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Energy distribution of residual stresses in bi-layer structure

Abstract. *The analysis of energy of elastic strain induced by the residual stresses in bonded structure in the framework of Timoshenko's model has been done. The analysis revealed the energy is due to the forces aimed at splitting the structure. The energy as well as the splitting forces strongly depended on the thickness ratio of the bonded layers and the total thickness of the structure. The dependence included the point where splitting forces were equal zero ($E_1h_1^2 = E_2h_2^2$). The latter allows reducing the debonding problem, often appearing in the case of bonded structures of dissimilar materials, which becomes peculiarly important in micro and nanofabrication processes.*

Streszczenie. *W pracy dokonano analizy rozkładu odkształceń elastycznych zainicjowanych przez naprężenia resztkowe przy użyciu modelu Timoshenko. Energia powstała wskutek działania sił w spojonej, dwuwarstwowej strukturze powodowała rozwarstwianie się materiału. Wartości energii oraz siły rozwarstwiającej były zależne od stosunku grubości spajanych materiałów oraz całkowitej grubości uzyskanej struktury. Zależność uwzględniała przypadek, w którym siły rozwarstwiające były równe zeru ($E_1h_1^2 = E_2h_2^2$). Pozwala to na zredukowanie problemu odklejania i rozwarstwiania jaki często ma miejsce w spajaniu materiałów o różnych właściwościach, co staje się szczególnie ważne w procesach mikro i nanofabrykacji. (**Rozkład energii naprężeń resztkowych w strukturach dwuwarstwowych**)*

Keywords: Bonding, crystal quartz, silicon, residual stress, surface processing, thin surface coatings,
Keywords: Połączenia spajalne , kwarc krystaliczny, silicon, naprężenia własne, obróbka powierzchni, powłoki cienkościenne,

Introduction

The development of the galvanic coating technology caused an urgent need for a proper theory describing the residual stresses in a structure consisting of the thin film deposited on the thick substrate. Stoney provided such theory in 1909 [1] under assumption of very small thickness of the film. More recently (1925) [2], Timoshenko developed a general theory of a bi-layer structure which imposed no restrictions on the thickness of the layers. His model was originally developed for analysis of a bi-metal strip thermostat operation and based on the use of the radius of curvature of a structure which is curved as result of a difference in the coefficients of thermal expansion. Timoshenko also mentioned about deviation from his theory at the edge of the bi-metal structure and appearance of additional shear stresses which are of local character. He predicted that the maximum intensity of these additional stresses may be of the same order as that of the normal stresses calculated in his main theory. Chan and Nelson (1979) used the shear lag model with intermediate layer at the interface and confirmed an importance of the shear stresses at the edge of the bonded structure [3]. Suhir developed a model without intermediate layer at the interface. He revealed the essential shear and pilling stresses at the edge of the bonded structure. The magnitude of the stresses is of the same order as that of the normal stresses and they concentrated near the edge at the distance comparable with the strip thicknesses [4], as predicted by Timoshenko.

As new composite materials and layered structure in microelectronics are developed, a problem of interfacial stresses which are responsible for the structural integrity is gaining in importance [5]. The known calculations describe the interfacial stresses at the edge of the bonded structure. But the interfacial stresses of the basic part of the interface are not described. The model presented in this paper considers new approach to solve this problem.

Energy of strain in bi-layer structure:

When free strip elongates due to temperature rising, the stresses do not arise and energy of strain is equal to zero because external forces are not applied. When the strips with different thermal expansion coefficients are bonded, the temperature changing creates the stresses even if the external forces are not applied. Regardless whether the temperature increases or decreases relatively to the equilibrium state, the thermal stresses are generated and the bonded structure stores energy of strain without application of the external forces. Each strip serves as a source of external forces in relation to other one and they interact through interface. Thermal stresses are the operational principle of the bi-metal thermostats. Timoshenko treated an influence of the thermal stresses on the bi-metal strip bending submitted to a uniform heating [2]. His approach has a clear physical justification and is based on detail consideration of the thermal constraining forces. The same Timoshenko's approach will be used below to calculate energy of the bi-material bonded structure and thereafter to reveal the interfacial stresses.

Considering the element of the structure, the internal forces acting on the interface can be reduced to axial forces P and bending moments M, as follows from the statics' (Fig.1).

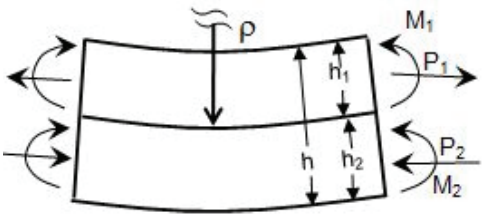

Fig.1. Deflection of a bonded bi-layer structure under residual stresses

The internal forces over any cross section of the structure must be in static equilibrium; therefore

$$P_1 = P_2 = P \tag{1}$$

$$P_1h_1/2 + P_2h_2/2 = M_1 + M_2 = E_1I_1/\rho + E_2I_2/\rho \tag{2}$$

Here $M_i = E_iI_i/\rho$, E_i is a Young modulus, I_i is a momentum of inertia of the single strip i, ρ is a radius of curvature of the deflected structure. The equations of statics alone are not enough to determine the variables P and ρ. Another equation should be derived from consideration of the deformation structure. In the Strength of Materials [6], such cases are called as statically indeterminate problems and respectively such structures are qualified as statically indeterminate systems.

An additional equation is obtained from the consideration of the strain at the interface; on the both sides of the interface the unit elongation in the longitudinal fibers of both materials must be equal:

(3) $\alpha_1 \Delta T + P_1/E_1 h_1 + h_1/2\rho = \alpha_2 \Delta T - P_2/E_2 h_2 - h_2/2\rho$

The solution of a system of equations (1)-(3) gives the desired analytical dependences ρ [2]

(4) $$\rho = \frac{h}{2(\alpha_1 - \alpha_2)\Delta T} + \frac{\left(E_1 h_1^3 + E_2 h_2^3\right)}{6h(\alpha_1 - \alpha_2)\Delta T}\left(\frac{1}{E_1 h_1} + \frac{1}{E_2 h_2}\right)$$

From (2) and (4) P is found to be

(5) $$P = \frac{(\alpha_1 - \alpha_2)\Delta T}{1/E_1 h_1 + 1/E_2 h_2 + 3h\left(E_1 h_1^3 + E_2 h_2^3\right)}$$

In the bonded structure (Fig.1), the axial force P_1 elongates strip 1 and P_2 compresses strip 2. During elongation (compression), work is done. Bending moments M_1 and M_2 bend the strips and the work is done as well. If the strain remains within the elastic limit, the work done by the axial forces and bending moments will be completely transformed into potential energy, therefore:

(6)
$$U_B = P^2/2E_1 h_1 + P^2/2E_2 h_2 + M^2/2E_1 I_1 + M^2/2E_2 I_2 =$$
$$= P^2/2E_1 h_1 + P^2/2E_2 h_2 + E_1 I_1/2\rho^2 + E_2 I_2/2\rho^2$$

Here ρ and P are expressed by equations (4) and (5) for the bonded structure shown in Fig.1.

Expression (6) represents the magnitude of potential strain energy of the bonded structure which is in steady state, and the internal forces P, M_1 and M_2 are such as to make the strain energy of the structure a minimum. But potential energy does not depend upon the order in which the forces are applied to the structure; it depends only upon their final values. Therefore, to reveal the interfacial forces, we will apply the forces in the order shown in Fig.2.

First, let two strips of unit length be the same as shown in Fig.1. But contrary to the case shown in Fig.1, they are free, not bonded. If the temperature increases from the initial value, the length of each strip will increase by $\alpha_1 \Delta T$ and $\alpha_2 \Delta T$ accordingly. Then, let us apply an eccentric load on the strip in such a way as shown in Fig.2. The arks of the adjacent surfaces are different, but they have a common chord C, or more generally, their chords are equal to each other (Fig.2, dashed line). The strips do not need to be fully touched to each other. The main condition is the equality of the length of the chords. The forces P'_1 and P'_2 are equal in magnitude and opposite in direction. Similar to the case shown in Fig.1, the eccentric load can be reduced to axial forces P'_1, P'_2 and bending moments M_1', M_2' (Fig.2, b). It is evident that in general case, axial forces P' and bending moments M_1', M_2' shown in Fig.2 do not equal to that shown in Fig.1.

The condition of the static equilibrium provides three equations, because the strips are free

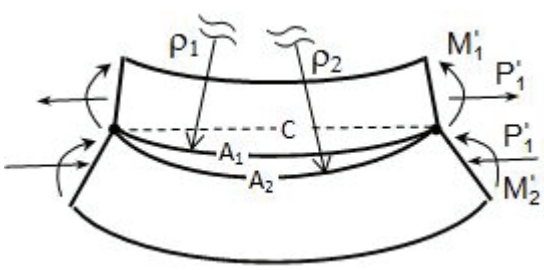

Fig.2. Deflection of a free strips structure under eccentric load

(7) $$P_1' = P_2' = P'$$

(8) $$P_1' h_1/2 = M_1' = E_2 I_2/\rho_1$$

(9) $$P_2' h_2/2 = M_2' = E_2 I_2/\rho_2$$

Here the radiuses of curvature ρ_1 and ρ_2 do not necessarily equal each other and ρ. Three equations (7)-(9) are not enough to determined four unknowns. The fourth equation can be obtained from the consideration, similar to that when the equation (3) was derived. But in the case of free strips, the chords joining the appropriate longitudinal fibers A_1 and A_2 (Fig.2) must be equal:

(10)
$$\rho_1 Sin\left[1 + \alpha_1 \Delta T + (1 + \alpha_1 \Delta T)P_1/E_1 h_1 + (1 + \alpha_1 \Delta T)h_1/2\rho_1\right] =$$
$$= 2\rho_2 Sin\left[1 + \alpha_2 \Delta T + (1 + \alpha_2 \Delta T)P_2/E_2 h_2 + (1 + \alpha_2 \Delta T)h_2/2\rho_2\right]$$

Equation (10) is implicit and unknowns P, ρ_1 and ρ_2 cannot be derived in analytical form therefore a numerical calculation should be used.

The expression for the stain energy of the free strips is identical in form to equation (6):

(11)
$$U_F = P'^2/2E_1 h_1 + P'^2/2E_2 h_2 + M_1'^2/2E_1 I_1 + M_2'^2/2E_2 I_2 =$$
$$= P'^2/2E_1 h_1 + P'^2/2E_2 h_2 + E_1 I_1/2\rho_1^2 + E_2 I_2/2\rho_2^2$$

The values of U_B, U_F, and difference $U_B - U_F$ versus normalized thickness $h_1/(h_1+h_2)$ of the strips are plotted in Fig.3.

As seen in Fig.3, the strain energy U_B of the bonded strips is essentially bigger than U_F of the free strips except the case where

(12) $$E_1 h_1^2 = E_2 h_2^2$$

In this condition $U_B = U_F$ and radii of curvature are equal: $\rho_1 = \rho_2 = \rho$ and the shape of the free structure becomes equal to the bonded one.

To complete the assembly of the structure from the free (except the case when $E_1 h_1 = E_2 h_2$). Because the steady state of the bonded structure is unique and the energy of strain in this state is in minimum, the work done by external forces must be equal to $P - P'$, $M_1 - M_1'$, and $M_2 - M_2'$. The forces will produce the deflections of the strips and by this means they will bring in touch the surfaces of the strips.

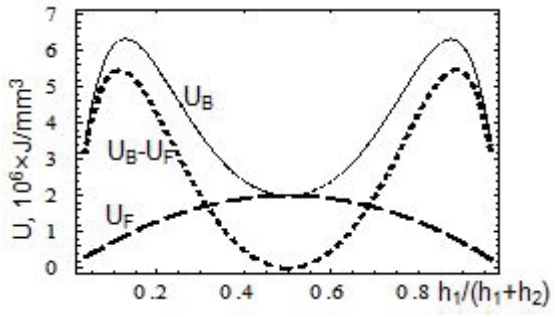

Fig.3. Elastic strain energy of the bonded strips U_B (solid line), free strips U_F (dashed line), and the difference $U_B - U_F$ (dotted line) versus the thickness ratio $h_1/(h_1+h_2)$; $E_1 = E_2 = 100$GPa, $\Delta\alpha T = 8*10^{-6}/^oC$, $\Delta T = 100^oC$.

The last step in constructing of the bonded structure from the free strips is the bonding itself. The bonding does not

produce new work, but it fixes the deflection at the state when the strips touch each other. Therefore, when the external forces are removed, the work, done by external forces, transforms to potential energy of the strain. In view of the fact that the deflections of the strip surfaces are directed opposite to each other (Fig.2), the forces are also directed opposite to each other and are oriented normally to the interface at any cross section of the structure. This energy is equal to U_B–U_F because the strain is within elastic limit. Thus, the bonding structure stores the energy of strain due to the forces aimed at splitting the structure.

Since the length of the strip in Fig.2 is taken as equal to unity and relative elongation is small, the splitting (pilling) force P_s at the interface can be approximately expressed as follows:

$$P_S = \sqrt{\frac{2(U_B - U_F)h_1 h_2 E_1 E_2}{h_1 E_1 + h_2 E_2}} \tag{13}$$

If instead of a narrow strip a plate of bi-material is bonded, the curvature in two orthogonal directions must be taken into consideration. In this case, it is necessary to substitute $E_1^*=E_1/(1-\nu_1)$ and $E_2^*=E_2/(1-\nu_2)$ instead of E_1 and E_2, where ν_1 and ν_2 are the Poisson's ratio of each material [2].

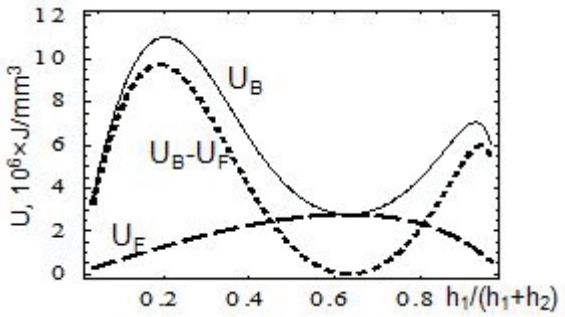

Fig.4. Elastic strain energy of the bonded wafers U_B (solid line), free wafers U_F (dashed line), and the difference U_B–U_F (dotted line) versus the thickness ratio $h_1/(h_1+h_2)$; E_1^*=86,4 GPa, E_2^*=256.9 GPa, $\Delta\alpha T$=8*10^{-6}/^{o}C, ΔT=100^{o}C.

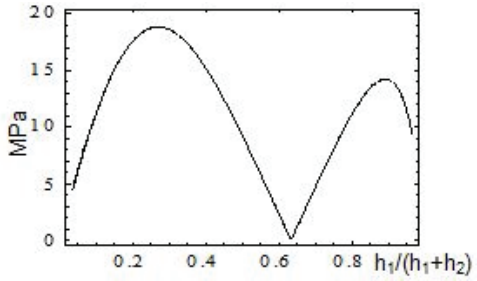

Fig.5. Splitting force calculated according (13) for silicon-quartz pair versus the thickness ratio $h_1/(h_1+h_2)$; E_1^*=86,4 GPa, E_2^*=256.9 GPa, $\Delta\alpha T$=8*10^{-6}/^{o}C, ΔT=100^{o}C.

Fig.4 shows the same curves as Fig.3 plotted by using elastic parameters for silicon-quartz pair. It is seen that the bonded structure and free structures have the same energies if $E_1h^2_1 = E_2h^2_2$ and this point approximately corresponds to the thickness ratio $h_1/(h_1+h_2)\approx$0.63. Fig.5 shows the splitting forces versus thickness ratio of the bonded silicon-quartz pair calculated according (13). It is seen, that the magnitude of the forces is of the same order as the bonding strength of many silicon-quartz structures reported so far and this might be the reason of unsuccessful bonding in many experiments. At the same time, there are three areas where the splitting forces do not essentially affect on the bonding strength: $h_1/(h_1+h_2)$–>0, $h_1/(h_1+h_2)$–>1, and $h_1/(h_1+h_2)\approx$ 0.63. Above result indicates a possibility to reduce the residual stresses in bonded structures by proper selection of the thickness ratio of the layers.

Conclusions

Energy of strain of bi-material bonded structure and the same but not bonded structure has been calculated. It is shown that the energy of the bonded structure includes the energy, which is due to the forces aimed at split the structure, oriented normally to the interface. The magnitude of the force depends on the total thickness of the structure and thickness ratio of the layers as well. The splitting force equals zero when $E_1h^2_1 = E_2h^2_2$ at any given total thickness of the structure. The latter permits creation of the bonded structures without the presence of splitting forces at the interface.

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