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Tuning of strongly damped angular velocity observers

Abstract. *The goal of the paper is to present a new method of observer parameter tuning. Nonlinear systems for which output injection observers exist are considered. Proposed method relies on guaranteeing that the equations of error dynamics generate a contraction semigroup i.e. that the estimation error norm is bounded by its initial value. Paper includes a proposition of a sufficient condition. It is also shown, that if its assumptions are not fulfilled appropriate gains can be chosen with the help of the Gershgorin theorem. Method is illustrated with examples of application for armature and field controlled separately excited DC motors and series DC motor.*

Streszczenie. *Celem pracy jest przedstawienie nowej koncepcji dobierania parametrów obserwatora. Rozważane są systemy nieliniowe, dla których można zaprojektować obserwator o liniowej dynamice błędu (tzw.* output injection*). Zaproponowana metoda opiera się na doprowadzeniu do sytuacji, gdy równanie błędu generuje półgrupę kontrakcji tj. sytuacji gdy norma błędu estymacji nie przekracza początkowej wartości. W pracy zawarto propozycję warunku dostatecznego. Pokazano również, że nawet gdy nie są spełnione założenia można z pomocą tw. Gerszgorina dobrać odpowiednie wzmocnienia. Zastosowaną metodę ilustrują przykłady dla silników elektrycznych prądu stałego: obcowzbudnego sterowanego od strony twornika, szeregowego i obcowzbudnego sterowanego od strony wzbudzenia.* ***(Strojenie silnie tłumionych obserwatorów prędkości obrotowej)***

Keywords: Nonlinear observer, output-injection, DC motor, velocity estimation, Gershgorin theorem
Słowa kluczowe: Obserwator nieliniowy, output-injection, silnik DC, estymacja prędkości, twierdzenie Gerszgorina

Introduction

A common problem in the control of electric motors is the lack of the angular velocity measurements. Typically we measure only the angular position via encoders or resolvers. Instead of the numerical differentiation of the position signal - which greatly increases disturbance effects on the system - the more sophisticated approach is to construct a state observer. This problem is continuously researched (see for example [15, 19]).Velocity observers for DC motors - described by continuous, discrete, linear and nonlinear models - were considered among the others in [1, 5, 9, 16, 18]. In this paper we will consider velocity estimation using the measurements of the angular position and the current. A very effective tool for state estimation is the full rank Luenberger observer (see for example [14, 16]) and its nonlinear generalizations. Let us consider systems described by the following equations

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{g}(\mathbf{C}\mathbf{x}(t), \mathbf{u}(t))\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n,\ \mathbf{u}(t) \in \mathbb{R}^r$ and $\mathbf{y}(t) \in \mathbb{R}^m$ with $\mathbf{A},\ \mathbf{B}$ and $\mathbf{C}$ of appropriate dimensions and $\mathbf{g}\colon \mathbb{R}^m \to \mathbb{R}^n$ and continuously differentiable. For this kind of systems a so called output-injection observer takes form

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}(\mathbf{y}(t) - \mathbf{C}\hat{\mathbf{x}}(t)) + \mathbf{g}(\mathbf{y}(t), \mathbf{u}(t)) \tag{2}$$

where the state estimate $\hat{\mathbf{x}}(t) \in \mathbb{R}^n$ and the gain matrix $\mathbf{G} \in \mathbb{R}^{n\times m}$. The estimation error given by $\mathbf{e} = \mathbf{x} - \hat{\mathbf{x}}$ evolves according to the equation

$$\dot{\mathbf{e}}(t) = (\mathbf{A} - \mathbf{G}\mathbf{C})\mathbf{e}(t) \tag{3}$$

which is linear. It is widely known, that if the pair $(\mathbf{C};\ \mathbf{A})$ is observable (that is $\operatorname{rank}(\mathbf{S}) = n$, where $\mathbf{S} = [\mathbf{C}^\mathsf{T}\ (\mathbf{C}\mathbf{A})^\mathsf{T}\ \ldots\ (\mathbf{C}\mathbf{A}^{n-1})^\mathsf{T}]$, see for example [13]) then one can find such $\mathbf{G}$ that results in a desired characteristic polynomial of $\mathbf{A} - \mathbf{G}\mathbf{C}$. Essentially $\mathbf{G}$ should be chosen in such way, that the dynamical system given by the equation (3) is asymptotically stable. In that case $\mathbf{e}$ will tend to zero, as time follows. In the case of $m = 1$ (systems with one output) choice of $\mathbf{G}$ that will result in chosen polynomial is unique. When $m > 1$ (many outputs) and system is observable from more than one of them the choice of $\mathbf{G}$ is much more problematic - infinite number of gains lead to the same characteristic polynomial. One can attempt to select such $\mathbf{G}$ which is optimal with respect to certain performance indexes (see for example [2,4,5,18]). When characterization of disturbances influencing the system is known (and they are gaussian) it is common to use the gain from Kalman filter, however because of output-injection linearization (canceling of nonlinear terms in error dynamics) its justification is debatable.

In this paper the focus is on a different approach. The intent is to guarantee, that the estimation error norm will be steadily reducing over time and does not exceed its initial value. For the remainder of the paper the spectral norm $\|\mathbf{A}\| = \max_{1\le i\le n} \sqrt{\lambda_i(\mathbf{A}^*\mathbf{A})}$ will be used, where $\mathbf{A}^*$ denotes the conjugate transpose of $\mathbf{A}$.

Motivation

It is a common conception that if a linear system such as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) \tag{4}$$

has only real negative eigenvalues it is well damped. It is usually explained that the absence of complex eigenvalues guarantees the lack of oscillations. Norm of the trajectory fulfills the relation

$$\|\mathbf{x}(t)\| \le \|\mathbf{x}(0)\|\ \|\mathrm{e}^{\mathbf{A}t}\|$$

Moreover the exponential stability of system guarantees that the norm of fundamental matrix $\|\mathrm{e}^{\mathbf{A}t}\|$ is bounded as follows (see for example [8] or [17] p. 133)

$$\begin{gathered}\forall\epsilon > 0, \exists c(\epsilon) > 0\colon\\ \|\mathrm{e}^{\mathbf{A}t}\| \le c(\epsilon)\mathrm{e}^{(\gamma+\epsilon)t}\\ 0 \le t < \infty\end{gathered} \tag{5}$$

where $\gamma = \max_i \operatorname{Re}\lambda_i(\mathbf{A})$. Let us consider the following matrix

$$\mathbf{A} = \begin{bmatrix} -1 & 20 & 0 & 0\\ 0 & -2 & 20 & 0\\ 0 & 0 & -3 & 20\\ 0 & 0 & 0 & -4\end{bmatrix} \tag{6}$$

It can be easily seen, that this matrix has real, distinct eigenvalues, so its trajectories should be well damped. However, see the figure 1 for the plot of the norm of the trajectory of the system (4) with the matrix (6) with the initial condition

$$x(0) = \begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}.$$

As one can see, the initial norm $\|\mathbf{x}(0)\| = 1$ is multiplied by about 150 at $t \approx 1.5$. That is why additional analysis of

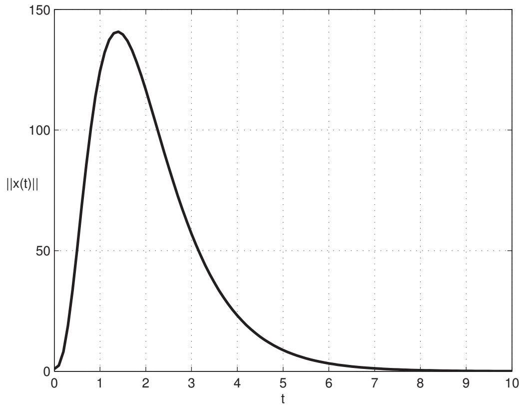

Fig. 1. Motivatory example - norm of a trajectory of a system with real negative eigenvalues

damping of the system is necessary. It should be noticed that the bound (5) still holds. Unfortunately, the multiplier $c(\varepsilon)$ in general case can be arbitrarily large, in this case it would be about few hundred.

One can easily come to the conclusion, that if error of the observer would behave in the similar way the control quality would be seriously impaired. Moreover analysis of these phenomena is desired because it is quite common for observers with high gains to generate large transitional errors. In literature it is called the peaking phenomena.

The best possible situation for the estimation quality would be for the error of estimation to decrease with time. In case of systems with linear error dynamics in order for this to happen, the evolution must behave as a contraction semigroup, that is $\|\exp((\mathbf{A}-\mathbf{GC})t)\| \leq \mathrm{e}^{-\alpha t},\ \alpha > 0$. It gives an exponential bound on the vanishing of error and guarantees that the error will not exceed the initial value.

Bound on the matrix exponential

In order to guarantee the fulfillment of

$$\|\exp((\mathbf{A}-\mathbf{GC})t)\| \leq \mathrm{e}^{-\alpha t},\ \alpha > 0 \quad t \geq 0 \tag{7}$$

one needs to find a way to asses the value of matrix exponential norm. It is rather obvious, that determination of such norm explicitly, especially as a function of observer parameters is difficult, if not entirely impossible. Let us introduce the following

Definition 1 (Damping coefficient)**.** A real number $\mu(\mathbf{A})$ will be called a damping coefficient of matrix exponential of $\mathbf{A}$ if

$$\mu(\mathbf{A}) = \lim_{\Delta\to 0_+} \frac{\|\mathbf{I}+\Delta\mathbf{A}\|-1}{\Delta}. \tag{8}$$

Norm of the matrix exponential fulfills the following bound

$$\|\mathrm{e}^{\mathbf{A}t}\| \leq \mathrm{e}^{\mu(\mathbf{A})t}. \tag{9}$$

Exact value of a damping coefficient depends on the norm used in definition 1. For spectral norm we have

$$\mu(\mathbf{A}) = \lambda_{\max}\left(\frac{\mathbf{A}+\mathbf{A}^{\mathsf{T}}}{2}\right) \tag{10}$$

More on damping coefficient (under the name logarithmic norm) see [12]. As it can be seen if damping coefficient is negative then matrix exponential of $\mathbf{A}$ is a contraction semigruoup. However determining the coefficients explicitly from (10) is not practical for tuning the observers - it leads to a non-smooth optimisation problem. On the other hand one can use the following

Theorem 1 (Gershgorin, see for example [20] p. 50)**.** *Let $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ be a matrix with complex elements a_{ij}. Let us denote by K_i a closed disk on the complex surface center of which is at a_{ii} with a radius*

$$r_i = \sum_{j=1, j\neq i}^{n} |a_{ij}|$$

Every eigenvalue of $\mathbf{A}$ lies inside or on the boundary of one of discs $K_1, K_2, \ldots, K_n$.

Using Gershgorin theorem a sufficient condition for a given matrix exponential to be a contraction semigroup will be proved

Proposition 2. *For $\mathbf{A} \in \mathbb{R}^{n\times n}$ such that*

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} \tag{11}$$

and for each $1 \leq i \leq n$ the following inequalities hold

$$a_{ii} < 0, \tag{12}$$

$$|a_{ii}| > \sum_{j=1,\ j\neq i}^{n} |a_{ij}|, \tag{13}$$

$$|a_{ii}| > \sum_{j=1,\ j\neq i}^{n} |a_{ji}|, \tag{14}$$

then a matrix exponential $\mathrm{e}^{\mathbf{A}t}$ is a contraction semigroup and its norm $\|\mathrm{e}^{\mathbf{A}t}\|$ is bounded by

$$\|\mathrm{e}^{\mathbf{A}t}\| \leq \mathrm{e}^{\alpha t}, \tag{15}$$

where

$$\alpha = \sup_{1\leq i\leq n}\left(a_{ii} + \sum_{j=1,\ j\neq i}^{n}\left|\frac{1}{2}(a_{ij}+a_{ji})\right|\right). \tag{16}$$

Proof. First one should see that Gershgorin discs of matrices $\mathbf{A}$ and $\mathbf{A}^{\mathsf{T}}$ are located strictly in left open complex half plane, it is assured by assumptions (13) and (14).The damping coefficient of $\mathbf{A}$ is

$$\mu(\mathbf{A}) = \lambda_{\max}(\mathbf{Q}), \tag{17}$$

$$\mathbf{Q} = \frac{\mathbf{A}+\mathbf{A}^{\mathsf{T}}}{2}. \tag{18}$$

Elements of $\mathbf{Q}$ are given by

$$q_{ii} = a_{ii}, \qquad q_{ij} = \frac{1}{2}(a_{ij}+a_{ji}). \tag{19}$$

It will be now shown, that Gershgorin discs of matrix $\mathbf{Q}$ are also located strictly in left open complex half plane. First, because the diagonals of (11) and (18) are equal, then centers of Gershgorin discs are located in negative part of real axis.

The radii of discs for given i are

$$\begin{aligned} r_i = \sum_{j=1,\, j\neq i}^{n} |b_{ij}| = \sum_{j=1,\, j\neq i}^{n} \left|\frac{1}{2}(a_{ij}+a_{ji})\right| \leq \\ \leq \frac{1}{2}\left(\sum_{j=1,\, j\neq i}^{n} |a_{ij}| + \sum_{j=1,\, j\neq i}^{n} |a_{ji}|\right) < \\ < \frac{1}{2}(|a_{ii}|+|a_{ii}|) = |a_{ii}| \end{aligned}$$

It means, that for every given i the radius of Gershgorin disc is lower than the distance of disc center from the imaginary axis. That is why, according to Gershgorin theorem, and because $\mathbf{Q}$ is symmetric, all its eigenvalues are located in the negative part of real axis. From (17) it concluded by that the damping coefficient of $\mathbf{A}$ is negative. In consequence it means, that according to (9) $\|\mathrm{e}^{\mathbf{A}t}\|$ is bounded by a vanishing exponential function.

The last thing is to give the bound on $\mu(\mathbf{A})$. Because all eigenvalues of $\mathbf{Q}$ are located in some closed interval (or sum of closed intervals), the upper bound on the largest eigenvalue will be the end of the interval closest to the imaginary axis. The value α of this point is given by (16). □

These results can be used as an useful tool for observer tuning. One can chose $\mathbf{G}$ in such way, that the proposition 2 for matrix $\mathbf{A}-\mathbf{GC}$ will hold. The assumptions of proposition 2 are unfortunately very restrictive. It is however possible to guarantee the contraction property without fulfilling them. One such way, is to locate the Gershgorin discs of matrix $\mathbf{Q} = 0.5((\mathbf{A}-\mathbf{GC})^{\mathsf{T}} + \mathbf{A}-\mathbf{GC})$ in left open complex half plane. Such approach will be considered in the following section.

Applications

Example 1 (Armature controlled separately excited DC motor)**.** Separately excited DC motors have two winding circuits: field circuit, used for establishing a magnetic field with the motor, and armature circuit, containing a current, interacting with the magnetic field to produce the electric torque resulting in the mechanical rotation. These drives are extensively used in industrial variable speed applications, such as mills, hoists, coolers and machine tools [4, 10]. Diagram of separately excited DC motor is presented in the figure 2.

$$\begin{aligned} \frac{\mathrm{d}\theta(t)}{\mathrm{d}t} &= \omega(t) \\ \frac{\mathrm{d}i_{\mathrm{t}}(t)}{\mathrm{d}t} &= \frac{u_{\mathrm{t}}(t)}{L_{\mathrm{t}}} - \frac{R_{\mathrm{t}}}{L_{\mathrm{t}}} i_{\mathrm{t}}(t) - \frac{K_{\mathrm{m}}L_{\mathrm{w}}}{L_{\mathrm{t}}} i_{\mathrm{w}}(t)\omega(t) \\ \frac{\mathrm{d}i_{\mathrm{w}}(t)}{\mathrm{d}t} &= -\frac{R_{\mathrm{w}}}{L_{\mathrm{w}}} i_{\mathrm{w}}(t) + \frac{u_{\mathrm{w}}(t)}{L_{\mathrm{w}}} \\ \frac{\mathrm{d}\omega(t)}{\mathrm{d}t} &= \frac{L_{\mathrm{w}}K_{\mathrm{m}}}{J} i_{\mathrm{w}}(t) i_{\mathrm{t}}(t) - \frac{B_{\mathrm{v}}}{J}\omega(t) - \frac{M_{\mathrm{Z}}(t)}{J} \end{aligned} \tag{20}$$

In this paper the general model (20) will be used multiple times with different notations. These changes in notations are similar to one another, but in order to avoid the possible confusion they will be explicitly presented.

In most of applications the field current $i_{\mathrm{w}} = u_{\mathrm{w}}/R_{\mathrm{w}}$ is kept constant during operation of the motor - it is called armature control. In such situations separately excited, armature controlled DC motor can be modelled by the following system of differential equations (see for example [9]) which is linear - a special case of (1)

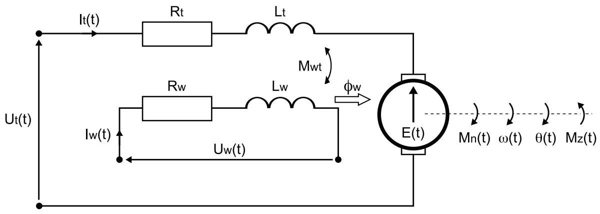

Fig. 2. Separately excited DC motor

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{Z}\tau(t) \tag{21}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & a & b \\ 0 & c & d \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{L_{\mathrm{t}}} \\ 0 \end{bmatrix} \qquad \mathbf{Z} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

where $u(t) = u_{\mathrm{t}}(t)$ is the armature voltage, $\mathbf{x} = [x_1\ x_2\ x_3]^{\mathsf{T}}$, $x_1 = \theta$ is the angular position, $x_2 = i_{\mathrm{t}}$ is the armature current and $x_3 = \omega$ is the angular velocity. Other parameters are given by the following formulas

$$a = -\frac{R_{\mathrm{t}}}{L_{\mathrm{t}}}, \quad b = -\frac{K_{\mathrm{m}}L_{\mathrm{w}}i_{\mathrm{w}}}{L_t}, \quad c = \frac{K_{\mathrm{m}}L_{\mathrm{w}}i_{\mathrm{w}}}{J}, \\ d = -\frac{B_{\mathrm{v}}}{J}, \quad \tau = \frac{M_{\mathrm{Z}}}{J}.$$

It is assumed, that armature current and angular position are measured so

$$\mathbf{y}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{x}(t) = \mathbf{C}\mathbf{x}(t) \tag{22}$$

In this situation observer (2) becomes a classical Luenberger observer and takes form

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}u(t) + \mathbf{Z}\tau(t) + \mathbf{G}(\mathbf{y}(t) - \mathbf{C}\hat{x}(t)) \tag{23}$$

with

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \end{bmatrix} \tag{24}$$

Estimation error of observer (23) evolves according to equation (3) with matrix

$$\mathbf{A} - \mathbf{GC} = \begin{bmatrix} -g_{11} & -g_{12} & 1 \\ -g_{21} & a-g_{22} & b \\ -g_{31} & c-g_{32} & d \end{bmatrix} \tag{25}$$

It can be easily verified, that for real motor parameters matrix (25) cannot fulfill the assumptions of proposition 2.

Considering matrix $\mathbf{Q} = 0.5((\mathbf{A}-\mathbf{GC})^{\mathsf{T}} + \mathbf{A}-\mathbf{GC})$ we get

$$\mathbf{Q} = \begin{bmatrix} -g_{11} & \dfrac{-g_{12}-g_{21}}{2} & \dfrac{1-g_{31}}{2} \\ \dfrac{-g_{12}-g_{21}}{2} & a-g_{22} & \dfrac{b+c-g_{32}}{2} \\ \dfrac{1-g_{31}}{2} & \dfrac{b+c-g_{32}}{2} & d \end{bmatrix}$$

It can be easily verified that setting gain matrix

$$\mathbf{G} = \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \\ 1 & b+c \end{bmatrix} \tag{26}$$

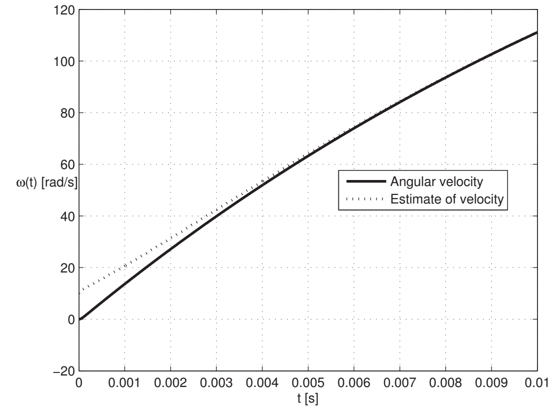

Fig. 3. State estimation in armature controlled separately excited DC motor - angular velocity and its estimate

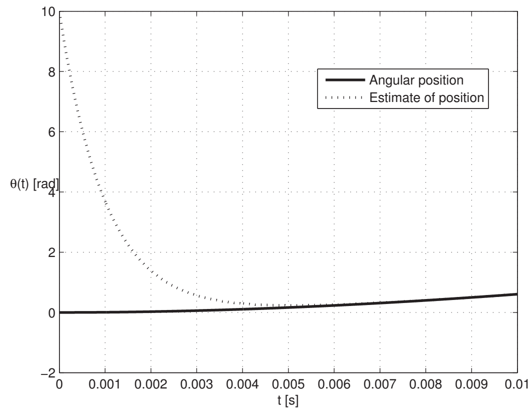

Fig. 4. State estimation in armature controlled separately excited DC motor - angular position and its estimate

with $g_{11} > 0$ and $g_{22} > a$ will locate the Gershgorin discs of $\mathbf{Q}$ in left open complex half plane. Two parameters are left which will not violate desired structure and can be used to tune the speed of observer convergence.

Arising observer (23) was tested with gain matrix (26), using motor parameters described in [11], through simulations. Tuning parameters were chosen as $g_{11} = 1000$ and $g_{22} = 0$, this choice was motivated by the desire of small disturbance augmentation, especially in fast and numerical instability prone electrical dynamics. The results of simulations are presented in figures 3–6. Especially interesting is the angular velocity plot in the figure (3). As it can be seen, initial error is quickly compensated. In the figure 6 norm of the estimation error is presented. It is divided by the initial error in order to show, that error vanishes exponentially.

Example 2 (Series DC motor)**.** A DC motor in which the field circuit is connected in series with the armature circuit is referred to as a series DC motor. In that case the motor current $i = i_{\mathrm{t}} = i_{\mathrm{w}}$. Due to this electrical connection, the torque produced by this motor is proportional to the square of the current (below field saturation), resulting in a motor that produces more torque per ampere of current than any other dc motor. Such a motor is used in applications that require high torque at low speed, such as subway trains and people movers. In fact, the series motor is the most widely used dc motor for electric traction applications [7]. A diagram for series DC motor is presented in the figure 7.

Series DC motor can be modeled by the following sys-

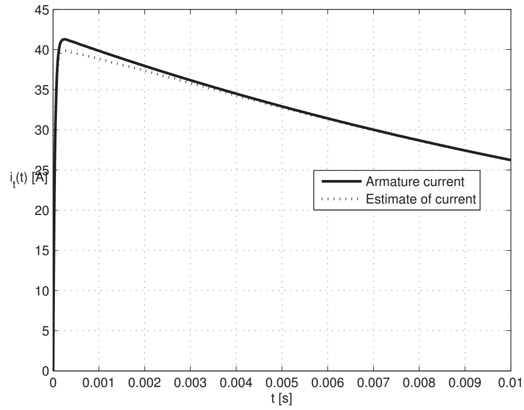

Fig. 5. State estimation in armature controlled separately excited DC motor - armature current and its estimate

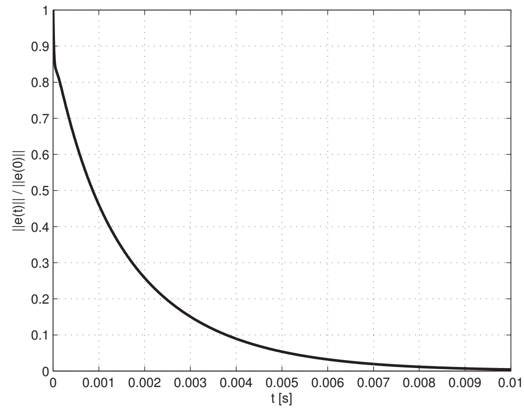

Fig. 6. State estimation in armature controlled separately excited DC motor - estimation error norm divided by initial error.

tem of differential equations (time argument was dropped)

$$\begin{aligned} \dot{x}_1(t) &= x_3(t) \\ \dot{x}_2(t) &= -a_1 x_2(t) - a_2 x_2(t) x_3(t) + b_1 u(t) \\ \dot{x}_3(t) &= c_1 x_2^2(t) - c_2 x_3(t) - \tau \end{aligned} \tag{27}$$

where $u(t)$ is the motor voltage, $\mathbf{x} = [x_1\ x_2\ x_3]^{\mathsf{T}}$, $x_1 = \theta$ is the angular position, $x_2 = i$ is the motor current and $x_3 = \omega$ is the angular velocity and parameters are:

$$\begin{aligned} a_1 &= \frac{R_{\mathrm{t}} + R_{\mathrm{w}}}{L_{\mathrm{t}} + L_{\mathrm{w}}}, & a_2 &= \frac{K_{\mathrm{m}} L_{\mathrm{w}}}{L_{\mathrm{t}} + L_{\mathrm{w}}}, & b_1 &= \frac{1}{L_{\mathrm{t}} + L_{\mathrm{w}}}, \\ c_1 &= \frac{K_{\mathrm{m}} L_{\mathrm{w}}}{J}, & c_2 &= \frac{B_{\mathrm{v}}}{J}, & \tau &= \frac{M_{\mathrm{Z}}}{J}. \end{aligned}$$

It is assumed that motor current and angular position are measured so

$$\mathbf{y}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{x}(t) = \mathbf{C}\mathbf{x}(t) \tag{28}$$

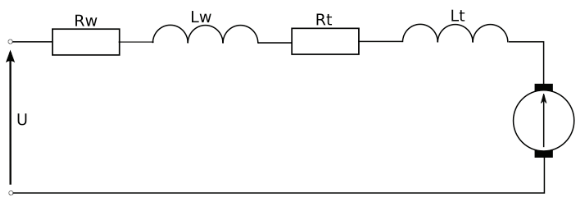

Fig. 7. Series DC motor

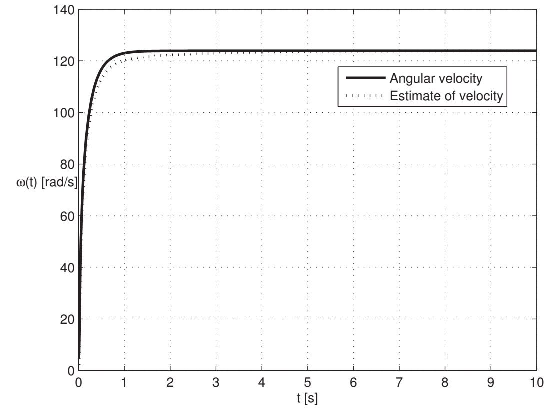

Fig. 8. State estimation in series DC motor - angular velocity and its estimate

The system (27) is transformable by the following change of coordinates (see [1, 6, 7, 18])

$$s_1 = x_1, \qquad s_2 = \ln x_2, \qquad s_3 = x_3$$

into a form (1) that allows output injection observer

$$\dot{\mathbf{s}}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{g}(s_2(t), u(t)) + \mathbf{Z}\tau$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -a_2 \\ 0 & 0 & -c_2 \end{bmatrix} \quad \mathbf{Z} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\mathbf{g}(s_2(t), u(t)) = \begin{bmatrix} 0 \\ -a_1 + b_1 \dfrac{u}{\exp(s_2)} \\ c_1 \exp(s_2)^2 \end{bmatrix}$$

which after the substitution of s_2 on the right side by $\ln x_2$ allows the construction of the following observer

$$\dot{\hat{\mathbf{s}}}(t) = \mathbf{A}\hat{\mathbf{s}}(t) + \mathbf{g}(\ln x_2(t), u(t)) + \mathbf{G} \begin{bmatrix} x_1 - \hat{s}_1 \\ \ln x_2 - \hat{s}_2 \end{bmatrix} \tag{29}$$

which is a nonlinear full rank observer estimating the angular position, natural logarithm of armature current and angular velocity. $\mathbf{G}$ has the same form as (24). Estimation error of the observer (29) evolves according to equation (3) with matrix

$$\mathbf{A} - \mathbf{G}\mathbf{C} = \begin{bmatrix} -g_{11} & -g_{12} & 1 \\ -g_{21} & -g_{22} & -a_2 \\ -g_{31} & -g_{32} & -c_2 \end{bmatrix} \tag{30}$$

The matrix $\mathbf{Q} = 0.5((\mathbf{A} - \mathbf{G}\mathbf{C})^{\mathsf{T}} + \mathbf{A} - \mathbf{G}\mathbf{C})$ takes form

$$\mathbf{Q} = \begin{bmatrix} -g_{11} & -\dfrac{g_{12} + g_{21}}{2} & \dfrac{1 - g_{31}}{2} \\ -\dfrac{g_{12} + g_{21}}{2} & -g_{22} & -\dfrac{a_2 + g_{32}}{2} \\ \dfrac{1 - g_{31}}{2} & -\dfrac{a_2 + g_{32}}{2} & -c_2 \end{bmatrix}$$

Similar to the previous example, a gain matrix, that will locate the Gershgorin discs of $\mathbf{Q}$ in left open half plane can be constructed. Such gain matrix has form

$$\mathbf{G} = \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \\ 1 & -a_2 \end{bmatrix} \tag{31}$$

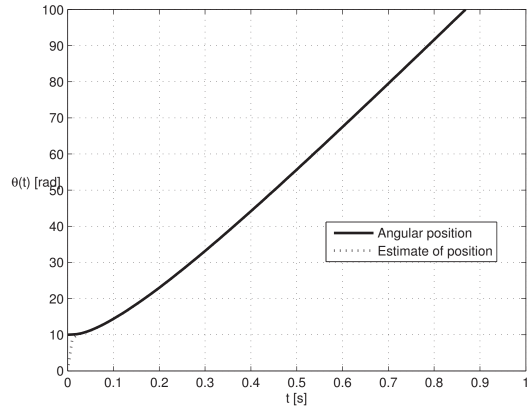

Fig. 9. State estimation in series DC motor - angular position and its estimate

with $g_{11} > 0$ and $g_{22} > 0$. Again g_{11} and g_{22} are tuning parameters, that do not violate the desired structure. Observer (29) was applied with gain (31). Tuning parameters were chosen as $g_{11} = g_{12} = 200$. Simulations were obtained for parameters from [7].

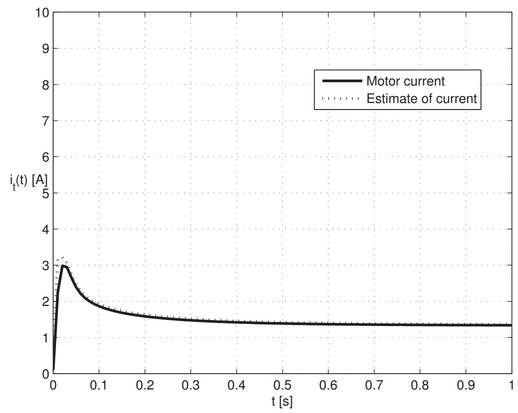

Fig. 10. State estimation in series DC motor - motor current and its estimate

Results are presented in figures 8–11. Because the observer convergence in the case of position and current is rapid, in order to increase legibility, the timescale of plots in figures 9 and 10 was shorter than in other plots. As it can be seen in the figure 11 the desired exponential vanishing of error was obtained. As earlier the norm of the error was divided by the norm of initial error. Speed estimation can be observed in the figure 8.

Example 3 (Field controlled separately excited DC motor - sensorless observer)**.** Different approach to control of separately excited DC motor is a so called field control. In this approach field current is also modified. Such approach can be used in order to improve the angular velocity, however at the cost of electromagnetic torque. In such situation, model (20) cannot be interpreted as linear. Let us assume that values of both currents in the DC motor (32) are available from the measurements, however there is no measurement of position.

Changing notation system (20) can be reformulated into

$$\begin{aligned} \dot{x}_1(t) &= -a_1 x_1(t) - a_2 x_2(t) x_3(t) + v_1(t) \\ \dot{x}_2(t) &= -b_1 x_2(t) + v_2(t) \\ \dot{x}_3(t) &= c_1 x_1(t) x_2(t) - c_2 x_3(t) - \tau(t) \end{aligned} \tag{32}$$

where $x_1 = i_{\mathrm{t}}$, $x_2 = i_2$, $x_3 = \omega$, $v_1 = u_{\mathrm{t}}/L_{\mathrm{t}}$, $v_2 = u_{\mathrm{w}}/L_{\mathrm{w}}$

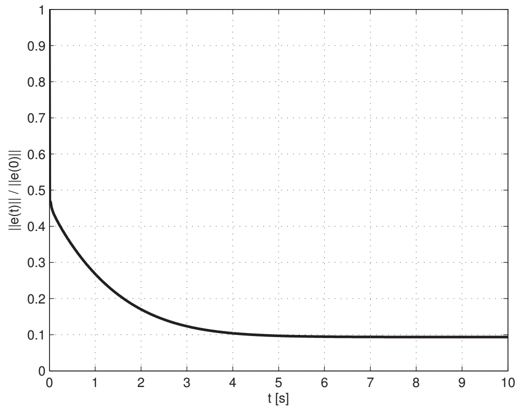

Fig. 11. State estimation in series DC motor - norm of estimation error divided by the initial error norm

and $\tau = M_Z/J$. And other parameters are

$$a_1 = \frac{R_t}{L_t}, \quad a_2 = \frac{K_m L_w}{L_t}, \quad b_1 = \frac{R_w}{L_w},$$
$$c_1 = \frac{L_w K_m}{J}, \quad c_2 = \frac{B_v}{J}, \quad \tau = \frac{M_Z}{J}.$$

The goal is to create formulate similar approach to for the separately excited DC motor in order to construct the observer with linear error dynamics for field controlled separately excited DC motor.

Let us introduce the following change of coordinates (see for example [3])

$$s_1 = \frac{x_1}{x_2}, \quad s_2 = x_2, \quad s_3 = x_3. \tag{33}$$

Under the change of coordinates (33) system (32) becomes

$$\dot{\mathbf{s}}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{g}(s_1(t), s_2(t), \mathbf{v}(t)) + \mathbf{Z}\tau(t) \tag{34}$$

where

$$\mathbf{A} = \begin{bmatrix} b_1 - a_1 & 0 & -a_2 \\ 0 & -b_1 & 0 \\ 0 & 0 & -c_2 \end{bmatrix}, \tag{35}$$

$$\mathbf{g}(s_1(t), s_2(t), \mathbf{v}(t)) = \begin{bmatrix} \frac{1}{s_2(t)} v_1(t) - \frac{s_1(t)}{s_2(t)} v_2(t) \\ v_2(t) \\ c_1 s_1(t) s_2^2(t) \end{bmatrix}, \tag{36}$$

$$\mathbf{Z} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \tag{37}$$

It should be noted, that because x_1 and x_2 were measurable also s_1 is measurable. Because of that the output is defined as

$$\mathbf{y}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{s}(t) = \mathbf{C}\mathbf{s}(t) \tag{38}$$

The following state observer is proposed

$$\begin{aligned} \dot{\hat{\mathbf{s}}}(t) &= \mathbf{A}\hat{\mathbf{s}}(t) + \mathbf{g}(s_1(t), s_2(t), \mathbf{v}(t)) + \\ &+ \mathbf{G}(\mathbf{y}(t) - \mathbf{C}\hat{\mathbf{s}}(t)) + \mathbf{Z}\tau(t) \end{aligned} \tag{39}$$

where $\mathbf{G}$ has the same form as (24). Estimation error of observer (23) evolves according to equation (3) with matrix

$$\mathbf{A} - \mathbf{GC} = \begin{bmatrix} b_1 - a_1 - g_{11} & -g_{12} & -a_2 \\ -g_{21} & -b_1 - g_{22} & 0 \\ -g_{31} & -g_{32} & -c_2 \end{bmatrix} \tag{40}$$

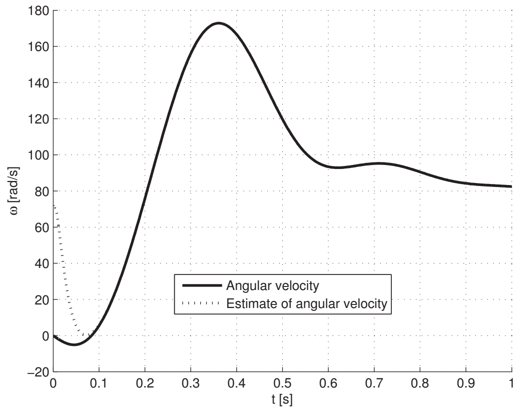

Fig. 12. State estimation in field controlled separately excited DC motor - angular velocity and its estimate

The matrix $\mathbf{Q} = 0.5((\mathbf{A} - \mathbf{GC})^{\mathsf{T}} + \mathbf{A} - \mathbf{GC})$ takes form

$$\mathbf{Q} = \begin{bmatrix} b_1 - a_1 - g_{11} & -\frac{g_{12} + g_{21}}{2} & -\frac{a_2 + g_{31}}{2} \\ -\frac{g_{12} + g_{21}}{2} & -b_1 - g_{22} & -\frac{g_{32}}{2} \\ -\frac{a_2 + g_{31}}{2} & -\frac{g_{32}}{2} & -c_2 \end{bmatrix}$$

As before a good choice of gain matrix $\mathbf{G}$ is

$$\mathbf{G} = \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \\ -a_2 & 0 \end{bmatrix} \tag{41}$$

as it guarantees that matrix $\mathbf{Q}$ has negative eigenvalues as long as $g_{11} > b_1 - a_1$ and $g_{22} > -b_1$. Because the matrix $\mathbf{A} - \mathbf{GC}$ has separated structure between modes for second state variable and the rest. That is why it was possible to obtain analytically values of g_{11} and g_{22} which not only guarantee the contraction semigroup property but also make the eigenvalues real and reasonably negative. The formulas for them are not presented, because they are relatively straightforward to obtain, but take a lot of space. For motor parameters from [9] eigenvalues of the error dynamics were

$$\lambda_{1,2} \approx -72, \quad \lambda_3 = -100.$$

Results of simulations are presented in figures 12 - 15. Velocity estimation is presented in the figure 12. As it can be seen the convergence is very quick and no overshoots of error are observed. In the figure 13 the estimate of auxiliary variable s_1 is presented along with its actual value computed by the division of currents. Both this variable and the field current in the figure 14 are estimated properly. The desired exponential vanishing of error can be observed in the figure 15, where norm of the estimation error divided by the norm of initial error is presented.

Conclusions

Some new results on state observer tuning were presented. Using the Gershgorin theorem, a sufficient condition for contracting observer error was proved. This result is

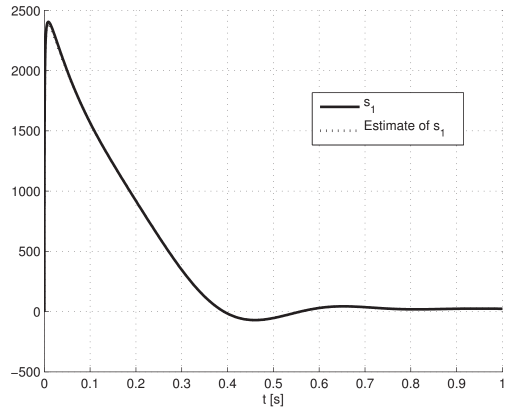

Fig. 13. State estimation in field controlled separately excited DC motor - s_1 and its estimate

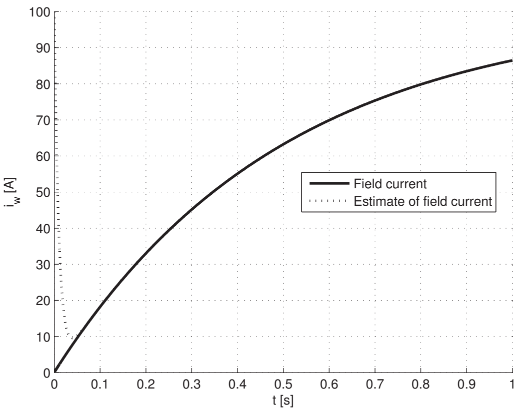

Fig. 14. State estimation in field controlled separately excited DC motor - field current and its estimate

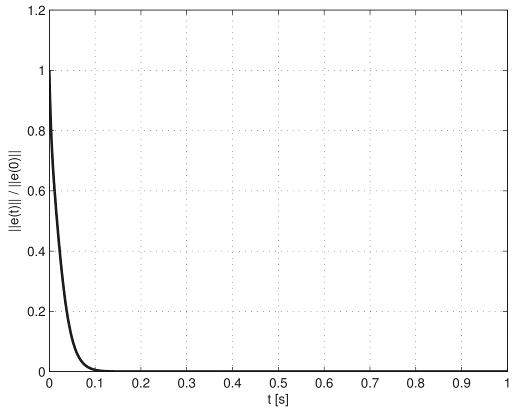

Fig. 15. State estimation in field controlled separately excited DC motor - norm of estimation error divided by the initial error norm.

unfortunately very conservative. However less conservative use of Gershgorin theorem allowed determining the gains of velocity observers for armature controlled separately excited DC motor, series DC motor and field controlled separately excited DC motor.

What is interesting, a method from linear control theory was used in order to choose parameters for observers of nonlinear systems. Specifically two observers for nonlinear DC motors were constructed and analysed.

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