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Defining and computing equivalent inductances of gapped iron core reactors

***Abstract**. The paper revisits the fundamental definitions and explores computational alternatives of equivalent inductances of gapped iron core reactors. Unlike in a transformer, the physical meaning of component inductances of a reactor is subject to uncertainty and open to different interpretations, leading to various equivalent circuits. It is argued that a definition based on flux distribution may not be reliable and calculations based on energy or co-energy are thus preferable.*

***Streszczenie.** W artykule omówiono podstawowe definicje i metody obliczania indukcyjności zastępczych dławików ze szczelinami powietrznymi. Używane są różne schematy zastępcze zależne od interpretacji fizycznej. Zasugerowano definicje oparte na energii i koenergii systemu zamiast rozkładu strumienia magnetycznego. (**Definicje i obliczanie indukcyjności zastępczych dławików ze szczelinami powietrznymi**)*

Keywords: magnetic fields, equivalent inductances, gapped iron core reactors.
Słowa kluczowe: pole magnetyczne, indukcyjności zastępcze, dławiki ze szczelinami powietrznymi

The Equivalent Inductance Model of a Gapped Iron Core Reactor

A conventional approach to defining equivalent inductances of a reactor normally relies on a similar argument as for a transformer and relates the component inductances to different portions or paths of the magnetic flux. Unlike in a transformer, however, such paths are less clearly defined and the notions of 'main' or 'leakage' flux are subject to uncertainty as there is only one winding and thus mutual coupling cannot be used as the criterion. Moreover, in a gapped iron core design, the flux often leaves and then re-enters the core, thus the concept of which 'flux tube' forms the main and which the leakage portion is further confused. Notwithstanding these difficulties, a definition based on magnetic flux is common as it provides clarity of the concept; this is illustrated by Fig. 1a where different flux paths are identified as a convenient approximation. This leads to various possible equivalent circuits with component inductances connected in series or parallel, with the most popular definition shown in Fig. 1b [1, 2]. In practice, the inductance associated with the flux in the iron core may often be neglected since it is very small (in a series model) or very large (in a parallel circuit) as the magnetic core is rarely saturated.

(a)

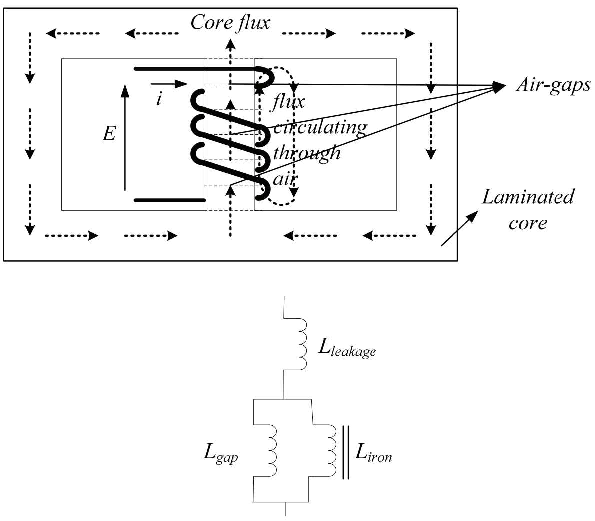

Fig. 1. Simplified definition of flux paths (a); the most popular equivalent circuit (b) [1, 2].

The simplistic definitions based on Fig. 1a are attractive but unfortunately also a source of difficulties in calculating the actual values. As mentioned already, the magnetic flux is unlikely to follow the strictly defined paths inside or outside the iron core and the analytical models in literature are often reliant on various empirical correction coefficients. The realistic field patterns are far more complicated as illustrated in Figs. 2a and 2b.

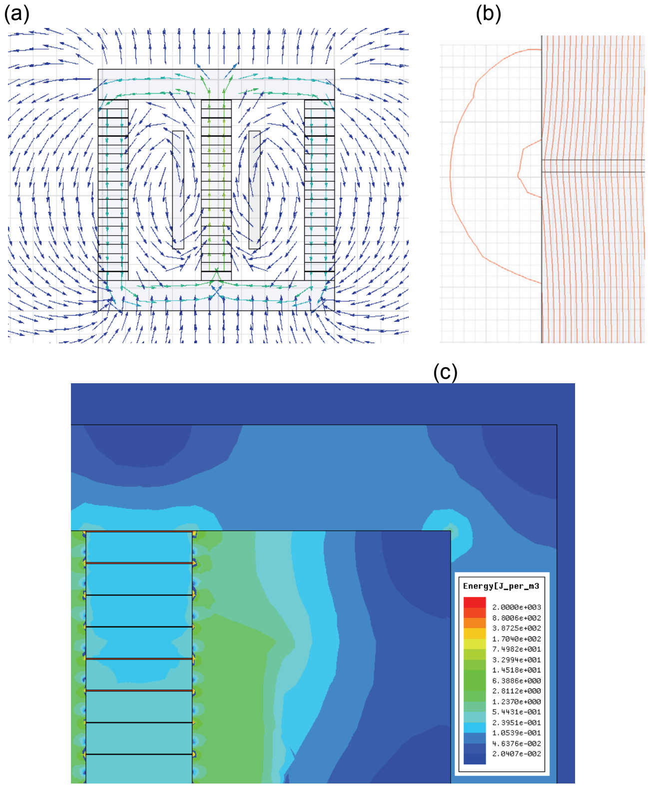

Fig. 2. Magnetic flux pattern in a three-limb reactor (a); a detail showing fringing (b); energy distribution (c).

Calculation of Equivalent Inductances

The difficulties associated with the analytical calculations are probably best illustrated by recalling the equations used for example in [3] based on the equivalent circuit of Fig. 1b:

(1) $$L_{gap} = \frac{\mu_0 N^2 A_{\mathrm{limb}}}{l_g k_g}; \qquad L_{leakage} = \frac{\mu_0 N^2 A_{\mathrm{leakage}} k_l}{l_l}$$

where N is the number of turns, A_{limb} the cross-section of the limb, A_{leakage} the cross-section of the leakage flux path, l_g the total length of the gaps, l_l the 'length' of the leakage path, k_g and k_l empirical correction coefficients (both less than 1). While L_{gap} has a reasonable chance of being estimated correctly as dominated by the well-defined gaps in the limbs, the calculation of the leakage inductance $L_{leakage}$ is subject to all sorts of uncertainties and inevitable simplifications. Moreover, the parallel/series connection of Fig. 1b may not necessarily be the best representation; obviously a parallel connection of inductances may always be combined as a single equivalent inductance, as can the series connection, but the physical association with different magnetic flux paths may be lost in the process. Finally – as clearly demonstrated by Figs. 2a and 2b – the magnetic flux paths cannot always be clearly identified as being purely leakage or main flux.

An alternative association can be made between the relevant component inductances and the energy (or co-energy) associated with different portions of the magnetic circuit. The total energy (which will be the same as co-energy if the magnetic circuit is unsaturated, which is normally the case in practical reactors) may be split into energy associated with the magnetic core, the gaps in the limbs and the rest of the volume surrounding the core (see an example in Fig. 2c); this leads naturally to three component inductances connected in series (as illustrated in Fig. 3) with clear interpretation of the role and contribution of each element. There is no longer a need to be concerned about the actual flux paths. Therefore

$$W = \frac{1}{2} L I^2 \tag{2}$$

$$\begin{aligned} W_{total} &= W_{gap} + W_{iron} + W_{leakage} = \\ &= \frac{1}{2} I^2 L_{total} = \frac{1}{2} I^2 \left(L_{gap} + L_{iron} + L_{leakage} \right) \end{aligned} \tag{3}$$

where W_{total} is the total energy of the device, W_{gap} is the energy associated with the volume filled by air-gaps in the limbs, W_{iron} is the energy related to the volume filled by iron core, and $W_{leakage}$ is the energy of the field in the rest of the system, respectively. Hence L_{gap}, L_{iron} and $L_{leakage}$ are the inductances calculated based on the energy corresponding to these volumes.

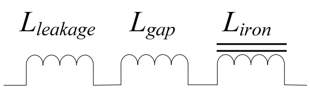

Fig. 3. Equivalent circuit consisting of three inductances connected in series.

However – as noted before – the magnetic circuit of the gapped iron core may also be described in terms of magnetic fluxes and associated reluctances, namely by a series connection of two reluctances, one given by the gap and the other by the iron path of the flux, as shown in Fig. 4c. Comparing the two equivalent circuit representations of the gapped iron core (Fig. 1b and Fig. 4c) it can therefore be argued that there exists one unique logical transformation of the series connection of inductances into a parallel connection, and vice versa. (A pure electrical circuit approach would obviously suggest that there is infinite number of possibilities of such a transformation, all resulting in identical total equivalent value of the inductance, which after all is what really matters.) As the leakage inductance ($L_{leakage}$) is always likely to be connected in series with the rest, attention needs to be focused on L_{gap} and L_{iron} as – depending on the model used, energy or flux based – a series or parallel representation may be more appropriate, respectively.

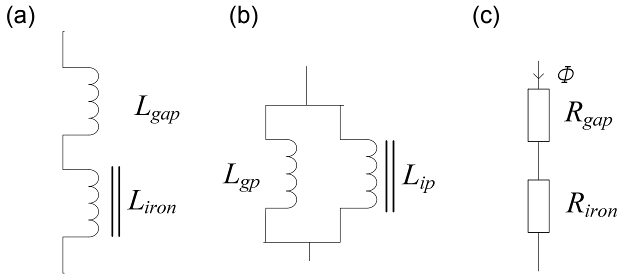

Fig. 4. Series equivalent circuit of inductances (a); parallel equivalent circuit of inductances (b); magnetic circuit of the limb in terms of reluctances (c).

The crux of the argument is therefore that the ratio of the energies corresponding to the two equivalent circuit representations should be preserved irrespectively of the connection. For the connection in series we can use inductances and associated current directly, whereas for the parallel connection work in terms of the flux and reluctances; thus

$$\frac{W_{gap}}{W_{iron}} = \frac{L_{gap} I^2}{L_{iron} I^2} = \frac{\Phi^2 R_{gap}}{\Phi^2 R_{iron}} \tag{4}$$

where R_{gap} and R_{iron} are the reluctances of the magnetic circuit of the reactor limb (these reluctances are connected in series as in Fig. 4c). Therefore, the circuits in Figs. 4a and 4c are equivalent in terms of associated energy. On the other hand, since inductance is inversely proportional to reluctance (obviously the square of the number of turns of the winding needs to be accommodated at some stage), the two reluctances connected in series can be expressed in terms of inductances as

$$R_{gap} + R_{iron} = \frac{1}{L_{gap}} + \frac{1}{L_{iron}} \tag{5}$$

which results in a parallel connection of the two equivalent inductances as depicted by Fig. 4b, where L_{gp} and L_{ip} are the inductance of the air-gap and the iron core of the parallel equivalent circuit, respectively. This simple scheme provides a unique transformation of parallel into series connection of equivalent inductances, or the other way round, while preserving the proportions of component energies. The resultant inductance is of course the same:

$$L_{gap} + L_{iron} = \frac{1}{\dfrac{1}{L_{gp}} + \dfrac{1}{L_{ip}}} \tag{6}$$

Finally, the relationship between the inductances of the parallel equivalent circuit and the series equivalent circuit, as given by the energy ratio, may be written as

$$\frac{W_{gap}}{W_{iron}} = \frac{L_{gap}}{L_{iron}} = \frac{\dfrac{1}{L_{gp}}}{\dfrac{1}{L_{ip}}} \tag{7}$$

Consequently, equations (6) and (7) can be used to easily convert from one representation into the other as desired to calculate the equivalent inductances of the gapped iron core reactor.

Simulation Results

The rated current of the gapped iron core reactor model used in simulations is 100 A; for illustrative purposes a particular design has been considered here but the actual values of inductance are of no consequence, only the relative proportions matter. The magnetic field of the reactor was simulated using Maxwell 3D software [4] for a series of values of the driving current from 1 A (1 per cent of the rated current, hence magnetic circuit completely linear, well below saturation) to 100,000 A (1000 times the rated current, thus magnetic circuit extremely highly saturated). For each case the equivalent inductances of the series equivalent circuit were calculated based on the energy values as described in the previous section (equations 2 and 3). Finally, this series equivalent circuit was converted into a parallel equivalent circuit (using equations 4 - 7).

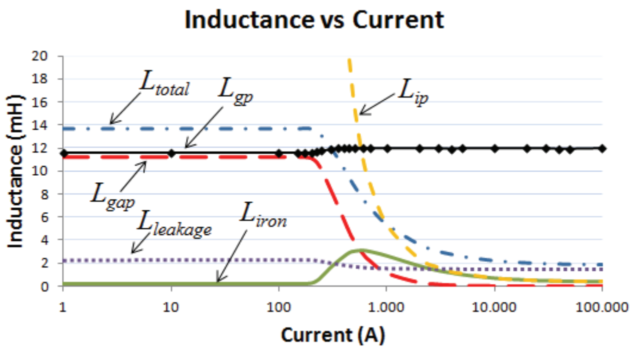

Fig. 5. Variation of inductances of a gapped iron core reactor with increasing current from completely unsaturated to highly saturated magnetic circuit.

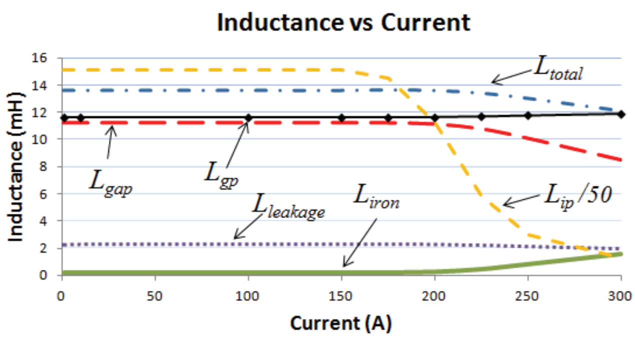

Fig. 6. Variation of inductances of a gapped iron core reactor with increasing current (as Fig. 5 but with linear scale and for smaller values of current).

The results are presented in Figs. 5 and 6, the former using a logarithmic scale to cover the full range of currents used (from unsaturated to highly saturated core), while the latter focusing on the range of currents of more practical interest using a linear scale but still allowing some saturation to be depicted as well. Up to about 150% of the rated current all inductances are effectively constant as expected in such designs. The more saturated region is perhaps of less practical importance but nevertheless shows interesting behaviour. Most notably, the leakage inductance (represented in the circuit as a series connection), and the inductance associated with the gaps in the limbs in the parallel circuit representation, both change very little throughout the whole range, whereas all other inductances (in either series or parallel representation) vary quite dramatically. As expected, the air-gap inductance totally dominates the 'section' responsible for the main flux (passing through the limbs and air gaps) below saturation and matches well the value calculated from (1). (It is also worth noting – which was to be expected – that the leakage inductance $L_{leakage}$ calculated from the simplified model of (1) is in very poor agreement with the reality.)

For higher values of current the magnetic saturation results in the inductance representing the iron (in both series and parallel representation) becoming important so that it cannot be omitted (as a short or open circuit respectively), although in reality this inductance can never be neglected if core losses are of interest (which in fact will constitute the continuation of this work). Under extreme saturation the reactor behaves like an air-cored device and the leakage inductance takes over.

The somewhat strange behaviour of the iron inductance L_{iron} (in the series circuit) at high saturation is perhaps difficult to explain, as first the value rises, then reaches a peak and finally starts to decrease, but this is simply a reflection of the fact that highly saturated iron acts like air and the final proportions of the 'gap' and 'iron' inductances will be related more to geometry rather than to magnetic properties.

The main purpose of the above numerical investigation was to demonstrate how the energy criterion allows for simple and unique transformation between parallel and series representations of the gapped limb of the reactor. In view of the recognised difficulties of defining the different components of flux (main, leakage, gap) it is therefore recommended that energy/co-energy is used throughout while relevant equivalent circuits naturally linked with such fluxes may be established 'retrospectively' by applying the transformation suggested in this paper.

Finally, as already mentioned, the next stage of this work will be to establish a reliable but simple way of estimating iron losses in gapped iron core reactors, hence the iron core inductance will be an important parameter. In preparation for this next step the dependence of this inductance on the maximum average flux density – as established using the 3D numerical simulation described in this paper and elsewhere [5] – has been investigated and simple curves fitted with the aim of subsequently using them in derivation of relevant analytical formulae. One such curve (overimposed on the original data obtained from the 3D simulation) is shown in Fig. 7.

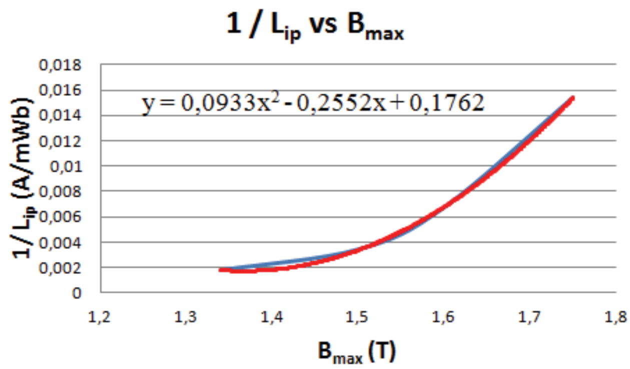

Fig. 7. An example curve fitted to represent the variation of iron core inductance as a function of flux density in the limb of a gapped core inductor.

Conclusions

In this paper we have argued that the equivalent inductance of a gapped iron core reactor may be reliably established by linking the components of this inductance with energy associated with three distinct parts of the device: iron core, air gaps in the limb and the rest of the system (so that the last component gives rise to the notion of a leakage inductance). By introducing an energy based criterion this series equivalent circuit may be easily transformed into a parallel circuit for the 'iron' and 'gap' components (with the leakage term added in series) which can then be related to the otherwise unreliable definitions of the inductance components traditionally expressed in terms of simplistic subdivision of the flux into main, iron and gap paths. The behaviour of all components in both series and parallel representations of the inductance has been studied numerically using 3D finite element simulation for a wide range of currents encompassing both linear regime and extreme saturation.

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