Zdenek HRUBOS

Brno University of Technology

Novel circuit implementation of universal and fully analog chaotic oscillator

Abstract. *It has been shown that the so-called dynamical system of class C can be used for studying chaotic motions. Even though we are speaking about a deterministic dynamical systems, theirs future evolution can hardly be predicted. It is due to the extreme sensitivity on tiny changes in the initial conditions. Everything follows from the general theory of the chaotic dynamics – one must have a large number of unstable initial conditions. This also suggests that a little change in the values of variable parameters can cause a dramatic change of the state space attractor's shape. This was also the main motivations to the construction of the optimized dynamical system of class C with piecewise-linear (PWL) feedback. Based on the given system of first-order differential equations, a fully analog chaotic oscillator works in hybrid mode has been discovered for laboratory measurements. Main contribution of this work is right in circuitry implementation of a fully analog chaotic oscillator with a new available active elements. The advantage is immediately evident. The smaller number of active elements is in the whole circuit. The proper function of the final circuit structure has been verified by means of the PSpice simulator as well as by a practical experiments on the real oscillator. The corresponding results are also given.*

Streszczenie. *Tak zwany dynamiczny system klasy C może być użyty do studiowania procesów chaotycznych. W pracy do badań zastosowano system klasy C z liniowym sprzężeniem zwrotnym PWL. Uzyskano analogowy generator przebiegów chaotycznych. Zaletą jego jest mała liczba elementów aktywnych. Układ sprawdzono eksperymentalnie i przez symulacje. (**Nowy układ uniwersalnego, analogowego generatora przebiegów chaotycznych**).*

Keywords: dynamic system, deterministic, chaos, attractor, universal, fully analog oscillator, class C, differential equations, optimized systems design, integrator synthesis, functional blocks, operational amplifiers, simulation, plane projection, time behavior, laboratory sample
Słowa kluczowe: przebiegi chaotyczne, generator, klasa C

Introduction

It is well known that many real physical systems with some evolution in the time domain can be described with the general system of non-linear differential equations. This case already represents some degree of idealization, describing the problem with a finite states number. These systems produce common types of behavior as limit cycles or fixed points. In addition produce complicated complex behavior known as chaos. This effect, which is characterized by a strange attractor in the state space, has been observed in chemistry, biology, optics, electronics and many other scientific disciplines [7]. In this article we refer circuit realization of dynamic system with using integrator synthesis. The integrator synthesis seems as a straightforward method for circuitry realization. Method is based on the knowledge of a dynamic system mathematical model. The advantage of this method is in the low number of necessary functional blocks; inverting integrator, differential (or summing) amplifier and two-port blocks with desired nonlinear transfer function. In contrast the large number of active and passive elements is disadvantage. This problem we can partially eliminate in current mode. However, here is an acute shortage of commercially available functional blocks processing input/output signals in current mode (transconductance amplifiers, current conveyors and their hypothetical mutations) [1]. Theoretically nor practically is not possible to create an electronic circuit representing all dynamic systems. In our paper we chose dynamical systems of class C because their type of saturation nonlinear global feedback function is easily electronically realizable. Main contribution of this work is right in circuitry implementation of a fully analog chaotic oscillator with a new available active elements. The advantage is immediately evident. The smaller number of active elements is in the whole circuit.

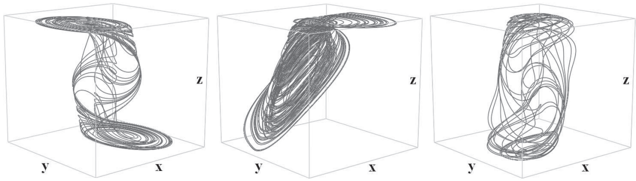

Fig. 1. 3-D visualization of numerically integrated system (1) for three different configurations from Tab. 1 and initial condition $ic = [0.05, 0, 0]^T$. DS-ECEC (left), CH_2-ECEC (center), CH_3-ECEC (right).

Mathematical model

We are consider dynamic system which can be written in following matrix form:

$$\dot{\mathbf{x}} = \mathbf{A}x + \mathbf{b}h\left(\mathbf{w}^T x\right), \quad (1)$$

where

$$\mathbf{A} \in \Re^{3x3}, \quad \mathbf{b} \in \Re^3, \quad \mathbf{w} \in \Re^3, \quad (2)$$

and $h()$ is a scalar odd-symmetric piecewise linear (PWL) function. In this case the scalar saturation nonlinear function

$$h\left(\mathbf{w}^T x\right) = \frac{1}{2}\left(|\mathbf{w}^T x + 1| - |\mathbf{w}^T x - 1|\right), \quad (3)$$

separates the state space by two parallel boundary planes into the three affine regions

$$\begin{aligned} \dot{x} &= \mathbf{A}_0 x, \ \mathrm{D}_0 \text{ region} \\ \dot{x} &= \mathbf{A}_1 x \pm \mathbf{b}, \ \mathrm{D}_{\pm 1} \text{ regions.} \end{aligned} \quad (4)$$

Therefore, the same corresponding eigenvalues of the characteristic polynomial describe dynamical motion in both outer segments as well as the geometry of the vector field. From the previous equations we can conclude that the vector field is symmetrical with respect to the origin [6]. Practically experiments proved that a function (PWL) can be smooth. Therefore, practical realization is considerably simplified, for example with using diodes etc. The PWL approximation is much more suitable because it leads to the linear section of the state space and allows us to generate qualitatively equivalent PWL dynamical systems of Class C [2], [3].

We can design the third-order model with Jordan's state matrix including complex decomposed second-order submatrix. However, we need know results of the second-order model similarity transformation to higher-order model [5]. Assume that one pair of the eigenvalues is complex conjugate and one eigenvalue is real. This definition applies for both outer respectively inner regions of the elementary PWL function (3) i.e.

$$v_{1,2} = v' \pm jv'', v_3 : \text{real}; \quad \mu_{1,2} = \mu' \pm j\mu'', \mu_3 : \text{real}. \quad (5)$$

Subsequently the state matrix and the vectors in (1) we rewrite in the following form

$$\mathbf{A} = \left[\begin{array}{cc|c} v' & -v'' & -\mu' + v' \\ v'' & v' & \frac{(\mu'-v')^2}{v''-\mu''K} \\ \hline 0 & 0 & v_3 \end{array} \right], \tag{6}$$

$$\mathbf{b} = \left[\begin{array}{c} \mu' - v' \\ \frac{(\mu'-v')^2}{v''-\mu''K} \\ \mu_3 - v_3 \end{array} \right], \mathbf{w} = \left[\begin{array}{c} 1 \\ \frac{v''-\mu''K}{\mu'-v'} \\ 1 \end{array} \right], \tag{7}$$

and the state matrix related with the inner region has the Jordan's matrix form

$$\mathbf{A}_0 = \left[\begin{array}{cc|c} \mu' & -\mu''K & 0 \\ \mu''K^{-1} & \mu' & 0 \\ \hline \mu_3 - v_3 & (\mu_3 - v_3)\frac{v''-\mu''K}{\mu'-v'} & \mu_3 \end{array} \right]. \tag{8}$$

The optimization coeficient K of similarity transformation we can express as the real root of the quadratic equation

$$\begin{gathered} K^2 - 2K(M+1) + 1 = 0, \\ \Downarrow \\ K_{1,2} = 1 + M \pm \sqrt{M(M+2)}, \end{gathered} \tag{9}$$

where the parameter M is described in the following form

$$M = \frac{(\mu' - v')^2 + (\mu'' - v'')^2}{2\mu''v''} > 0, \; (\mu'', v'' \neq 0). \tag{10}$$

This system model have a very low eigenvalue sensitivity in both the outer and inner regions of the PWL feedback function [2]. Using a simple transformation of state variables we can describe behavior of the dynamic system of class C in the four configurations.

CDCD - dynamic system contain a complex decomposed second-order submatrix,

$$\begin{gathered} e_{11} = v'', \; e_{12} = -u', \; e_{13} = -v', \; e_{21} = -v'', \\ e_{22} = -v', \; e_{23} = -\frac{(\mu'-v')^2}{v''-\mu''K}, \; e_{31} = v_3 \\ e_{32} = -u_3, \; e = \frac{v''-\mu''K}{\mu'-v'}, \end{gathered} \tag{11}$$

ECEC - dynamic system contain elementary canonically decomposed second-order submatrix,

$$\begin{gathered} e_{11} = 1, \; e_{12} = -2u', \; e_{13} = -2v', \\ e_{21} = -{v'}^2 - {v''}^2, \; e_{22} = 0, \\ e_{23} = -\frac{(\mu'-v')^2}{v''-\mu''K}, \; e_{31} = v_3, \; e_{32} = -u_3, e = 0., \end{gathered} \tag{12}$$

Last two configurations are combination of two previous.

ECCD

$$\begin{gathered} e_{11} = 1, \; e_{12} = -u', \; e_{13} = -2v', \\ e_{21} = -{v'}^2 - {v''}^2, \; e_{22} = 0, \\ e_{23} = -\frac{(\mu'-v')^2}{v''-\mu''K}, \; e_{31} = v_3, \; e_{32} = -u_3, \\ e = \frac{v''-\mu''K}{\mu'-v'}. \end{gathered} \tag{13}$$

CDEC

$$\begin{gathered} e_{11} = kv'', \; e_{12} = -2u', \; e_{13} = -v', \\ e_{21} = -K^{-1}v'', \; e_{22} = -v', \\ e_{23} = -\frac{(\mu'-v')^2}{v''-\mu''K}, \; e_{31} = v_3, \; e_{32} = -u_3, \\ e = \frac{v''-\mu''K}{\mu'-v'}. \end{gathered} \tag{14}$$

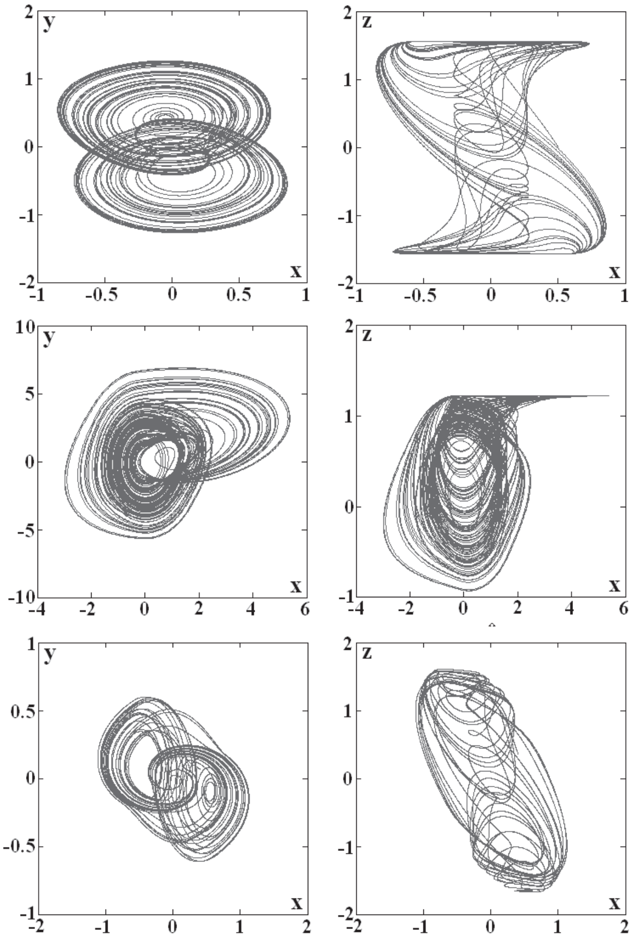

Fig. 2. Numerical analysis of three different system (1) configurations from Tab. 1 - projection X-Y. Initial condition $ic = [0.05, 0, 0]^T$, DS-ECEC (top), CH_2-ECEC (center), CH_3-ECEC (bottom).

Finally the complete state equations of the optimized third-order PWL autonomous system for circuit realization are given as

$$\begin{aligned} -RC\frac{du_1}{d\tau} &= \varepsilon_{11}u_2 + \varepsilon_{12}\left[(u_1 + \varepsilon u_2 + u_3) - u_3\right] + \\ &+\varepsilon_{13}\left[u_1 + u_3 - h\left(u_1 + \varepsilon u_2 + u_3\right)\right], \\ -RC\frac{du_2}{d\tau} &= \varepsilon_{21}u_1 + \varepsilon_{22}u_2 + \\ &+\varepsilon_{23}\left[h\left(u_1 + \varepsilon u_2 + u_3\right) - u_3\right], \\ -RC\frac{du_3}{d\tau} &= \varepsilon_{31}\left[u_3 - h\left(u_1 + \varepsilon u_2 + u_3\right)\right] + \\ &+\varepsilon_{23}\left[h\left(u_1 + \varepsilon u_2 + u_3\right)\right]. \end{aligned} \tag{15}$$

Numerical analysis

Embedded Runge-Kutta fourth order method in MathCAD environment was used for numerical integration of differential equation system. Parameters of numerical integration are consistent. Time interval $t(0, 500)$ and step $\Delta t = 10^{-2}$. The plane 2-D and 3-D projections associated with a numerical integration of the mathematical model shows Fig. 1, resp. Fig. 2.

Bifurcation diagram and Poincaré section

Bifurcation diagram and Poincaré section are visualisation methods presented behavior of the dynamical systems. Bifurcation is define as a qualitative change in the dynamical behavior of a system, of its phase portrait, as one or more parameters are changed. Figure 3 (left side) shows bifurcation diagram for three chosen dynamic system configurations, where e_{32} is adopted as a bifurcation parameter. Any point in the parameter set, where the behavior of dynamical system is unstable is called bifurcation point, and the set of these points is called a bifurcation set. For the sufficiently high resolution graph it is necessary to use very small parameter step as well as to numerically integrate the state space trajectory for the time long enough.This set can contain infinite number of points but usually has zero measure [7].

The Poincaré section are other very useful to the qualitative analysis of nonlinear dynamical systems, since they provide a lower dimensional system that still captures the essential features of the original dynamics [11]. In the case of nonautonomous systems, the Poincare section of a periodic solution is calculated easily because the Poincaré mapping can be defined as a mapping whose period is identical to the period of forcing signal. While the case of autonomous systems, the period of the limit cycle is changed as the parameter changes, so it is not suitable to analyze the limit cycle just as nonautonomous system. Therefore we should provide a cross-section called the Poincaré section and define the corresponding Poincaré mapping. This method implicitly requires the accurate location of the point at which the periodic orbit started from the cross-section returns. The transition surface must be perpendicular to the flow [13]. Figure 3 (right side) shows the Poincaré section of three chosen dynamic system configurations.

Circuitry realization

Main contribution of this work is right in circuitry implementation of a fully analog chaotic oscillator. Circuitry realization is novel in the sense, that this realization using new available active elements (AD844 [17], MAX435 [18]), which are simplifies whole circuitry solution. Now let's focus attention right on the circuitry implementation based on the equation (15). Integrator synthesis was used [10], [14], [15], [16]. The schematic in Fig. 5 shows oscillator with three integrators, one summing amplifier, one PWL function and works in voltage mode. An operational amplifiers TL084, monolithic operational amplifiers AD844 and wideband transconductance amplifier MAX435 were used for circuitry implementation of mathematical model. The PWL function forms a connection of dual-diode limiters with operational amplifiers TL084. Values of used passive elements were chosen $C_1 = C_2 = C_3 = 100nF,\ R_1 - \ R_{24} = 1k\Omega,\ R_{25} = 2.7k\Omega,\ R_{26}\ and\ R_{27} = 10k\Omega,\ R_{28}\ and\ R_{29} = 140k\Omega.$

Block in Fig. 4 represents a dynamic system parameter e_x and can be considered as a bifurcation parameter. Circuit is powered by symmetrical $\pm 5V$ voltage source. There were

Table 1. Parameteres of different dynamical systems.

*	e_{11}	e_{12}	e_{13}	e_{21}	e_{22}	e_{23}	e_{31}	e_{32}	e	configuration
DS	1	0,319	-0,061	1	-0,061	-0,358	-1,29	-0,728	-1,062	CDCD
DS	1	0,638	-1,122	-1,004	0	0,092	-1,29	-0,728	0	ECEC
DS	1	0,319	-0,122	-1,004	0	-0,35	-1,29	-0,728	-0,911	ECCD
DS	0,844	0,638	-0,061	-1,185	-0,061	0,273	-1,29	-0,728	0,223	CDEC
CH_1	1	-0,299	-0,3	-1	-0,3	-99	-3	-0,202	$-1{,}01.10^{-5}$	CDCD
CH_1	1	-0,598	-0,6	-1,09	0	-99	-3	-0,202	0	ECEC
CH_1	0,999	-0,598	-0,3	-1,001	-0,3	-99,09	-3	-0,202	$-3{,}028.10^{-3}$	CDEC
CH_2	1	-0,058	-0,29	-1	-0,29	-6,79	-1,33	-0,3	-0,034	CDCD
CH_2	1	-0,115	-0,58	-1,084	0	-6,648	-1,33	-0,3	0	ECEC
CH_3	1	0,136	-0,045	-1	-0,045	0,839	-0,409	-0,272	0,216	CDCD
CH_3	1	0,272	-0,091	-1,002	0	0,816	-0,409	-0,272	0	ECEC
CH_4	1	0,049	-0,034	-1	-0,034	0,08	-1,04	-1,474	1,042	CDCD
CH_4	1	0,097	-0,069	-1,001	0	$-1{,}195.10^{-3}$	-1,04	-1,474	0	ECEC
CH_5	1	-0,024	0,124	-1	0,124	0,883	0,277	0,45	-0,167	CDCD
CH_5	1	-0,047	0,248	-1,015	0	0,895	0,277	0,45	0	ECEC
CH_7	1	-0,058	-0,29	-1	-0,29	-6,79	-1,33	-0,44	-0,034	CDCD
CH_7	1	-0,116	-0,58	-1,084	0	-6,648	-1,33	-0,44	0	ECEC
DS … double scroll *CH_i … chaotický atraktor										

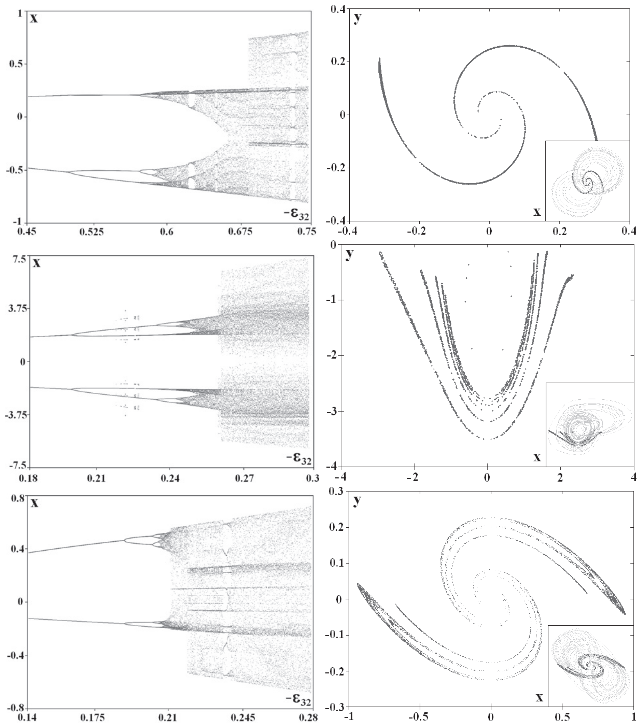

Fig. 3. Bifurcaion diagrams (left) and Poincaré section (right) of three selected system (1) configurations from Tab. 1, where e_{32} is adopted as a bifurcation parameter. DS-ECEC (top), CH_2-ECEC (center), CH_3-ECEC (bottom).

used identical values of passive elements from E24 product line for simulation purposes and also for experimental measurements. State variables represented output voltage of integrators and therefore are easily measurable. The parasitic properties of the active components are not critical because we adjusted time constant (RC) in the low-frequency band.

Simulation and measurement results

The circuitry implementation functionality was first successfully tested in PSpice simulator. Figure 6 shows simulated plane projections associated with a designed. Correct function of the dynamical system was also verified experimentally. Plane projections of the selected signals were measured by means of HP 54603B oscilloscope. Figure 7 shows photo of plane projection. These measured results are in a very good accordance with theoretical expectations, i.e. numerical integration of the given mathematical model. During experimental measurement we have verified that the time constant can not be much lower than $\tau\ =\ 10\mu s.$

Conclusion

The advantage of the chaotic oscillator, proposed in this paper, is an easy relation between parameters of the mathematical model (equivalent eigenvalues) and the circuit elements. Main contribution of this work is right in circuitry implementation of a fully analog chaotic oscillator with a new available active elements. The advantage is immediately evident. The smaller number of active elements is in the whole circuit. Moreover, a huge number of the simulation and laboratory experiments proved a good agreement between numerical integration and practical simulation and measurement; see Fig. 6

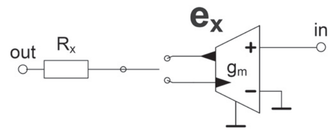

Fig. 4. Example of block for setting system parameters e_x.

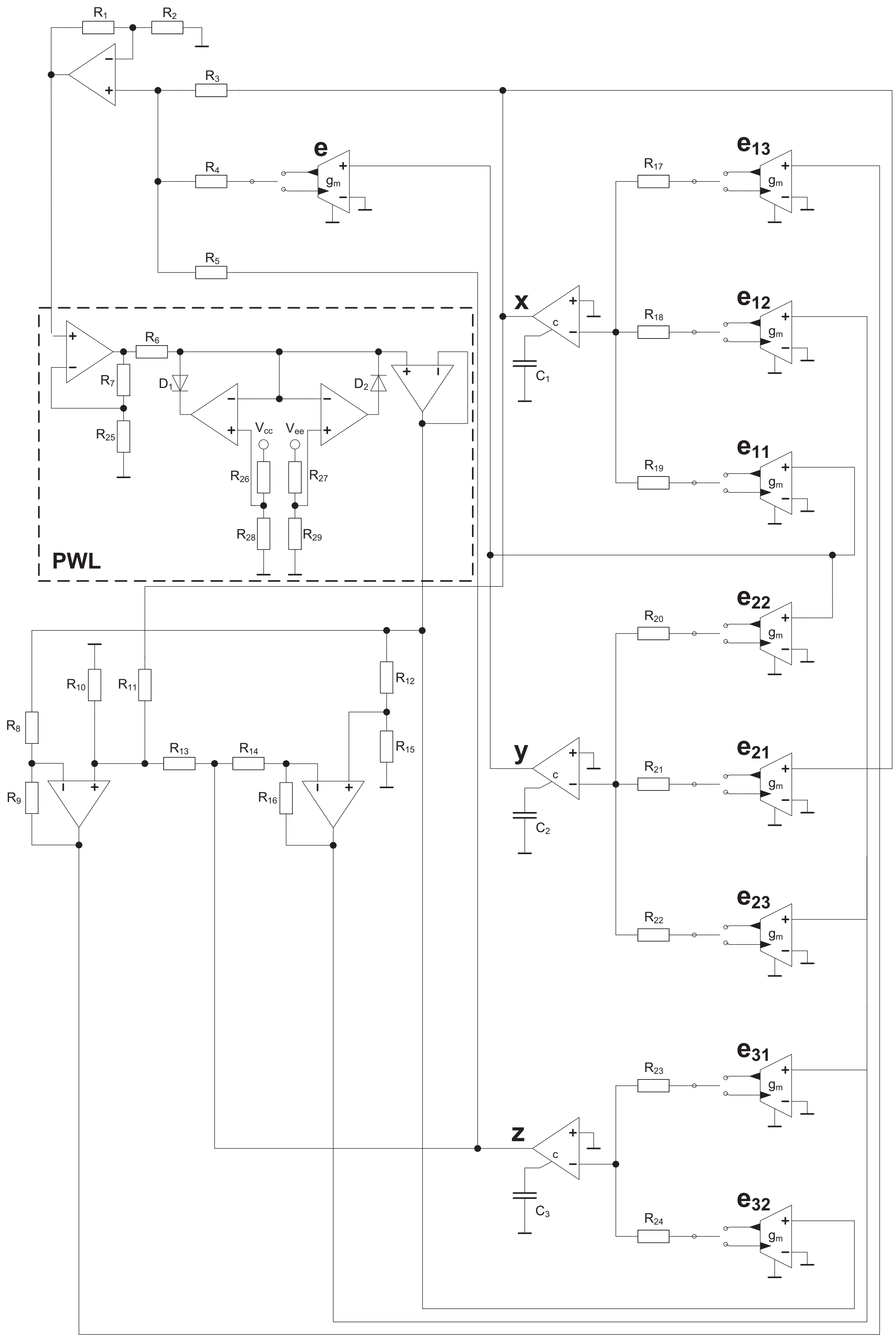

Fig. 5. Schematic of universal and fully analog oscillator.

and Fig. 7. The oscillator proved the versatility of the circuit synthesis based on the integrator block schematic. The rather extensive list of the references provides an opportunity to study chaotic motion from many different viewpoints. The algebraically simple dynamical systems with minimum terms [7] are suitable for practical training in the analog chaotic oscillator design. Circuitry realization of such systems using only analog elements is bounded by dynamical ranges and represents almost impossible task. The presented approach can be utilized to investigate such problems.

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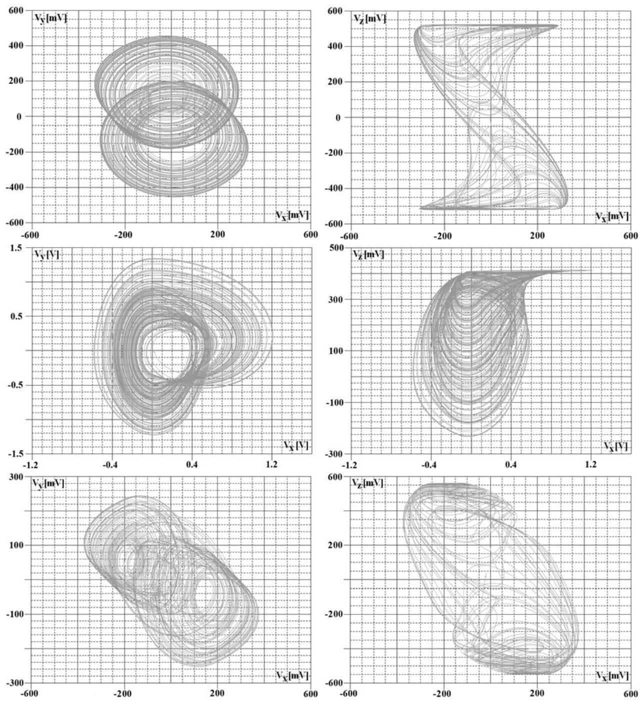

Fig. 6. Simulation results of chaotic attractors for three different system (1) configurations from Tab. 1 - projection X-Y. Initial condition $ic = [0.05, 0, 0]^T$, DS-ECEC (top), CH_2-ECEC (center), CH_3-ECEC (bottom).

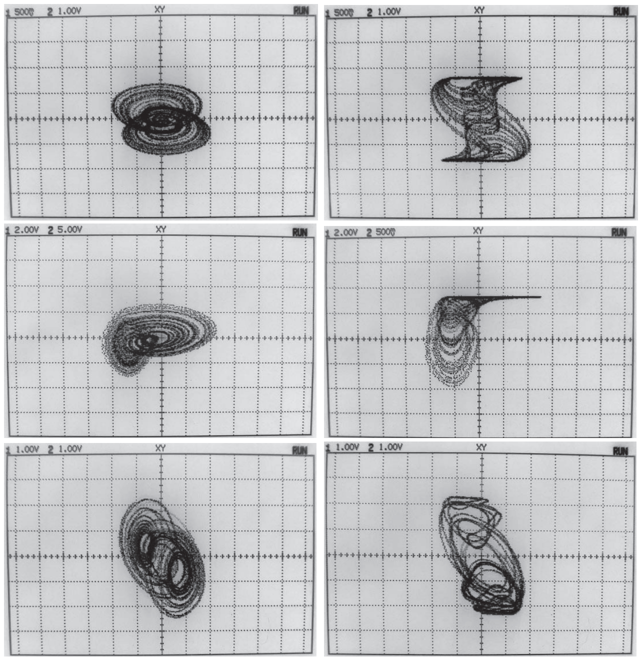

Fig. 7. Measured results of chaotic attractors for three different system (1) configurations from Tab. 1 - projection X-Y. Initial condition $ic = [0.05, 0, 0]^T$, DS-ECEC (top), CH_2-ECEC (center), CH_3-ECEC (bottom).

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Authors: MSc. Zdenek Hrubos Department of Radio Electronics, Faculty of Electrical Engineering and Communication, Brno University of Technology, ul. Purkynova 118, 612 00 Brno, Czech Republic, email: xhrubo00@stud.feec.vutbr.cz