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Numerical analysis of electrohydrodynamic air flow in dc corona field

***Abstract**: Digital model for analysis of two-dimensional direct corona field and induced electrohydrodynamic air flow field in wire-to-plane electrode system is discussed. Finite-difference method in polar coordinate system is used for corona field computation. The digital model of electrohydrodynamic air flow consists of finite-difference approximation of the Navier-Stokes equation and the continuity equation in Cartesian coordinates. Values of air flow velocities are measured by using the hot-wire anemometer.*

***Streszczenie.** Rozpatrzono cyfrowy model do analizy dwuwymiarowego pola korony oraz elektrohydrodynamicznego pola strumieniu powietrza w drutowo-płaszczyznowym systemie elektrod. Ten model zawiera aproksymację równania Navier-Stokes'a metodą różnic skończonych oraz równanie ciągłości we współrzędnych kartezjańskich.(**Analiza numeryczna elektrohydrodynamicznego strumieniu powietrza pola korony**).*

Keywords: field strength, spatial charge density, Coulomb force per unit volume, velocity components.
Słowa kluczowe: natężenie pola, gęstość przestrzenna ładunku, siła kulombowska na jednostkę objętości, składowe szybkości.

Introduction

Ion movement in corona field causes the movement of electrically neutral air molecules, this phenomenon is called electrohydrodynamic air flow movement. It is a useful phenomenon in some technologies and undesirable in others. It is necessary to know both of corona field and electrohydrodynamic air flow parameters to design more accurate and cost-effective devices.

There are many experimental investigations [1, 2] or numerical models for computation of electrohydrodynamic air flow [3]. Because of mathematical models consist of partial difference equations all these models require special expensive program instruments and scientific computers with powerful software.

Procedure of numerical analysis of the field consists of two stages. The first stage comprises of the numerical solution of the Poisson and charge conservation equations describing the direct-current corona discharge. The second stage performs computation of electrohydrodynamic flow field by use of Navier-Stokes and flow continuity equations.

Analysis of direct current corona discharge electric field

We use the system of equations for direct current corona discharge reduced to Poisson and charge conservation equations [2]:

(1) $$\mathrm{div}\,\mathrm{grad}V = -\frac{\rho}{\varepsilon},$$

(2) $$\mathrm{grad}\rho \cdot \mathrm{grad}V = \frac{\rho^2}{\varepsilon},$$

where *V* is corona field potential, ρ is the space charge density, and ε is the dielectric permittivity of the space,

Selected electrode system is shown in figure 1. Polar coordinate system is used because of round wire and intensive variation of corona field strength near the wire electrode. Therefore we have a sufficient amount of information how the field quantities vary near the zone of ionization.

The numerical model of the corona field comprises of Poisson's equation (1) and the charge conservation equation (2). Algorithm is based on the raising of the wire voltage from the 0 value with step δV , all potentials of wire surface nodes are increased at the same time. Potentials of all nodes of computational grid are calculated by use the Poisson's equation.

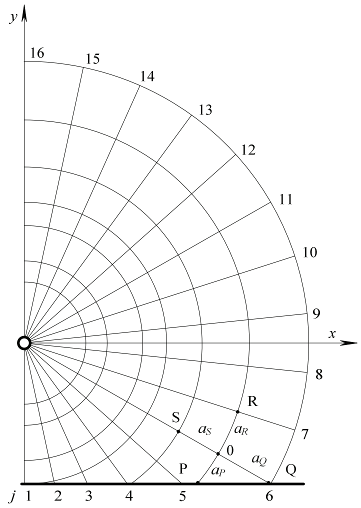

Fig. 1. Polar grid for wire-to-plane electrodes

Boundary condition for the space charge density at any point of the surface of corona electrode corresponding to the given value of the voltage is related with the Kaptzov's assumption which claims that the electric field strength on the surface of wire is constant, and is determined by Peek's law. If the electric field strength on the surface of wire corresponding to given voltage value exceeds the value of initial field strength the spatial charge originates at the nodes of wire surface. It reduces the value of electric field strength to initial one, and Kaptzov's assumption is satisfied. To satisfy this condition the value of spatial charge density at all wire surface nodes, i.e. at the first node of the each radius of polar coordinate system, is increased iteratively by the step $\delta\rho$. When spatial charge originates on the surface of discharge electrode distribution of space charge density in all the area between electrodes is computed by use the charge continuity equation. The computation algorithm is shown in Figure 2.

Computation ends when Kaptzov's assumption is satisfied and wire surface voltage is reached the given voltage value *U*. If any condition is unsatisfied then computation begins again from the beginning of the algorithm.

The main acting force to the air molecules in corona discharge is Coulomb force which is the product of space charge density and electric field strength. It is needed a new computation to compute electrohydrodynamic air flow.

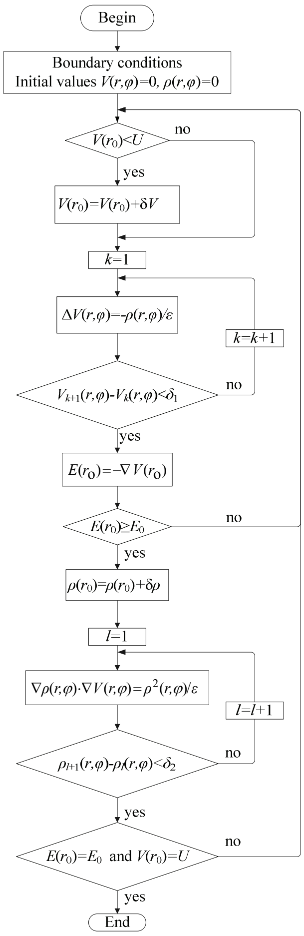

Fig. 2. Numerical algorithm of corona discharge field analysis

Computation of electrohydrodynamic air flow

The radius of the wire $r_0 = 0{,}05$ mm is very small in comparison with other dimensions of the field, it wouldn't affect any meaningful resistance to air flow, therefore the wire can be assumed as elementary node of the computational grid. If elements of computational grid would be equal that would simplify application of flow continuity equation. Therefore the grid in Cartesian system of coordinates can have an advantage in comparison with polar system of coordinates used for computation of velocity components.

We assume that electrohydrodynamic air flow field is 2-dimensional, i.e. flow equations are being solved in two dimension space, there is no flow in *z* direction because of only space force components F_x and F_y causes the air movement. Therefore we solve two Navier-Stokes equations (3, 4) and one flow continuity equation (5) with two unknown air flow velocity components w_x and w_y. We suppose that air is still in moment of time $\tau = 0$, and air flow velocity *w* components are equal to zero. The next assumption simplifying the solution of flow continuity equation is incompressibility of air, $\rho = \mathrm{const}$ for all nodes of computational area. Navier-Stokes and flow continuity equations are of the following form for the plane flow field in Cartesian coordinates:

$$(3)\quad \frac{\partial w_x}{\partial \tau} + w_x \frac{\partial w_x}{\partial x} + w_y \frac{\partial w_x}{\partial y} = a_x + \nu \nabla^2 w_x \,,$$

$$(4)\quad \frac{\partial w_y}{\partial \tau} + w_y \frac{\partial w_y}{\partial y} + w_x \frac{\partial w_y}{\partial x} = a_y + \nu \nabla^2 w_y \,,$$

$$(5)\quad \frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} = 0 \,,$$

where a_y, a_x are the sums of mass forces depending on free fall acceleration and electric field spatial force components, ∂x, ∂y are the distances between nodes, *x* and *y* are coordinate axes, w_x, w_y are the air flow velocity *x* and *y* components, $\partial \tau$ is the change of time between iterations of computation, ν is the kinematic viscosity coefficient, determined as the ratio of dynamic viscosity coefficient and air density. There we assume kinematic viscosity coefficient $\nu = 1{,}507 \cdot 10^{-5}$ m^2/s = const.

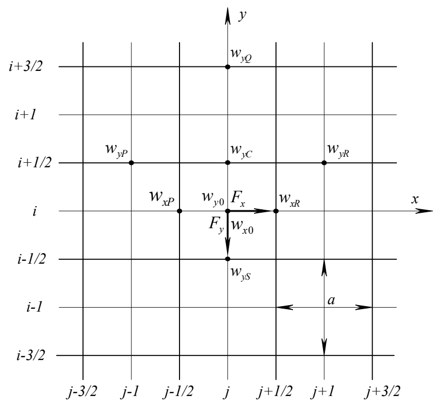

Fig. 3. Computational grid with square elements

There is no flow across *y* axis because the field of the spatial forces is symmetric to *y* axis. Movement of the air on the symmetry axis exists only in direction of *y* axis.

At the surface of the plane, for *y* = 0 components of velocity are equal to 0, $w_y = 0$ and $w_x = 0$ (principle of fluid adhesion).

The principle of numerical algorithm is to calculate *y* axis velocity component at horizontal sides of rectangular grid element and *x* axis component at vertical sides (Fig. 3). Components of flow velocity are calculated by Navier-Stokes equations (3, 4). Components of force and average velocity are determined in the centre of the square element. Friction velocity components are taken from the sides of square. The result is velocity component at the centre of the side of square element. Principle of computation is to compute how much increase or decrease components of air flow velocity in the square, inside of which we know the acting force, average velocity and velocity components of neighbour nodes, which affect the friction. In other words we have information how much air flow velocity components at elementary square sides increase or decrease per time interval in dependence on acting forces. However these results are underestimating of flow continuity equation. To estimate influence of this equation to flow recalculation is needed.

Flow continuity equation states – what is the balance of incoming and outgoing flow in the volume at the each time instant under the assumption that air density is steady. If flow grows to *x* axis, that determines deceleration to *y* axis at the same time. This principle is applied to each iteration by performing re-count of solved Navier-Stokes equations to satisfy a requirement the sum of incoming and outgoing flows to be equal to zero over the sides of square element. The computation algorithm is shown in Fig. 4.

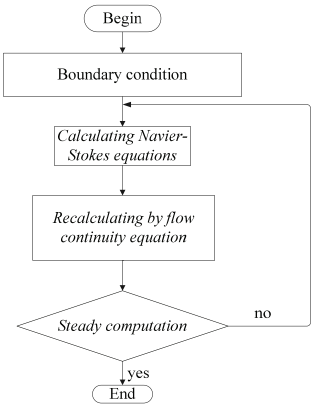

Fig. 4. Computation scheme for numerical analysis of air flow

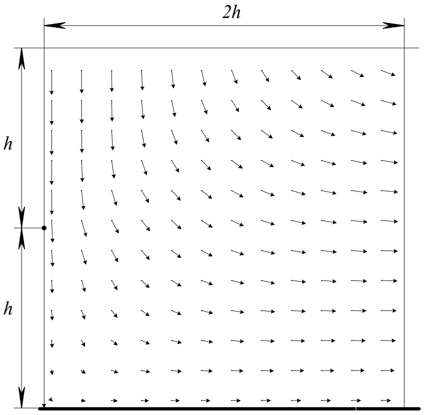

Fig. 5. Electrohydrodynamic air flow distribution in the electrode system for *h* = 12 mm

Visual representation of air flow velocities is given by vectors in Fig. 5. The length of each vector is proportional to modulus of velocity, and the angle of the vector depends on the ratio of the vector components.

Experimental investigation

Experimental modelling is made for the following parameter values of wire-to-plane electrode system with corona discharge: radius of wire r_0 = 0,05 mm, distance between wire and plane electrode *h* = 12,0 mm, discharge electrode voltage *U* = 10,0 kV, metal plane dimensions 200x200x10 mm, electrode system length 20 cm. Measurements are carried out under normal weather conditions: atmospheric pressure $p = p_0 = 101{,}3\ \mathrm{kPa}$, ambient temperature $T = T_0 = 293\ \mathrm{K}$. Position of probe in electrode system and parameters is shown in Figure 6.

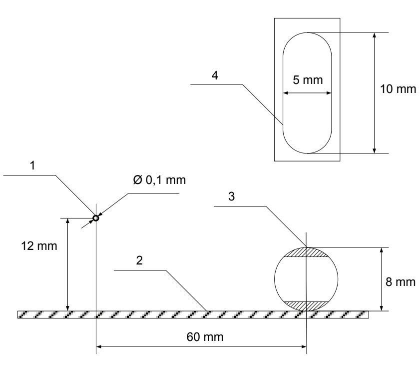

Fig. 6. Electrode system experimental investigation scheme for electrohydrodynamic air flow

Meanings of numerals noticed in Figure 6 are the following: 1 – corona discharge electrode – wire, 2 – earthed metallic plane, 3 – hot wire anemometer probe, 4 – hot wire anemometer probe measuring hole. It is clear from figure 6 that using this scheme we get the averaged value of air flow velocity *x* component near the flat electrode surface.

Dependence of experimental and computed air flow velocity values upon corona discharge voltage in the point of measurement is shown in figure 7.

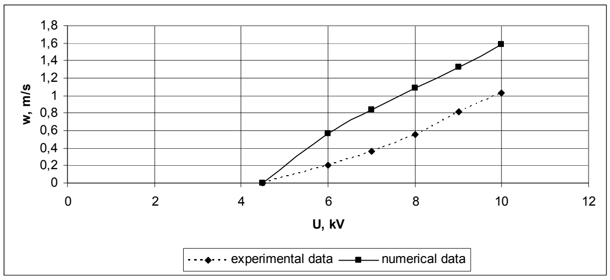

Fig. 7. Experimental and computed air flow velocity in the point of measurement

Theoretical air velocity values averaged in the range from $y = 0$ to $y = 8$ mm (the diameter of the probe) is 1,57 m/s, corresponding experimental value is 1,02 m/s. Numerical and experimental air velocity dependence on discharge voltage curves are of analogue character. Assuming the distortion of air flow and corona fields by hot wire anemometer probe we may predicate that experiment qualitatively prove numerical analysis data.

Conclusions

Maximum value of air flow velocity above the wire corresponding to the *U* = 10 kV is approximately 4 m/s. Measured and computed values of air flow velocities coincide qualitatively.

It is determined that Coulomb force strength has an influence to air velocity, but almost no influence to vectors direction, because the ratio of spatial force components vary insignificantly, changes only their module.

Averaged theoretical air velocity values at hot wire anemometer probe top and low points is 1,57 m/s, (experiment value is 1,02 m/s). Experimental and computed curves of velocity dependence upon voltage are of similar character, but differ in velocity values. This difference is influenced by hot wire anemometer probe distortion of flow and corona fields, therefore we may predicate that experiment qualitatively prove numerical analysis data.

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