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An Optimal Heat Line Simulation Method to Calculate the Steady-Stage Temperature and Ampacity of Buried Cables

***Abstract**. A numerical study based on an optimal heat line simulation method is proposed to calculate the steady-state temperature distribution of underground power cables surrounded by sand backfill and soil. Virtual heat lines are used to replace the heat in the cable conductors in this method. The heat and location of these virtual heat lines are determined by solving Laplace's and Poisson's equations with boundary conditions satisfied based on the genetic algorithm. An experiment was carried out to verify the validity of the proposed method. Through comparison with the experimental results, the proposed model is proved to be effective to calculate the temperature distribution of buried power cables. Furthermore, the ampacity of buried power cables is calculated by iteration method.*

*Streszczenie. W artykule zaprezentowano metodę symulacji nagrzewania podziemnego kabla. Wykonano obliczenia wykorzystując równania Laplace i Poissona przy warunkach granicznych bazujących na algorytmie genetycznym. Określono też przeciążalność prądową kabla. (**Symulacja nagrzewania kabla podziemnego w celu określenia rozkładu temperatury i przeciążalności prądowej**)*

Keywords: Optimal heat line simulation method, genetic algorithm, buried power cables, steady-state temperature, ampacity.
Słowa kluczowe: kabel podziemny, nagrzewania, przeciażalność prądowa**.**

Introduction

The thermal analysis of buried power cables is crucial to determine their ampapcity. An accurate evaluation of the heat dissipation through the cables and the surroundings, sand and soil, enables electricity utilities to overcome the conservativeness typically employed in buried cable designation, and thus to achieve a better cable utilization.

The traditional method adopted by the most electricity utilities is IEC60287 which is based on the Neher and McGrath's theory that the surroundings were homogeneous and had uniform thermal conductivities. While, buried power cables are always surrounded by 200mm height of sand backfill. This means that the power cables are not placed in a homogeneous materials. For complexity systems, it is difficult to solve the temperature field by IEC60287[1]-[4].

In recent years, numerical calculation methods, such as the finite element method (FEM), have been developed to calculate the temperature distribution and the ampacity of buried power cables. Numerical calculation methods not only allow better representation of the interaction of the heat between different power cables and outer heat sources, but also permit more accurate modeling of the region's boundaries. However, numerical calculation methods often result in a very large number of algebraic equations, and their solution is a problem in itself[5]-[10]. Therefore, it is useful to find a more simpler method without decreasing the accuracy.

The charge simulation method has been widely used to calculate the electric field and its effective has been proved in solving open field. It has less computation effort with sufficient precision [11] [12]. In this paper, we describe a mathematical model by using the optimal heat line simulation method to calculate the temperature distribution of buried power cables. This method has less algebraic equations and computation effort than FEM.

The model of buried power cables

One of the most used configurations of buried power cables consists of three individual cables at the same level and there is a typical separation between the centers of buried power cables as shown in Fig.1. These power cables are buried in sand and soil.

Usually, there are two type of losses generated in a cable: current-dependent losses and voltage-dependent losses. The current-dependent losses, including the conductor losses Q_1, the sheath losses Q_2 and the armor losses Q_3, refer to the heat generated in the metallic components. The voltage-dependent losses refer to dielectric losses. Dielectric losses can be neglected for distribution voltages, and the limiting voltage levels for each insulation type can be found in IEC 60287.

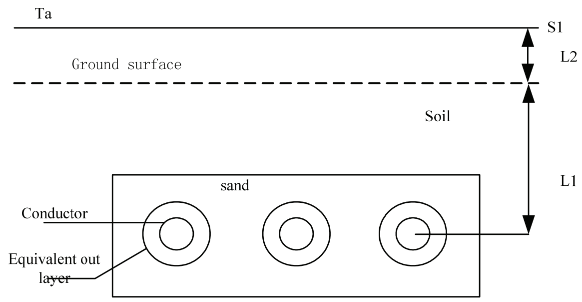

Fig.1. Geometric model of a typical buried cable installation

A power cable usually consists of conductor, insulation, sheath, armor and external cover and losses are distributed in conductor, sheath and armor. If the losses is only in conductor and the construction is simplified to two layers, the calculation will be easy to conduct. In Fig.1, insulation, sheath, armor and external cover of cable are substituted by an equivalent out layer using method as follow:

If we move the sheath losses Q_2 into conductor, the thermal resistance of insulation must be changed to Eq.(1) in order to keep the temperature of conductor and sheath unchanged

(1) $$R_T{}' = \left[\frac{Q_1}{Q_1 + Q_2}\right] R_T$$

where R_T is the original thermal resistance of insulation and R_T'the modified thermal resistance.

Other losses can be moved into conductor by the same way. After these steps, there is not losses in other layers except conductor. In order to reduce the complexity of the cable structure, we replace all the layers except conductor by an equivalent out layer. The thermal conductivity of the equivalent out layer can be calculated by.

(2) $$\lambda_T = \frac{\ln(r_c/r_1)}{2\pi} \Big/ \sum_{i=1}^{n} \frac{\ln(r_i/r_{i-1})}{2\pi\lambda_i}$$

where λ_T is the equivalent thermal conductivity, λ_i is the thermal conductivity of the ith layer, r_c is the outer diameter of cable, r_1 is the diameter of conductor, r_i and r_{i-1} are the diameter of adjacent layers.

At the ground surface, the heat is transferred between soil and air with heat convection mode and the convection losses can be calculated by Newton's law:

(3) $$Q_{ca} = h(T - T_a)$$

Where h is the convective heat transfer coefficient $(W/(m^2 \cdot °C))$, T is the temperature at the surface, and T_a is the air temperature.

The convection heat transfer can be replaced by equivalent depth of soil. Taking into account Fourier's law, we obtain the transmission loss in the soil:

(4) $$Q_{cs} = \frac{\lambda}{\delta}(T - T_a)$$

where δ is the depth of soil.

Let's $Q_{ca} = Q_{cs}$, the convection heat transfer at the ground surface can be substituted by equivalent depth of soil (L3) and air can be substituted by soil. At the same time, the temperature of any point at line S1 is equal to T_a .

If we take line S1 as the temperature isotherm shown in Fig.1, the temperature of any point in the solved region can be calculated by image method.

Image method

Image method can be used to calculate the temperature distribution of a half infinite field[13,14]. As shown in Fig.2, there is a temperature isotherm (x -axis) and line heat (q) in the field.

The relative temperature of point C in the solved region can be defined as:

(5) $$\theta = T_C - T_s$$

where T_s is the temperature of isotherm and T_C is the temperature of point C .

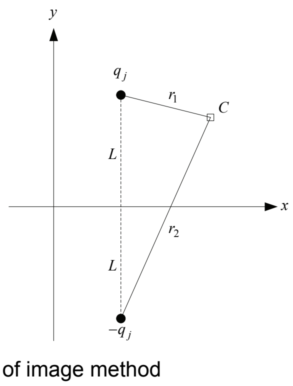

Fig.2. Sketch map of image method

Given that $-q$ is the mirror image of q . The relative temperature of point C can also be described as:

(6) $$\theta = \frac{q}{2\pi\lambda} \ln \frac{r_2}{r_1} = Pq$$

where λ is the thermal conductivity and $P = \frac{1}{2\pi\lambda} \ln \frac{r_2}{r_1}$.

Optimal Heat Source Simulation Method

In Fig.1, power cables are surrounded by multilayer soil. In order to calculate the temperature field in underground cables by image method, the heat must be substituted by virtual heat line sources and solving equations must be subject to the following boundary conditions.

In the separation between adjacent different materials, such as different layer of the cable, the surface of the cables and the surrounding medium, the different materials surrounding the cables, calorific flow continuity is satisfied at the separation surface:

(7) $$\lambda_1 \frac{\partial T}{\partial n} = \lambda_2 \frac{\partial T}{\partial n}$$

In most cases, the current which flow through the cables is known and the heat in conductors is known. If the heat in the conductor is substituted by n virtual heat line sources arranged inside the conductor, the total heat of n virtual heat line sources should be equal to the heat in conductor Q_C.

(8) $$\sum_{j=1}^{n} q_j = Q_C$$

Because of the high thermal conductivity, the conductor can be taken as an isotherm body. The temperature of any points at the surface of conductor is equal to each other.

(9) $$T_j = T_{j+1}$$

In this paper, the virtual heat source is heat line sources. The arrangement of the heat line sources and the contour points are shown in Fig.3.

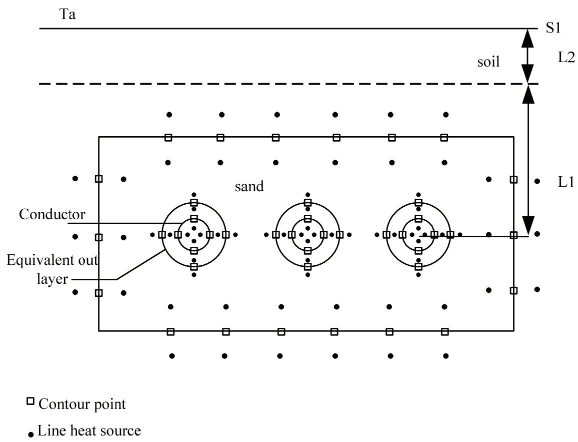

Fig.3. Sketch map of simulation heat charge and contour point location

Heat line sources 1~4 simulate the conductor of left cable, and 5~12 simulate the interface between equivalent out layer of left cable and sand. Similarly, heat line sources 13~24 simulate the middle cable and 25~36 simulate the right cable. Heat line sources 37~72 simulate the interface between sand and soil. Furthermore, we set 42 contour points in the solved region.

By (6), the relative temperatures of contour points can be calculated based on the uniqueness theorem of the static temperature field.

(10) $$\begin{cases} \theta_i = \sum_{j=1}^{4} P_{ij}Q_j + \sum_{j=9}^{12} P_{ij}Q_j & i = 1,2,3,4 \\ \theta_i = \sum_{j=13}^{16} P_{ij}Q_j + \sum_{j=21}^{24} P_{ij}Q_j & i = 9,10,11,12 \\ \theta_i = \sum_{j=25}^{28} P_{ij}Q_j + \sum_{j=33}^{36} P_{ij}Q_j & i = 17,18,19,20 \end{cases}$$

$$(11)\quad \begin{cases} \theta_{i1} = \sum\limits_{j=1}^{4} P_{ij}Q_j + \sum\limits_{j=9}^{12} P_{ij}Q_j & i = 5,6,7,8 \\ \theta_{i1} = \sum\limits_{j=13}^{16} P_{ij}Q_j + \sum\limits_{j=21}^{24} P_{ij}Q_j & i = 13,14,15,16 \\ \theta_{i1} = \sum\limits_{j=25}^{28} P_{ij}Q_j + \sum\limits_{j=33}^{36} P_{ij}Q_j & i = 21,22,23,24 \end{cases}$$

$$(12)\quad \begin{cases} \theta_{i2} = \sum\limits_{j=5}^{8} P_{ij}Q_j + \sum\limits_{j=17}^{20} P_{ij}Q_j \\ \quad + \sum\limits_{j=29}^{32} P_{ij}Q_j + \sum\limits_{j=55}^{72} P_{ij}Q_j \\ i = 5,6,7,8,13,14,15,16,21,22,23,24 \end{cases}$$

$$(13)\quad \begin{aligned} \theta_{i1} = \sum_{j=5}^{8} P_{ij}Q_j + \sum_{j=17}^{20} P_{ij}Q_j \\ + \sum_{j=29}^{32} P_{ij}Q_j + \sum_{j=55}^{72} P_{ij}Q_j \end{aligned} \qquad i = 25\cdots42$$

$$(14)\quad \theta_{i2} = \sum_{j=37}^{54} P_{ij}Q_j + \sum_{j=73}^{75} P_{ij}Q_j \qquad i = 25\cdots42$$

The constraint equations which satisfy the boundary condition are as follow:

$$(15)\quad \begin{cases} \theta_i = \theta_{i+1} & i = 1,2,3 \\ \theta_i = \theta_{i+1} & i = 9,10,11 \\ \theta_i = \theta_{i+1} & i = 17,18,19 \end{cases}$$

$$(16)\quad \begin{cases} \sum\limits_{j=1}^{4} Q_j = Q_1 \\ \sum\limits_{j=13}^{16} Q_j = Q_2 \\ \sum\limits_{j=25}^{28} Q_j = Q_3 \end{cases}$$

$$(17)\quad \begin{cases} \theta_{i1} = \theta_{i2} \\ \lambda_1 \dfrac{\partial \theta_{i1}}{\partial l} = \lambda_4 \dfrac{\partial \theta_{i2}}{\partial l} \\ i = 5,6,7,8 \end{cases}$$

$$(18)\quad \begin{cases} \theta_{i1} = \theta_{i2} \\ \lambda_2 \dfrac{\partial \theta_{i1}}{\partial l} = \lambda_4 \dfrac{\partial \theta_{i2}}{\partial l} \\ i = 13,14,15,16 \end{cases}$$

$$(19)\quad \begin{cases} \theta_{i1} = \theta_{i2} \\ \lambda_3 \dfrac{\partial \theta_{i1}}{\partial l} = \lambda_4 \dfrac{\partial \theta_{i2}}{\partial l} \\ i = 21,22,23,24 \end{cases}$$

$$(20)\quad \begin{cases} \theta_{i1} = \theta_{i2} \\ \lambda_4 \dfrac{\partial \theta_{i1}}{\partial l} = \lambda_5 \dfrac{\partial \theta_{i2}}{\partial l} \\ i = 25,26,\cdots,42 \end{cases}$$

where λ_1 , λ_2 and λ_3 represent the thermal conductivities of equivalent out layers of three cables, and λ_4 and λ_5 represent the thermal conductivities of sand and soil, respectively. Q_1 , Q_2 and Q_3 represent the total heat of three cables, respectively.

When the locations of the heat line sources are known, we will obtain the quantities of heat line sources and the temperatures of contour points by solving equation (15)-(20) together. In order to verify the accuracy of the results, we calculate the temperature of check points between contour points and compare it with the temperature of contour points. When the error is acceptable, the result is credible. The error depends on the locations of the heat line sources.

Based on well studied and analyzed, the locations of heat line sources can be determined by minimizing certain objective function by using genetic algorithm as shown in Fig.4.

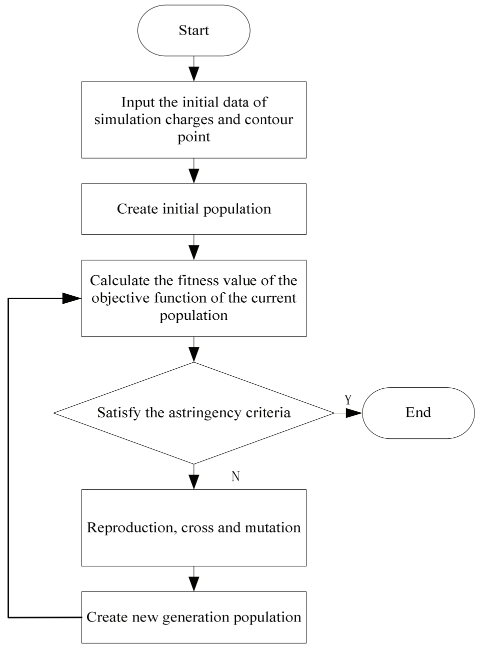

Fig.4. Flowchart of the optimal heat line sources simulation method

The objective function is the least square error method as below:

$$(21)\quad f = \sum_{i=1}^{n} \left(T_{ci} - T_{ci}' \right)^2 \le \varepsilon$$

where T_{ci} and T_{ci}' are, respectively, the temperature of contour points and check points, ε is a very small positive value.

Method validation

In order to verify the effective of the optimal heat line source simulation method, a model of buried heat pipe covered with rubber is calculated by the optimal heat line sources simulation method and the result is compared with the trail. The trial model is shown in Fig.5.

The heat pipe is placed horizontally at 0.5m depth. The length of heat pipe is 2.5m and the radius is 0.006m. The thickness of rubber is 0.004m. The thermal conductivity of soil and rubber are 1.664 $W/(m^2 \cdot °C)$ and 0.25 $W/(m^2 \cdot °C)$, respectively. The temperature of air is 30°C.

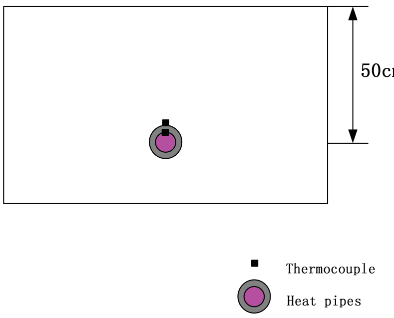

Fig.5. The trial model of heat pipes

When the power of heat pipe is 30W, the temperature of heat pipe at steady-state is 43 °C and the thermal conductivity of soil near heat pipe changed to 0.7893 $W/(m^2 \cdot °C)$ because of soil dry-out. The result calculated by the optimal heat line sources simulation method is 42.6 °C. The error is 0.4 °C. So, the optimal heat line sources (OHLS) simulation method is effective to calculate the temperature distribution in buried cables.

Simulation result

The typical single-loop buried cable system is shown in Fig.1. In this paper, 110kV YJLW02 power cable ratings are given in Table 1.

Table 1. Cable Ratings

Cable size (mm^2)	800
XLPE insulation (mm)	20
Sheath (mm)	2
PVC external covering (mm)	5
Rated frequency (Hz)	50
Operating current (A)	500

These cables are placed horizontally at 0.7m depth and separated 0.2m between the centers. The temperature of air is 40°C. The required thermal properties for each different material are listed in Table 2.

Table 2. Material Properties for Cable System

component	Thermal Conductivity ($W/(m^2 \cdot °C)$)
Soil	1.0
Sand	0.5
Conductor	386
Insulation	0.2857
Sheath	236
PVC external covering	0.1667

The computation purpose is to obtain the temperature of conductor under operating current and the ampacity of the buried cables. When the sheaths are bonded at one point, the conductor losses and the sheath losses can be calculated by finite element method [10]. The calculation results solved by the optimal heat line sources simulation method are shown in Table 3 and Fig.6. The calculation results are compared with the results calculated by finite element method and the error is small.

Table 3. Calculation Result of OHLS for one System Cables with Sheaths Bonded at one Point

Method	Conductor Temperature ($°C$)		
	A	B	C
Result of OHLS	48.3982	49.7304	48.4117
Result of FEM	48.606	49.839	48.62
Error	-0.2078	-0.1085	-0.2083

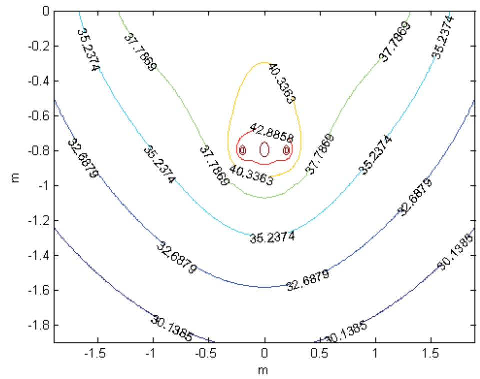

Fig.6. Distribution of isotherm for one system cables with sheaths bonded at one point

When the sheaths are bonded at both ends, the calculation results solved by the optimal heat line sources simulation method are given in Table 4 and Fig.7.

Table 4 Calculation Result of OHLS for one System Cables with Sheaths Bonded at Both Ends

Method	Conductor Temperature ($°C$)		
	A	B	C
Result of OHLS	64.9432	65.9230	62.2023
Result of FEM	64.765	65.976	62.345
Error	0.1782	-0.0530	-0.1427

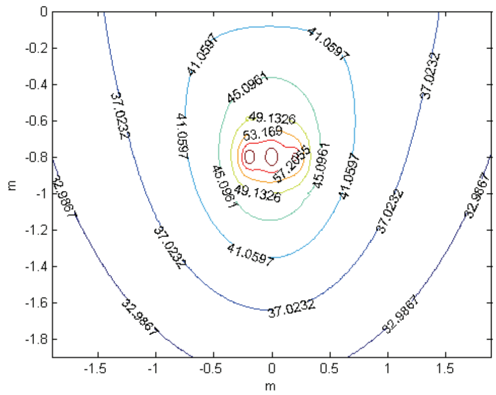

Fig.7. Distribution of isotherm for one system cables with sheaths bonded at both ends

For two systems of six cables placed horizontally at 0.7m depth and separated 0.2m between the cable centers, the cable properties are the same as above. When the sheaths are bonded at one point, the calculation results are given in Table 5.

Table 5. Calculation Result of OHLS for two System Cables with Sheaths Bonded at one Point

Method	Conductor Temperature($°C$)					
	A1	B1	C1	A2	B2	C2
OHLS	58.43	61.69	62.31	62.42	61.68	58.25
FEM	58.55	61.70	62.35	62.47	61.68	58.37
Error	-0.12	-0.01	-0.04	-0.05	0.00	-0.12

When the sheaths are bonded at both ends, the calculation results are given in Table 6.

Table 6. Calculation Result of OHLS for two System Cables with Sheaths Bonded at Both Ends

Method	Conductor Temperature($°C$)					
	A1	B1	C1	A2	B2	C2
OHLS	74.25	78.99	80.67	80.94	79.58	77.41
FEM	73.81	78.75	80.64	80.96	79.40	77.22
ERROR	0.44	0.24	0.03	-0.02	0.18	0.19

Discussion

Through comparison, the results of OHLS is not equal to the results calculated by finite element method. The error may be caused by followed reasons:

In order to simplify the computation process, the cable structure is equivalent to only two layers. This process is based on the assumption that there is only normal heat transfer inside the cables. Actually, heat flux may be different in different normal direction because of the external environment. The center conductor temperature calculated by finite element method is 49.839 $°C$ before cable equivalent and 49.932 $°C$ after cable equivalent. The error is 0.107 $°C$.

In order to eliminate the convection coefficient in the calculation process, we elevate the ground surface and replace the convection heat transfer with the heat conduction of equivalent depth soil to keep ground surface isotherm. Actually, the temperature of air may be not exactly the same everywhere. The center conductor temperature calculated by finite element method is 49.839 $°C$ before convection coefficient equivalent and 49.837 $°C$ after convection coefficient equivalent. The error is 0.02 $°C$.

In the OHLS, the heat line sources simulation are required to satisfy the boundary conditions only at a selected number of ‘contour’ points. Because the number of such point is kept small in order to economize the computer memory and computational time requirement, it is essential to set some ‘check’ points between ‘contour’ points. When the precision can't be improved by adjusting the position of heat line sources, we have to increase the quantities of heat line sources. Compared with FEM, the relationship between precision and quantities of heat line sources is shown in Table 7.

Table 7. The Relationship Between Error and Simulation Heat Line Quantities

Conductor	4 points	8 points	16 points
A	1.3297	0.2078	0.0415
B	1.3489	0.1085	0.0415
C	1.3313	0.2083	0.0729

Conclusion

The optimal heat line sources simulation method is successfully used for calculating the temperature field of underground cable systems. Compared with FEM, the optimal heat line source simulation method has simplified the process model and greatly increased the computation speed and greatly decreased the computer requirements with sufficient precision. The optimal heat line source simulation method has been proved to be an effective method for electricity utilities to calculate the temperature of underground cable systems.

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