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General mathematical model of the drive system for a polymerization reactor based on the induction motor with a pipe body

***Abstract**. In the paper a general mathematical model of the drive system for a polymerization reactor based on the induction motor with a pipe body is presented. The abovementioned model may be considered as an universal model for the drive systems made in a vertical version dealing with a composite form of the torques loading a system. The examples of dependencies that allow for consideration of the composite torques in the mathematical model are given.*

***Streszczenie**. W artykule przedstawiono ogólny model matematyczny układu napędowego reaktora polimeryzacji z asynchronicznym silnikiem indukcyjnym w wykonaniu rurowym, który może być modelem uniwersalnym dla układów napędowych w wersji pionowej w których występuje złożona forma momentów oporowych obciążających układ. Podano przykładowe zależności umożliwiające uwzględnienie w modelu matematycznym złożonych momentów oporowych. (**Ogólny model matematyczny układu napędowego reaktora polimeryzacji z asynchronicznym silnikiem indukcyjnym w wykonaniu rurowym**).*

Keywords: polymerization reactors, specially designed motors, mathematical modelling, transient responses analysis.
Słowa kluczowe: reaktory polimeryzacji, silniki specjalnego wykonania, modelowanie matematyczne, analiza przebiegów czasowych.

Introduction

Polymerization reactors play the most significant role in technological line of the polyethylene production. The drive system of the mixer for polymerization process works in two chambers. The operating conditions of the drive system are specific due to the necessity of keeping the constant temperature, the ethylene atmosphere and the high working pressure up to 2800 atm. The motor of the drive system has nonstandard dimensions and construction due to the socket fastening in the upper part of the reactor chamber in the vertical position. The drive system may be directly fed by the grid or the motor-generator set or the frequency converter. Drive systems for polymerization reactors often break down due to the exceptional operating conditions, including feeding the motor via the pressure electrodes providing a trouble-free operation under the high difference in pressure. The specially designed motor with a pipe body was made as a result of the carried out designing efforts and alternative elaborations. A large-size slide bearing made of sintered carbides was used in the abovementioned motor. The rated parameters of the motor are as follows [1, 2]: P_n = 55 kW, U_{1n} = 380 V, f_n = 50 Hz, M_n = 374 Nm, M_{max} = 842,85 Nm, n_n = 1420 rpm, p_b = 2, I_{1n} = 108 A, J = 1,02 kg·m^2 , G = 385 kg.

Kinematic structure of the drive system

The kinematic structure of the drive system for polymerization reactor based on the specially designed induction motor with a pipe body is presented in Fig. 1.

The drive system for polymerization reactor based on the induction motor with a pipe body (Fig. 1) may be presented in the simplified form (Fig. 2) considering the inertial moments of the respective elements and the existing load torques.

Considering the introduced symbols of the inertial moments (Fig. 2), the analytical inertial moments may be determined as follows:

$$(1)\quad J_1^{'} = 0{,}5J_s\,,\; J_2^{'} = 0{,}5J_s + 0{,}5J_1\,,\; J_3^{'} = 0{,}5J_1 + J_2 + 0{,}5J_3$$
$$J_4^{'} = 0{,}5J_3 + J_4 + 0{,}5J_5\,,\; J_5^{'} = 0{,}5J_5 + J_6 + 0{,}5J_7\,,$$
$$J_6^{'} = 0{,}5J_7 + J_8 + 0{,}5J_9\,,\; J_7^{'} = 0{,}5J_9 + J_{10} + 0{,}5J_{11}\,,$$
$$J_8^{'} = 0{,}5J_{11} + J_{12} + 0{,}5J_{13}\,,\; J_9^{'} = 0{,}5J_{13} + J_{14} + 0{,}5J_{15}$$

Considering the dependencies (1) as well as the kinematic structure of an exemplary drive system for a polymerization reactor based on the induction motor with a pipe body (Fig. 1), the analytical kinematic structure of the drive system for a polymerization reactor was determined (Fig. 3).

The analytical kinematic structure of the drive system (Fig. 3) was reduced into the rotating rigid plates with the inertial moments $J_1' \div J_9'$ of the respective elastic elements with the elastic factors $c_1 \div c_8$ and the damping coefficients $b_1 \div b_8$, respectively.

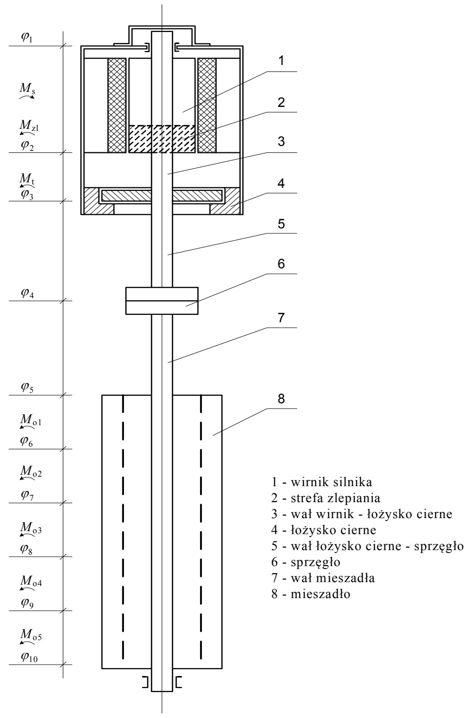

Fig. 1 The kinematic structure of the exemplary drive system for polymerization reactor based on the induction motor with a pipe body [4], where: 1 is rotor, 2 is area of agglutination, 3 is shaft between rotor and bearing, 4 is bearing, 5 is shaft between bearing and clutch, 6 is clutch, 7 is shaft of mixer, 8 is mixer

The position angles $\varphi_1' \div \varphi_9'$ were related to each rotating element with determined inertia. Considering the assumed symbols, the interior torques may be determined for any subassembly. That is meaningful in order to determine the foredesigning assumptions during the designing activities deal with this type of systems.

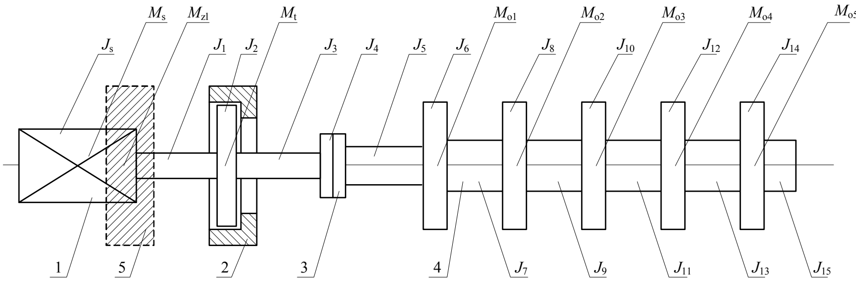

Fig. 2 The simplified kinematic structure of the drive system (Fig. 1), where: 1 is motor, 2 is the set of a large-size slide bearing, 3 is clutch of the mixer, 4 is mixer, 5 is the area of occurrence of the agglutination torque between stator and rotor of the motor [4]

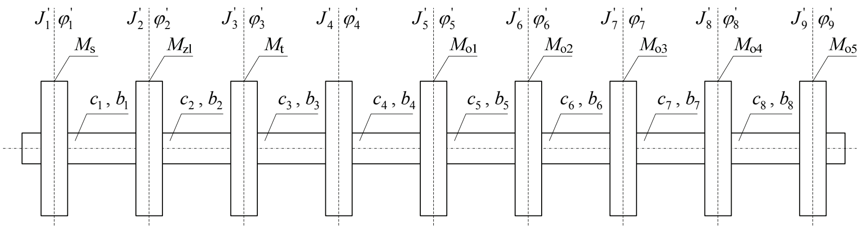

Fig. 3 The analytical kinematic structure of the drive system for a polymerization reactor based on the induction motor with a pipe body

Mathematical model

The drive system for a polymerization reactor based on the induction motor with a pipe body considered as a climatic system is simple, however the phenomena occurring when the system is in use do not occur in standard drive systems, e.g. the following phenomena having direct impact on the operation of the drive system as its loads:

- the agglutination of the rotor of the induction motor with its stator from the driving side as a result of polymerization in this area of the polymerization reactor
- the sliding friction in the large-size slide bearing made of sintered carbides and cooled with the ethylene stream via ducts of the cooling sets
- the filling of the mixer construction with the polyethylene in the mixing chamber of a polymerization reactor
- the sliding friction of the mixer filled with polyethylene in the area of the polyethylene charge of the mixing chamber of a polymerization reactor
- mixing the ethylene stream by the mixer in the lower chamber of a polymerization reactor in the part unfilled with polyethylene or if the drive system works with the uncharged mixer.

Considering the assumed symbols of the rotation angles (Fig. 1) the column matrix for the respective angles corresponding with the inertial moments is determined:

(2) $$\mathbf{q}^{\mathrm{T}} = [\varphi_1' \ \varphi_2' \ \varphi_3' \ \varphi_4' \ \varphi_5' \ \varphi_6' \ \varphi_7' \ \varphi_8' \ \varphi_9']$$

In the analytical kinematic structure (Fig. 3) the rigid sections containing the reduction of rotation do not occur, therefore the inertial moments may be directly introduced into the diagonal matrix of the inertial moments corresponding with the rotation angles in the column matrix (1). The diagonal matrix of the inertial moments corresponding with the independent rotation angles is given as follows:

(3) $$\mathbf{D} = \mathrm{diag}\left[J_1^* \ J_2^* \ J_3^* \ J_4^* \ J_5^* \ J_6^* \ J_7^* \ J_8^* \ J_9^*\right]$$

where: $J_k^* = J_k'$, $k = 1, 2, ..., 9$.

The dependencies (4) determining: the entire kinetic energy of the studied drive system, the entire potential energy and the loss of energy are taken into account in order to constitute the system of equation of the mathematical model.

(4) $$T = 0{,}5\sum_{j=1}^{9} J_j^*\left(\dot{\varphi}_j'\right)^2, \quad V = 0{,}5\sum_{j=1}^{8} c_j\left(\varphi_j' - \varphi_{j+1}'\right)^2$$

$$\Phi = 0{,}5\sum_{j=1}^{8} b_j\left(\dot{\varphi}_j' - \dot{\varphi}_{j+1}'\right)^2$$

The general mathematical model of the drive system for a polymerization reactor based on the induction motor with a pipe body is given as follows:

(5)
$$
\begin{aligned}
&J_1^*\ddot{\varphi}_1' + b_1\left(\dot{\varphi}_1' - \dot{\varphi}_2'\right) + c_1\left(\varphi_1' - \varphi_2'\right) = M_s \\
&J_1^*\ddot{\varphi}_1' + b_1\left(\dot{\varphi}_1' - \dot{\varphi}_2'\right) + c_1\left(\varphi_1' - \varphi_2'\right) = M_s \\
&J_2^*\ddot{\varphi}_2' - b_1\left(\dot{\varphi}_1' - \dot{\varphi}_2'\right) + b_2\left(\dot{\varphi}_2' - \dot{\varphi}_3'\right) - \\
&- c_1\left(\varphi_1' - \varphi_2'\right) + c_2\left(\varphi_2' - \varphi_3'\right) = -M_{zlw} \\
&J_3^*\ddot{\varphi}_3' - b_2\left(\dot{\varphi}_2' - \dot{\varphi}_3'\right) + b_3\left(\dot{\varphi}_3' - \dot{\varphi}_4'\right) - \\
&- c_2\left(\varphi_2' - \varphi_3'\right) + c_3\left(\varphi_3' - \varphi_4'\right) = -M_{tw} \\
&J_4^*\ddot{\varphi}_4' - b_3\left(\dot{\varphi}_3' - \dot{\varphi}_4'\right) + b_4\left(\dot{\varphi}_4' - \dot{\varphi}_5'\right) - \\
&- c_3\left(\varphi_3' - \varphi_4'\right) + c_4\left(\varphi_4' - \varphi_5'\right) = 0 \\
&J_5^*\ddot{\varphi}_5' - b_4\left(\dot{\varphi}_4' - \dot{\varphi}_5'\right) + b_5\left(\dot{\varphi}_5' - \dot{\varphi}_6'\right) - \\
&- c_4\left(\varphi_4' - \varphi_5'\right) + c_5\left(\varphi_5' - \varphi_6'\right) = -M_{o1} \\
&J_6^*\ddot{\varphi}_6' - b_5\left(\dot{\varphi}_5' - \dot{\varphi}_6'\right) + b_6\left(\dot{\varphi}_6' - \dot{\varphi}_7'\right) - \\
&- c_5\left(\varphi_5' - \varphi_6'\right) + c_6\left(\varphi_6' - \varphi_7'\right) = -M_{o2} \\
&J_7^*\ddot{\varphi}_7' - b_6\left(\dot{\varphi}_6' - \dot{\varphi}_7'\right) + b_7\left(\dot{\varphi}_7' - \dot{\varphi}_8'\right) - \\
&- c_6\left(\varphi_6' - \varphi_7'\right) + c_7\left(\varphi_7' - \varphi_8'\right) = -M_{o3} \\
&J_8^*\ddot{\varphi}_8' - b_7\left(\dot{\varphi}_7' - \dot{\varphi}_8'\right) + b_8\left(\dot{\varphi}_8' - \dot{\varphi}_9'\right) - \\
&- c_7\left(\varphi_7' - \varphi_8'\right) + c_8\left(\varphi_8' - \varphi_9'\right) = -M_{o4} \\
&J_9^*\ddot{\varphi}_9' - b_8\left(\dot{\varphi}_8' - \dot{\varphi}_9'\right) - c_8\left(\varphi_8' - \varphi_9'\right) = -M_{o5}
\end{aligned}
$$

where: M_s is electromagnetic torque of the motor, M_{zlw} is anti-torque coming from a partial agglutination of the rotor and stator occurring in drive systems for polymerization reactors [6], M_{tw} is anti-torque coming from friction in the large-size slide bearing of specially designed induction motors [5], $M_{o1} \div M_{o5}$ are anti-torques in the working part of the system.

In the system of equations (5) the load torques applied to the drive are determined accordingly with their representation in Fig. 1 to 3, respectively. The torques $M_{o1} \div M_{o5}$ have general meaning and concern the load of the system caused by the torques applied to the respective sections of the mixer i.e. friction torques and ventilation torques caused by working mixer. The friction torques come from sliding friction between the surface layer of polyethylene and given section of the mixer filled with polyethylene, whereas the ventilation torques occur alternately for given section of the mixer depending on the state of a polymerization process in the lower chamber of the polymerization reactor. The inertial moments $J_5^* \div J_9^*$ have the composite form that results from a polymerization process causing a variation of these moments in time as a result of filling with polyethylene the respective sections of the mixer. The inertial moments of the mixer differ from the inertial moments of the fillings of the mixer due to the cross-sectional areas considered during determination of the inertial moments as well as due to the different mass densities of the mixer and the filling of the mixer [6, 7].

In the proposed mathematical model the following assumption is considered: the filling of the mixer with polyethylene is homogeneous up to charge level of the mixer in the lower chamber of a polymerization reactor. In practice the filling of the mixer often has the random character that is difficult to analytical determination due to the fact the polymerization is carried out depending on the requirements concerning the process with normal parameters or the processes with the exceeded parameters.

The linear system of differential equations was derived as a result of the transformation of the system (5). The derived system of equations (6) describes the mechanical part of the drive system for a polymerization reactor.

(6)
$$\ddot{\varphi}_1' + \frac{b_1}{J_1^*}\dot{\varphi}_1' - \frac{b_1}{J_1^*}\dot{\varphi}_2' + \frac{c_1}{J_1^*}\varphi_1' - \frac{c_1}{J_1^*}\varphi_2' = \frac{M_s}{J_1^*}$$

$$\ddot{\varphi}_1' + \frac{b_1}{J_1^*}\dot{\varphi}_1' - \frac{b_1}{J_1^*}\dot{\varphi}_2' + \frac{c_1}{J_1^*}\varphi_1' - \frac{c_1}{J_1^*}\varphi_2' = \frac{M_s}{J_1^*}$$

$$\ddot{\varphi}_2' - \frac{b_1}{J_2^*}\dot{\varphi}_1' + \frac{b_1+b_2}{J_2^*}\dot{\varphi}_2' - \frac{b_2}{J_2^*}\dot{\varphi}_3' - \frac{c_1}{J_2^*}\varphi_1' + \frac{c_1+c_2}{J_2^*}\varphi_2' - \frac{c_2}{J_2^*}\varphi_3' = -\frac{M_{zlw}}{J_2^*}$$

$$\ddot{\varphi}_3' - \frac{b_2}{J_3^*}\dot{\varphi}_2' + \frac{b_2+b_3}{J_3^*}\dot{\varphi}_3' - \frac{b_3}{J_3^*}\dot{\varphi}_4' - \frac{c_2}{J_3^*}\varphi_2' + \frac{c_2+c_3}{J_3^*}\varphi_3' - \frac{c_3}{J_3^*}\varphi_4' = -\frac{M_{tw}}{J_3^*}$$

$$\ddot{\varphi}_4' - \frac{b_3}{J_4^*}\dot{\varphi}_3' + \frac{b_3+b_4}{J_4^*}\dot{\varphi}_4' - \frac{b_4}{J_4^*}\dot{\varphi}_5' - \frac{c_3}{J_4^*}\varphi_3' + \frac{c_3+c_4}{J_4^*}\varphi_4' - \frac{c_4}{J_4^*}\varphi_5' = 0$$

$$\ddot{\varphi}_5' - \frac{b_4}{J_5^*}\dot{\varphi}_4' + \frac{b_4+b_5}{J_5^*}\dot{\varphi}_5' - \frac{b_5}{J_5^*}\dot{\varphi}_6' - \frac{c_4}{J_5^*}\varphi_4' + \frac{c_4+c_5}{J_5^*}\varphi_5' - \frac{c_5}{J_5^*}\varphi_6' = -\frac{M_{o1}}{J_5^*}$$

$$\ddot{\varphi}_6' - \frac{b_5}{J_6^*}\dot{\varphi}_5' + \frac{b_5+b_6}{J_6^*}\dot{\varphi}_6' - \frac{b_6}{J_6^*}\dot{\varphi}_7' - \frac{c_5}{J_6^*}\varphi_5' + \frac{c_5+c_6}{J_6^*}\varphi_6' - \frac{c_6}{J_6^*}\varphi_7' = -\frac{M_{o2}}{J_6^*}$$

$$\ddot{\varphi}_7' - \frac{b_6}{J_7^*}\dot{\varphi}_6' + \frac{b_6+b_7}{J_7^*}\dot{\varphi}_7' - \frac{b_7}{J_7^*}\dot{\varphi}_8' - \frac{c_6}{J_7^*}\varphi_6' + \frac{c_6+c_7}{J_7^*}\varphi_7' - \frac{c_7}{J_7^*}\varphi_8' = -\frac{M_{o3}}{J_7^*}$$

$$\ddot{\varphi}_8' - \frac{b_7}{J_8^*}\dot{\varphi}_7' + \frac{b_7+b_8}{J_8^*}\dot{\varphi}_8' - \frac{b_8}{J_8^*}\dot{\varphi}_9' - \frac{c_7}{J_8^*}\varphi_7' + \frac{c_7+c_8}{J_8^*}\varphi_8' - \frac{c_8}{J_8^*}\varphi_9' = -\frac{M_{o4}}{J_8^*}$$

$$\ddot{\varphi}_9' - \frac{b_8}{J_9^*}\dot{\varphi}_8' + \frac{b_8}{J_9^*}\dot{\varphi}_9' - \frac{c_8}{J_9^*}\varphi_8' + \frac{c_8}{J_9^*}\varphi_9' = -\frac{M_{o5}}{J_9^*}$$

The agglutination of rotor and stator should not occur in normal operation of the technological line. In construction of the pipe motor it was assumed that the large-size slide bearing is cooled with the use of the shunted ethylene stream. The aforementioned cooling often causes the phenomenon of polarization in the slide bearing as well as in the lower part of the air-gap of the motor. This phenomenon causes the agglutination of rotor and stator. The polymerization in the lower part of the motor is shown in the picture (Fig. 4) of the disassembled motor SAR-55/1500/09 after operating tests in a polymerization reactor.

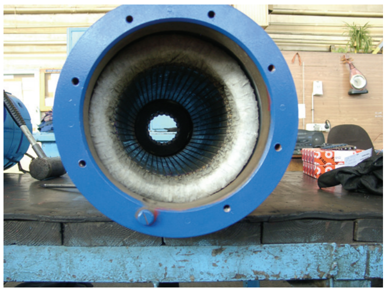

Fig. 4 Stator of the induction motor SAR-55/1500/09 after tests in a polymerization reactor with polymerization occurring from the side of the slide bearing [1, 2]

In the system of equations (6) there are the load torques that deal with the real phenomena occurring in drive systems working vertically and used among others in polymerization reactors, mixers, mills, etc. The abovementioned load torques are given as follows:

(7)
$$M_{zlw} = F_{ip}\frac{\pi}{2}d_w r_p(\delta)\, l_w, \qquad J_{kji}^* = \vartheta_m + j\left(\frac{\vartheta_m'}{i}\right)$$

$$M_{tmji} = j\left(\frac{M_{tm}}{i}\right), \; M_{wmji} = (i-j)\left(\frac{M_{wm}}{i}\right)$$

$$M_{wm}'(v) = \alpha M_c\left(1-\frac{l'}{l_m}\right)\left[\left(\frac{v}{d_m}\right)^2\left(\frac{p_b}{f}\right)^2\frac{1-\alpha}{\pi^2\alpha}+1\right]$$

where: F_{ip} is the unit measurement agglutination force of a polymerization process, d_w is the external diameter of the rotor, $r_p(\delta)$ is the phase agglutination factor, δ is size of the air-gap between rotor and stator of the motor, l_w is length of agglutination of rotor and stator, l' is charge level of the mixer, l_m is length of the mixer, ϑ_m is moment of mixer inertia, ϑ_m' is moment of the charged mixer inertia, M_{tmji} is anti-torque coming from friction of the mixer for the j-th section of the charged mixer, M_{wmji} is the ventilation anti-torque for the j-th section of the uncharged mixer, M'_{wm} is the ventilation anti-torque dependent on the charge level of the mixer, M_c is the entire ventilation torque for rotational speed $n = n_o$, v is linear velocity of the point belonging to the exterior surface of the mixer, d_m is the external diameter of the mixer, p_b is number of the pole couple, f is frequency of the feeding voltage, α is coefficient depending on a construction of the centrifugal fan [3], $\alpha = 0{,}05 \div 0{,}20$.

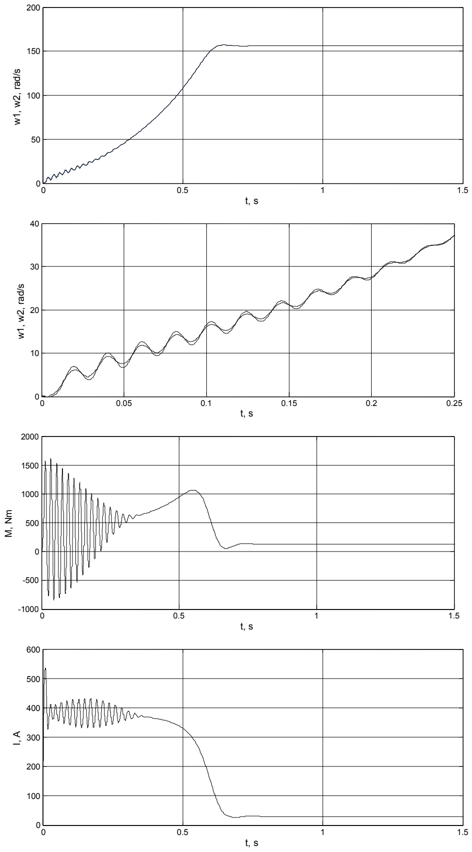

Fig. 5 Numerical time-dependencies of the following quantities: rotational speed of the motor and mixer w1 = w(t), w2 = w(t), torque of the motor M_s = M(t) and phase stator current I_1 = I(t) in the dynamical states for the Example P1 and constructional data [1, 2]

The system of equations (6) and dependencies (7) allow for total and penetrating computational analysis of the dynamical states considering the true load torques occurring in industrial operating systems.

Examples of numerical time-dependencies

The simulations of the working drive system for a polymerization reactor based on the specially designed induction motor with a pipe body and a double-row thrust ball bearing [1, 2, 4, 7] have been made for the following cases:

- starting the unloaded system with the uncharged mixer (Example P1)
- starting the unloaded system with the charged mixer (Example P2)
- starting the system with the loaded mixer as a result of the charge in the lower chamber of a polymerization reactor (Example P3).

The computational time-dependencies made for Example P1 are presented in Fig. 5. The comparison of the results of calculations for Examples P1, P2 and P3 is given in Table 1.

Table 1

No.	Quantity	Example of calculations		
		P1	P2	P3
1	2	3	4	5
1.	Rotational speed in steady state [rad/s]	155,9	155,9	155,9
2.	Time required to stabilize the rotational speed [s]	0,60	0,70	0,95
3.	Electromagnetic torque in steady state [Nm]	122,4	151,0	207
4.	Time required to stabilize the torque [s]	0,75	1,05	1,20
5.	Phase current of the stator in steady state [A]	28,8	31,6	37,9
6.	Time required to stabilize the current [s]	0,65	0,85	1,05

Conclusions

The mathematical model given as the system of equations (6) together with the dependencies (7) that determine the additional anti-torques of the system allows for the numerical analysis of dynamical states with the consideration of the phenomena occurring in the industrial operating conditions.

Determination of the anti-torque coming from friction in large-size slide bearing (the third equation of the system 6) allows for consideration of the additional interior anti-torque in motors for various constructions of thrust bearings [5].

Comparing the results of the exemplary calculations (Table 1) it follows that the consideration of the additional anti-torques causes the particularization of the calculations. Simultaneously, the relations among the results for the respective examples are fulfilled. The relations justify the consideration of the additional anti-torques in general mathematical model. It can be observed mainly in the range of the significant values of the phase current of stator in steady state (no. 5).

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