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Full graph method of switched-capacitor circuit analysis

***Abstract**. Circuits with switched capacitors are described by a capacitance matrix and seeking voltage transfers then means calculating the ratio of algebraic supplements of this matrix. As there are also graph methods of circuit analysis in addition to algebraic methods, it is clearly possible in theory to carry out an analysis of the whole switched circuit in two-phase switching exclusively by the full graph method as well. In this case the summary graph can be constructed by the transformation graph or two graphs and can be simply evaluated by the Mason's relation.*

***Streszczenie.** Przedstawiono metodę wykorzystania grafów do analizy obwodów z przełączanym kondensatorem. Sumaryczny graf był konstruowany z wykorzystanie grafów transformacji i metody Masona. (**Wykorzystanie metody grafów do analizy układów z przełączanym kondensatorem**)*

Keywords: M-C graph, summary graph, transformation graph, two graphs.
Słowa kluczowe: obwody z przełączanym kondensatorem, teoria grafów.

Introduction

The analysis of the electric circuits is necessary not only for computing of circuit properties but also for understanding their principles. The computer methods are a powerful tool for symbolic analysis of circuit parameters. But they can be considered only tools.

Thanks to its clarity, the graphic method is extremely suitable even for understanding these networks. A clearly arranged set of transformation graphs derived for different types of switching circuits can be used for analyzing capacitor switched networks and, of course, for understanding them, too. The M-C signal flow graphs are used to design and analyze continuous time circuits as well as periodically switched linear circuits. Transformation graphs [1] and two-graphs [2, 3] are commonly used for assembling the final matrix considering all phases of solving circuits. The matrix is calculated by algebraic minors. It means that this method is a combination of both graph and numerical methods.

However, only selected circuits can be solved by graphs as described below, and also the circuits which contain switched capacitors. The summa graph can be constructed by transformation graphs (T-graphs) or by two-graphs as the resulting graph and is evaluated by Mason's formula.

Solving Circuits Considering Operational Amplifier with Diffraction of its Frequency Characteristics of Amplification by Transformation Graphs

This calculation will be illustrated by an example of solving a circuit with a switched capacitor whose schematic wiring diagram is shown in Fig.1.

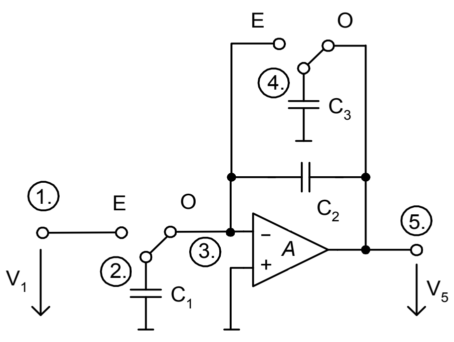

Fig.1. Circuit diagram for the solving

The phases are indexed as E for even and O for odd, because the nodes are numbered. The circuit in Fig.1 has five nodes; therefore the starting graph of the circuit in Fig.2 has five nodes, too.

The operational amplifier is connected to the third node by its inverting input and into the fifth node by its output. Consequently, the branch with the charge transfer of the transformation graph goes from node 3, the branch with the voltage transfer of the transformation graph enters node 5 and the branch with the voltage transfer expressing the final amplification of the operational amplifier by the value $1/A$ enters node 3 [1, 4].

Following this transformation graph, the capacity C_2 connected between nodes 3 and 5 then transforms into the resulting capacity of the amount $(1/A)C_2\text{-}C_2$. The capacitor C_2 is now connected to node 3 by one of its ends, therefore the inherent loop at this node has transfer C_2 and is transformed according to the equation $C = a^V.\widetilde{C}.a^Q.\alpha$, to the inherent loop $(1/A)C_2$, where $a^v = 1/A$.

The branch between nodes 3 and 5 with transfer C_2 is transformed to the inherent loop with transfer $-C_2$, because in the relation $C = a^V.\widetilde{C}.a^Q.\alpha$ [5, 6] is now $\alpha = -1$, as the branch of the original graph converts to the inherent loop in the resulting transformed graph. In the odd phase OO, by closing the switch, nodes 2 and 3 will be connected, which will be demonstrated in the graph by their transformation – uniting into a single node 2O.=3O. At the same time, this resulting node is the input node of the operational amplifier. Therefore the branch with charge transfer a^Q of the operational amplifier's transformation graph issues from this node.

In the remaining phases EO and OE, we start, according to the equation $C = a^V.\widetilde{C}.a^Q.\alpha$, along the branch with voltage transfer a^V from the resulting node to the original node. We enter back to the resulting node along the branch with charge transfer a^Q. The transformation graphs for all the four phases are in Fig.2.

The summary graph obtained from the partial transformed graphs from Fig.2 by the above mentioned procedure is then shown in Fig.3. First the results of the transformed graphs for EE and OO phases are plotted (in case of this example only) as three nodes: 1E.=2E., 3E.=5E., 3O.=5O. with the transfers C_1, $\frac{1}{A}(C_2 + C_3) - C_2$, $\frac{1}{A}(C_1 + C_2) - C_2$.

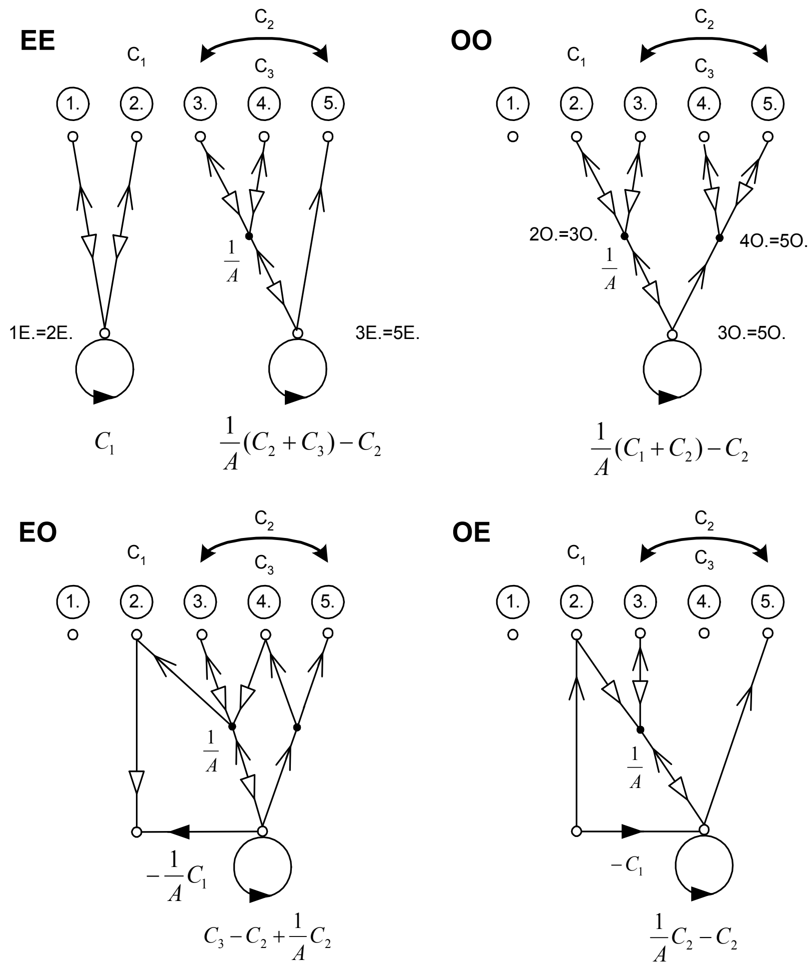

Fig.2 Transformation graphs for EE, OO, EO and OE phases

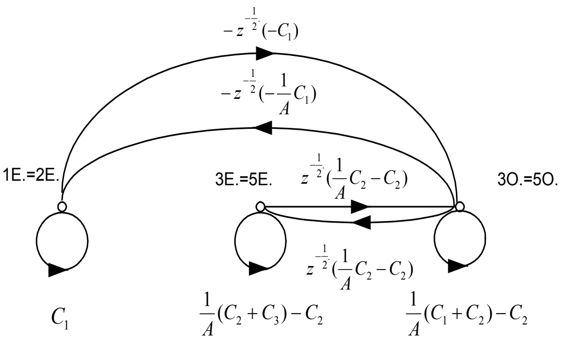

Fig.3 The summary graph of the SC circuit from Fig.1.

In the next step, the results of the transformed graph for the EO and OE phases multiplied by $-z^{-\frac{1}{2}}$ or $z^{-\frac{1}{2}}$ are then drawn between these nodes as branches, i.e. the branch with the transfer $-z^{-\frac{1}{2}}.(-C_1)$ between the nodes 1E.=2E. and 3O.=5O. and the branches with the transfers $z^{-\frac{1}{2}}.(\frac{1}{A}C_2 - C_2)$ between the nodes 3E.=5E. and 3O.=5O.

By evaluating this summary graph which is done by substitution into the Mason's formula $T = \frac{\sum p_{(i)}.\Delta_{(i)}}{V - \sum S^{(K)}.V^{(K)}}$ [5, 7] we get the following final results this way:

From the graph it is obvious that the entry node is 1E or the first node in the even phase, therefore there will only be transfers from the even phase of the first node. It is further evident from the graph that the exit (i.e. fourth) node exists here both in the even phase as 5E (5E.=3E.) and in the odd phase as 5O (5O.=3O.).

It is thus possible to express in numbers the following transfer $\frac{V_{4O}}{V_{1E}}$ by Mason's formula, for which it holds that:

(1) $$\frac{V_{5O}}{V_{1E}} = \frac{\sum p_{(i)}\Delta_{(i)}}{V - \sum S^{(K)}V^{(K)}} =$$

$$= \frac{-z^{-\frac{1}{2}}(-C_1)\left[\frac{1}{A}(C_2 + C_3) - C_2\right]}{\left[\frac{1}{A}(C_2 + C_3) - C_2\right]\left[\frac{1}{A}(C_1 + C_2) - C_2\right] - \left[z^{-\frac{1}{2}}(\frac{1}{A}C_2 - C_2)z^{-\frac{1}{2}}(\frac{1}{A}C_2 - C_2)\right]}$$

where V_{IJ} is the voltage and V or $V^{(K)}$ is the transfer of the loop, by cancelling out and removing the complex fractions.

By substituting $A = \frac{A_0.\omega_T}{\omega_T + sA}$ or $A = \frac{A_0.\omega_T}{\omega_T + sA}.\frac{\omega_2}{\omega_2 + s}$ we can calculate the frequency dependence.

Solving Circuits Considering Operational Amplifier with Diffraction of its Frequency Characteristics of Amplification by Two-Graphs

A solution of a circuit by the described method will be shown by solving a particular circuit with two switched capacitors, whose wiring diagram is in Fig.1. First we draw a partial diagram for the even phase and for the odd phase separately; these diagrams are shown in Fig.4. The node numbers in the squares are the numbers of the nodes of the charge graph and the node numbers in triangles are the numbers of the nodes of voltage graph after renumbering the nodes. For orientation there are the original numbers of nodes from the diagram in Fig.1.

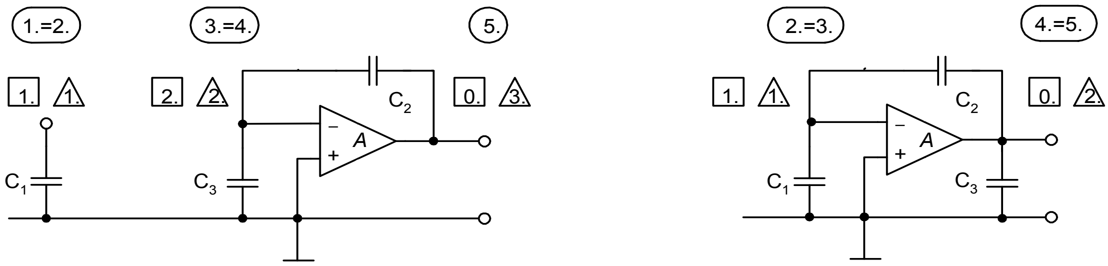

Fig.4. Diagrams of circuits for even and odd phases

For both even and odd phases it is necessary to draw a special voltage (V-graph) and charge (Q-graph) graphs. These graphs are in Fig.5 and include capacitors only. A summary MC-graph is now constructed by first finding the incomplete common skeletons of the V-graph and the Q-graph in the EE, OO, EO and OE phases by formula (2).

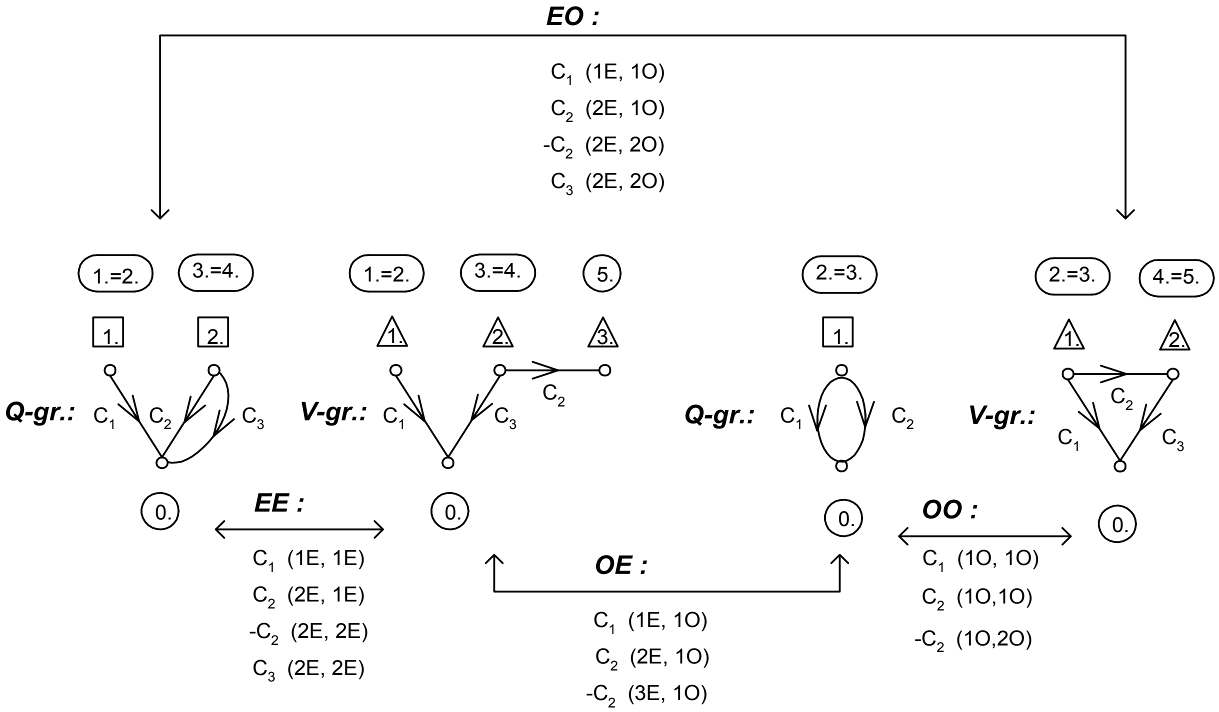

Fig.5. Two-graphs for even and odd phases

As described in [8], [9] determinant Δ of the matrix $\mathbf{Y}$ i.e. the matrix $\mathbf{C}$ in this case of switched capacitors circuits is given by (2)

$$\Delta = \sum_{C} \pm (product \quad of \quad capacitors) \tag{2}$$

where

$$C = \begin{Bmatrix} set \quad of \quad spanning \\ trees \quad of \quad V - graph \end{Bmatrix} \cap \begin{Bmatrix} set \quad of \quad spanning \\ trees \quad of \quad Q - graph \end{Bmatrix}.$$

In other words, there is a term in the expression for Δ corresponding to each spanning tree that is common to the charge (Q-gr.) and voltage graphs (V-gr.).

This principle is used for constructing summary MC-graph by two-graphs.

First we draw the nodes in both phases. Because graph in Fig.5 has three nodes (1.=2., 3.=4., 5.) in the even phase and two nodes (2.=3., 4.=5.) in the odd one, the summary MC-graph in Fig.6 has three nodes in the even phase and two nodes in the odd one, too.

In the second step, between thus obtained nodes 2E, 3E, 1O, 2O we will consequently draw branches and inherent loops of the VCVS, according to the rules stated in []. This step is illustrated in Fig.6.

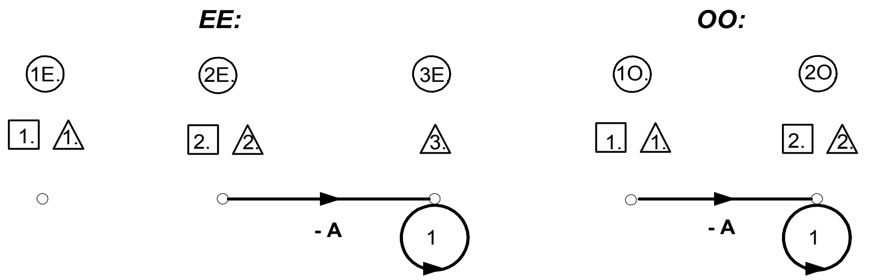

Fig.6 Graph with VCVS after second step

Between thus obtained nodes and branches, we will consequently draw branches and inherent loops as the results of finding the incomplete common skeletons of the V-graph and the Q-graph in the event phase and in the odd one.

After completing the summary MC-graph we will get the form shown in Fig.7.

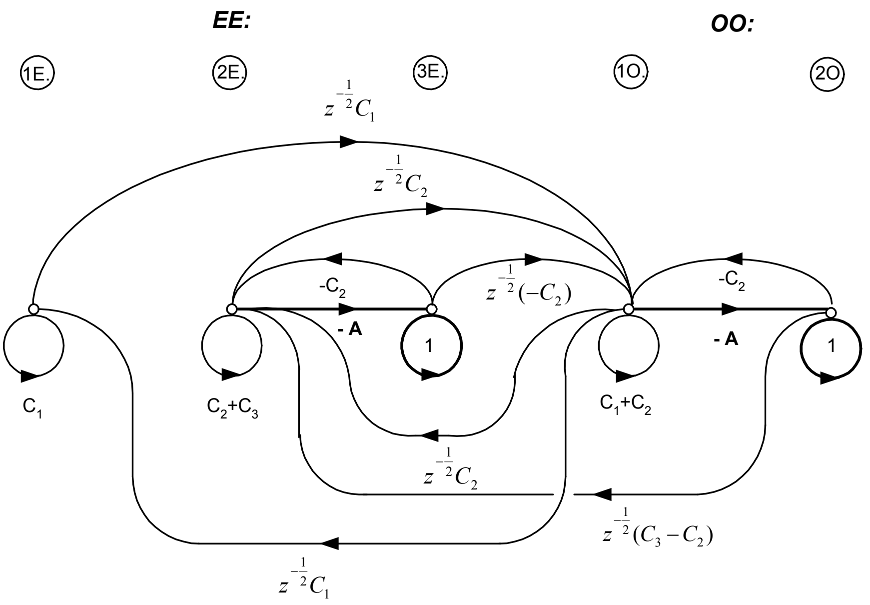

Fig.7 Resulting summary MC-graph

The voltage transfer for example $\frac{V_{5O}}{V_{1E}}$ (2) will now be obtained from a shortened graph, i.e. a graph in which there will not be the entry node's own loop and branches going into entry node, by means of the Mason rule [5, 7]:

$$\frac{V_{5O}}{V_{1E}} = \frac{\sum p_{(i)}\Delta_{(i)}}{V - \sum S^{(K)}V^{(K)}} = \quad (3)$$

$$= \frac{z^{-\frac{1}{2}}.C_1.(-A).\{(C_2+C_3).1-(-A).(-C_2)\}}{\begin{aligned}&(C_2+C_3).(C_1+C_2)-(-A).(-C_2).\{(C_1+C_2).1\}-(-A).(-C_2).\{(C_2+C_3).1\}-z^{-\frac{1}{2}}.C_2.z^{-\frac{1}{2}}.C_2.\{1.1\}\\&+z^{-\frac{1}{2}}.C_2.(-A).z^{-\frac{1}{2}}.(C_3-C_2).\{1\}+(-A).z^{-\frac{1}{2}}.(-C_2).z^{-\frac{1}{2}}.C_2-(-A).z^{-\frac{1}{2}}.(-C_2).(-A).z^{-\frac{1}{2}}.(C_3-C_2)\\&+(-A).(-C_2).(-A).(-C_2)\end{aligned}}$$

By substituting $A = \frac{A_0.\omega_T}{\omega_T + sA}$ or $A = \frac{A_0.\omega_T}{\omega_T + sA}.\frac{\omega_2}{\omega_2 + s}$

we can calculate the frequency dependence.

This formula (3) is much more complicated than (1). However, the results must be the same. Therefore, quite a complicated adjustment is needed. After this relatively laborious adjustment (3) the result will be the same as (1).

Conclusion

While in case of using the graph method, first a graph was indicated, then a transformation graph was plotted, and from its results a summary graph was drawn and evaluated by the Mason's rule, after which the result was obtained by an easy simplification, in case of solving by the matrix calculus the procedure was much more complicated. First a partial capacitance matrix had to be composed, in the next step it was modified by an operational amplifier. From four matrices obtained by this a capacitance matrix was constructed and was reduced by the activity of switches; from the reduced matrix three algebraic complements were made up and they had to be expressed by means of an expansion because they were of a higher grade than 3. After an elaborate simplification in four steps, the same result was reached. In case of using the graph method a graph was solved, but modified nodal method is rather difficult.

While in case of using the graph method, first a graph was indicated, then voltage and charge graphs were plotted and from these two-graphs a summary graph was drawn and evaluated by the Mason´s rule, after which the result was obtained, in case of solving by the matrix calculus the procedure was much more complicated. First a partial capacitance matrix had to be composed, in the next step it was modified by an operational amplifier. From four matrices obtained by this, a capacitance matrix was constructed and was reduced by the activity of switches.

In literature [2], 3, 8] etc. we can find a description of a solution procedure by a method of two graphs, which leads to the construction of a matrix, from which the desired voltage transfers for corresponding phases are calculated by the method of algebraic complements.

However, the above described method enables us to carry out the whole solution procedure in the graphic form. With regard to the evaluation of the resulting summary MC graph it is suitable for manual solution of rather simple circuits. Its certain disadvantage is the renumbering of node in different ways for the charge graph and for the voltage graph, which may seem complicated.

While in case of using a transformation graph for plotting the summary graph, the result is gained from Mason's formula without simplification, in case of the two graphs evaluation is more complicated, because simplification of the result from Mason's formula is necessary. The t-graph is much more advantageous for the assembling of the summa graph.

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