sufficiently large number of wavelengths applied (N = 2201), spaced evenly every 0.0001 nm relative to λ_0, ensures accuracy of the results.

Lorenz-Mie theory provides the exact separation-of-variables solution of Maxwell's equations to the problem of monochromatic plane wave scattering [3, 23]. The case of Gaussian beam scattering has been also examined recently [24, 25]. Following the solution proposed by Bohren and Huffman [3], the scattered ($I_{\|s}$) and incident ($I_{\|i}$) field intensities are related through:

(3) $$I_{\|s}(\theta) = \frac{2}{\pi kr}\left|b_0 + 2\sum_{n=1}^{\infty} b_n \cos(n\theta)\right|^2 I_{\|i}$$

where $k = 2\pi/\lambda$ is the propagation constant and λ is the wavelength of the incident wave. The scattered field expansion coefficient, b_n, is obtained from matching the boundary conditions at the surface of the fibre and depends on its shape, size, and refractive index [3].

In the first numerical experiment, the influence of the spectral linewidth on the field scattered in the vicinity of a primary rainbow has been assessed. According to Fig. 3, together with the increase in *fwhm* (0.1-35 nm), a gradual reduction of the ripple structure superimposed on the Airy fringes is observed, despite the fact that there is no physical separation of the scattered components responsible for the Airy fringes formation from others. Reduction of the ripples is related to the incoherent (i.e. phase independent) superposition of monochromatic waves comprising the scattered field. Such a superposition corresponds to averaging over the scattering angle or, equivalently, to low-pass filtration of the intensity pattern excited for the incident wave of length λ_0.

Macroscopic effects of scattering in the form of Airy fringes, give rise to the question whether the part of cognition process dedicated to the physical interpretation of low-coherent radiation scattering may be restricted to these fundamental physical phenomena that determine the formation of Airy fringes? Some insight into the physics of Airy fringes may be achieved by means of Debye series decomposition [26], applied to the scattered wave of length λ_0. The Debye solution provides a possibility to interpret the scattered field in terms of scattering orders, as in geometrical optics. Selected results of the Debye series calculations shown in Fig.4 indicate, that formation of the Airy rainbow is due to a vector sum of geometrical components of p = 2 order, i.e. rays suffering one internal reflection (see the ray trajectory sketch in Fig.4). What makes the issue interesting is that the Debye solution resembles the intensity pattern achieved in the experiment with low-coherent radiation scattering of 20 nm *fwhm*, see Fig.4. In fact, the angular minima positions are the same with resolution of 0.01°. Consequently, when the macroscopic effects of scattering are assessed, it is legitimate to claim that the Airy rainbow due to low-coherent illumination may be regarded, under certain conditions outlined above, as if it was caused by p = 2 order rays interference, corresponding to a monochromatic wave scattering of length λ_0. This conclusion is of importance when an approximate solution to the problem of low-coherent radiation scattering is considered.

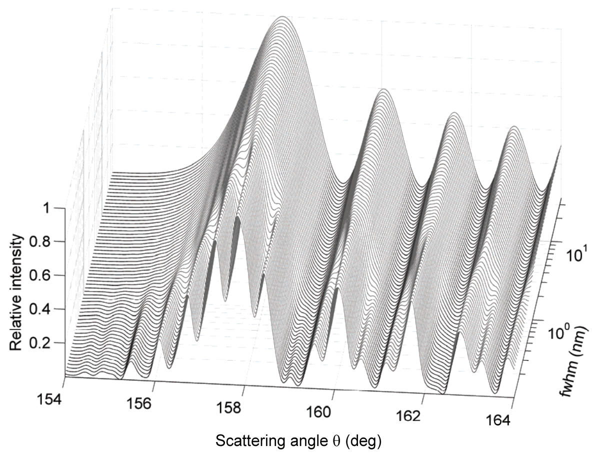

Fig.3. Influence of the spectral linewidth *fwhm* on the scattering intensity near the primary rainbow for a glass fibre (d = 125 μm, λ_0 = 0.6328 μm, $m(\lambda_0)$ = 1.4957 + i1E-8, TM-TM polarization).

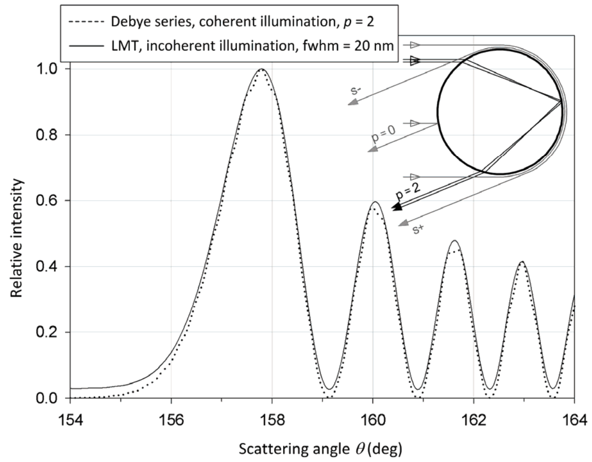

Fig.4. Scattered intensity vs. scattering angle for a glass fibre in the vicinity of the primary rainbow according to (···) Debye series calculation for monochromatic incident beam due to contribution of p = 2 order rays, (–) LMT calculations for incoherent illumination of 20 nm *fwhm*, (d = 125 μm, λ_0 = 0.6328 μm, $m(\lambda_0)$ = 1.4957 + i1E-8, TM-TM polarization).

The Airy approximation for fibre diameter characterization

The concept of indirect measurement adopted in this paper is to convert measurement data into unambiguous information about the diameter of a homogeneous glass fibre. To address this issue, a simple causal model (i.e. forward model) will be introduced on the basis of Airy rainbow theory. This model will be then transformed into an inverse relation for the characterization of fibre diameter.

Airy, using the concepts of geometrical optics, approximated the scattered wavefront shape in the vicinity of the rainbow with a cubic function, seeking then a solution to the scalar wave equation with appropriate boundary conditions using the theory of diffraction [27]. An analytic expression for the rainbow intensity, in terms of so-called Airy integral, is as follows [4, 28]:

(4) $$I_{\parallel s}(\theta) \propto \mathrm{Ai}^2\left[-x^{2/3}\left(\theta-\theta^D\right)/h^{1/3}\right],$$

where: Ai is the Airy function [29], $x = \pi d/\lambda$ is the fibre size parameter, θ^D is the scattering angle of the Descartes ray, i.e. the ray of light which after one internal reflection leaves the fibre at the smallest possible angle of deviation from the direction of incidence:

(5) $$\theta^D = (p-1)\pi + 2\theta_i - 2p\theta_t,$$

with:

(6) $$\cos(\theta_i) = \left[\left(n^2-1\right)/\left(p^2-1\right)\right]^{1/2}, \quad \sin(\theta_t) = n^{-1}\sin(\theta_i),$$

where p = 2 for the primary rainbow considered, θ_i, θ_t are the angles relative to the normal to the cylinder surface at which the Descartes ray enters and leaves the fibre respectively. The factor h in (4) governs the shape of the far-zone scattered wavefront in the vicinity of the rainbow:

(7) $$h = \left[\left(p^2-1\right)^2\left(p^2-n^2\right)^{1/2}\right]/\left[p^2\left(n^2-1\right)^{3/2}\right].$$

According to Wang and Hulst [28], the angular spacing between the Airy fringes is dependent to a significant extent on the fibre diameter, and much smaller on its refractive index. Moreover, the cubic approximation of the wavefront is valid for small deviations from θ^D, i.e. for a few first fringes [28]. These conclusions lead to propose a model for data inversion, for which the angular spacing between the first two successive minima/maxima will be an input.

Let θ_i and θ_j denote the angular positions of two arbitrary fringes and z_i, z_j – the Airy function arguments in (4) corresponding to θ_i, θ_j respectively. The difference of z_i and z_j is given by:

(8) $$z_i - z_j = x^{2/3}\left(\theta_j - \theta_i\right)\big/h^{1/3}.$$

Under the above formula, the fibre diameter is simply related to the fringe spacing by:

(9) $$d = \frac{\lambda}{\pi}\left[\left(z_i - z_j\right)\big/\left(\theta_j - \theta_i\right)\right]^{3/2} h^{1/2}$$

Angular locations of the fringe minima and maxima must occur at critical points of the Airy function $Ai^2(z)$ in (4). The function's arguments corresponding to these points may be found in Abramowitz and Stegun [29], for instance.

To assess the validity of the inverse formula (9), calculations of the fibre diameter limiting error vs. spectral linewidth has been performed and the results shown in Fig. 5. The calculations refer to three different measurement ranges, i.e. 70-80, 120-130, and 170-180 μm. Furthermore, the angular spacing between the first two successive minima/maxima has been provided as an input to (9). Synthetic measurement data, i.e. θ_i, θ_j, has been acquired from the scattering intensity patterns calculated with the use of Lorenz-Mie formalism for scattering of low-coherent radiation in the vicinity of a primary rainbow, as introduced above.

The accuracy of diameter measurement in the present case is determined by two factors - the approximate nature of the forward model and, to a much lesser extent, by the limited resolution of fringe position readings (0.001°). As for the first factor, the Airy approximation tends to be more adequate, the larger diameter in relation to the wavelength is. It is of importance to note, that the choice of measurement data is essential for the error plot. Position of the primary bow, as the closest to the angle of Descartes, is addressed best by the Airy's theory. In effect, the measurements associated with the primary bow are most accurate. On the other hand, it is apparent from Fig. 5 that the bright fringes are more sensitive to the ripple structure and resonances than dark ones.

It is crucial to realize, that the mathematical approach of the rainbow in terms of the Airy's theory, applied to the discussed case of low-coherent radiation scattering, does not require knowledge of the refractive index changes due to material dispersion, i.e. $n(\lambda)$. Only the refractive index corresponding to the peak wavelength of the incident wave, $n(\lambda_0)$, is necessary.

Conclusions

The theoretical considerations and numerical studies included in this study suggest, that the method of low-coherent, high-angle scattering may be used in the assessment of glass fibre diameter. At the core of the method is the idea to influence some spectral features of the incident radiation towards intuitive physical representations of the scattered field described at simple level of mathematical sophistication. With this methodology, it became possible to adapt the classical model of the primary rainbow to characterize the field properties scattered on a glass fibre of a small diameter relative to the wavelength.

From the perspective of empirical research, it becomes necessary to verify the classical solutions of the measuring systems designed for laser light scattering and acquisition. A key issue is forming a high-intensity beam of low temporal coherence radiation in order to obtain the scattering pattern of a good quality. The activities in this regard will be aimed towards the use of light emitting diodes in radiation emitters of prototype structures. Achievements to date address the problem of heat dissipation and temperature stabilization of a light emitting diode [30, 31]. Besides, acquisition and processing of a weak-intensity rainbow is a challenge.

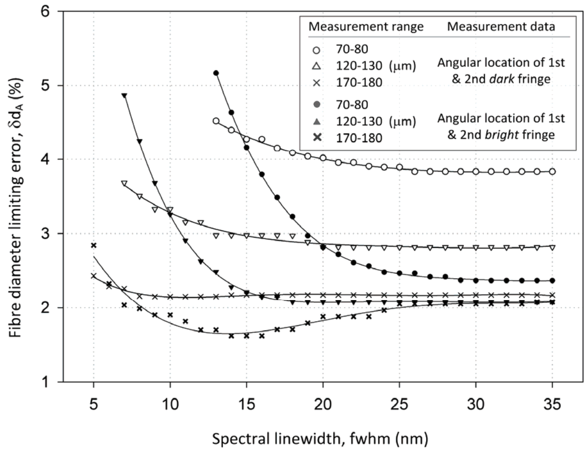

Fig.5. Fibre diameter limiting error vs. spectral linewidth for different measurement ranges and measurement data (λ_0 = 0.6328 μm, $m(\lambda_0)$ = 1.4957 + i1E-8, TM-TM polarization).

Grateful acknowledgement is made to the Ministry of Science and Higher Education for providing financial support for this work under grant No. N N505 557539.

REFERENCES

[1] Adam, J. A., "The mathematical physics of rainbows and glories," *Physics Reports,* vol. 356 (2002), pp. 229-365.
[2] Lee, R. L. and Fraser, A. B., *The Rainbow Bridge: Rainbows in Art, Myth and Science*. Pennsylvania State University Press (2001).
[3] Bohren, C. F. and Huffman, D. R., *Absorption and Scattering of Light by Small Particles*. New York: John Wiley & Sons (1983).
[4] van de Hulst, H. C., *Light scattering by small particles*. New York: Dover Publications (1981).
[5] Nussenzveig, H. M., "Complex angular momentum of the rainbow and the glory," *Journal of optical Society of America,* vol. 69 (1979), pp. 1068-1079.
[6] Mroczka, J. and Szczuczyński, D., "Inverse problems formulated in terms of first-kind Fredholm integral equations in indirect measurements," *Metrology and Measurement Systems,* vol. 16 (2009), pp. 333-357.
[7] Mroczka, J. and Szczuczyński, D., "Improved regularized solution of the inverse problem in turbidimetric measurements," *Applied Optics,* vol. 49 (2010), pp. 4591-4603.
[8] van Beeck, J.*, et al.*, "Global Rainbow Thermometry for Mean Temperature and Size Measurement of Spray Droplets," *Particle & Particle Systems Characterization,* vol. 18 (2001), pp. 196-204.
[9] Vertrano, M. R.*, et al.*, "Global rainbow thermometry: improvements in the data inversion algorithm and validation technique in liquid-liquid suspension," *Applied Optics,* vol. 43 (2004), pp. 3600-3607.
[10] Onofri, F.*, et al.*, "Critical angle refractometry and sizing of bubble clouds," *Optics Letters,* vol. 32 (2007), pp. 2070-2072.
[11] Onofri, F.*, et al.*, "Optical characterization of bubbly flows with a near-critical-angle scattering technique " *Experiments in Fluids,* vol. 47 (2009), pp. 721-732.
[12] Onofri, F.*, et al.*, "Near-critical-angle scattering for the characterization of clouds of bubbles: particular effects," *Applied Optics,* vol. 50 (2011), pp. 5759-5769.
[13] Girasole, T.*, et al.*, "Cylindrical fibre orientation analysis by light scattering: Part 1: Numerical aspects," *Particle & Particle Systems Characterization,* vol. 14 (1997), pp. 163-174.

[14] Girasole, T., *et al.*, "Cylindrical fibre orientation analysis by light scattering: Part 2: Experimental aspects," *Particle & Particle Systems Characterization,* vol. 14 (1997), pp. 211-218.
[15] Girasole, T., *et al.*, "Fiber orientation and concentration analysis by light scattering: experimental setup and diagnosis," *Review of Scientific Instruments,* vol. 68 (1997), pp. 2805-2811.
[16] van Beeck, J. P. A. J. and Riethmuller, M. L., "Rainbow phenomena applied to the measurement of droplet size and velocity and to the detection of nonsphericity," *Applied Optics,* vol. 35 (1996), pp. 2259-2266.
[17] Roth, N., *et al.*, "Refractive-index measurements for the correction of particle sizing methods," *Applied Optics,* vol. 30 (1991), pp. 4960-4965.
[18] Han, X. e., *et al.*, "Characterization of initial disturbances in a liquid jet by rainbow sizing," *Applied Optics,* vol. 37 (1998), pp. 8498-8503.
[19] Mèés, L., *et al.*, "Scattering of laser pulses (plane wave and focused gaussian beam) by spheres," *Applied Optics,* vol. 40 (2001), pp. 2546-2550.
[20] Mèés, L., *et al.*, "Scattering of a Gaussian beam by an infinite cylinder with arbitrary location and arbitrary orientation: numerical results," *Applied Optics,* vol. 38 (1999), pp. 1867-1876.
[21] Bach, H. and Neuroth, N., Eds., *The Properties of Optical Glass*. Berlin: Springer-Verlag (1998).
[22] Schott, "Optical Glass. Data Sheets," in *Optics for devices*, S. AG, Ed. (2007).
[23] Barber, P. W. and Hill, S. C., *Light Scattering by Particles: Computational Methods.* Advanced Series in Applied Physics, vol. 2. Singapore: World Scientific Publishing (1990).
[24] Ren, K. F., *et al.*, "Scattering of a Gaussian beam by an infinite cylinder in the framework of Generalized Lorenz-Mie Theory: formulation and numerical results," *Journal of Optical Society of America A,* vol. 14 (1997), pp. 3014-3025.
[25] Mroczka, J. and Wysoczański, D., "Plane-wave and Gaussian-beam scattering on an infinite cylinder," *Optical Engineering,* vol. 39 (2000), pp. 763-770.
[26] Lock, J. A. and Adler, C. L., "Debye-series analysis of the first-order rainbow produced in scattering of a diagonally incident plane wave by a circular cylinder," *Journal of Optical Society of America A,* vol. 14 (1997), pp. 1316-1328.
[27] Airy, G. B., "On the intensity of light in the neighbourhood of a caustic," in *Transactions of the Cambridge Philosophical Society*. vol. 6, University Press (1838).
[28] Wang, R. T. and van de Hulst, H. C., "Rainbows: Mie computations and the Airy approximation," *Applied Optics,* vol. 30 (1991), pp. 106-117.
[29] Abramowitz, M. and Stegun, I. A., *Handbook of Mathematical Functions* 9th ed. Washington DC: National Bureau of Standards (1970).
[30] Mroczka, J., "Temperature stabilisation of light-emitting diode radiation," *Journal of Physics E: Scientific Instruments,* vol. 21 (1988), pp. 306-309.
[31] Mroczka, J. and Parol, M., "Methods of temperature stabilization of light-emitting diode radiation," *Review of Scientific Instruments,* vol. 65 (1994), pp. 803-806.

Author: Grzegorz Świrniak, Ph.D., Chair of Electronic and Photonic Metrology, Wrocław University of Technology, ul. Prusa 53/55, 50-317 Wrocław, Poland.
e-mail: Grzegorz.Swirniak@pwr.wroc.pl.