Antonín PŘEDOTA, Zdeňka BENEŠOVÁ, Lukáš KOUDELA
University of West Bohemia

Analysis of Transients in Transformer Winding Respecting Space-varying Inductance

***Abstract**. The paper deals with the influence of the space-varying inductance on transients in the transformer winding. The method for the inductance evaluation respecting the manner of turn layout is presented. Obtained results have been implemented in our algorithm for the numerical analysis of transients in the circuit model with distributed parameters which has been solved in the time domain. The comparison of the time-space voltage distribution for the case with the homogenous inductance and the non-homogenous inductance has been carried-out.*

***Streszczenie.** W artykule analizowano wpływ zmieniającej się w przestrzeni indukcyjności na zjawiska przejściowe w uzwojeniu transformatora. Opracowano algorytm numerycznej analizy zjawisk przejściowych przy rozproszonych parametrach. Analizowano czasowy rozkład napięcia dla przypadku indukcyjności jednorodnej i niejednorodnej. (**Analiza stanów przejściowych w uzwojeniu transformatora z uwzględnieniem przestrzennego rozkładu indukcyjności**)*

Keywords: Inductance, transformer winding, distributed parameters, non-homogenous isolation.
Słowa kluczowe: uzwojenia transformatora, stany przejściowe, przestrzenny rozkład indukcyjności.

Introduction

Very fast transient phenomena produced by switching-on and switching-off processes can cause a very high overvoltage in the transformer winding. It can result in damaging of this winding. The adequate model of the transformer winding is needed to determine the dangerous place at the winding and the dangerous voltage peak value. The algorithm published in [1] provides the time-space voltage and current distribution which allows receiving such data. But, for obtaining useful and reliable results it is necessary to determinate correct values of model parameters which respect not only the high frequency of surge phenomena but also the arrangement of turns. In [2], [3] was carried-out an evaluation of inductances and capacitances but authors used it for analysis in the frequency domain. They are focused on frequency dependences of these parameters. Our analysis is carried-out in the time domain [4] where the influence of the space-varying capacitance was studied. In this presented paper is introduced an algorithm for an evaluation of the space-varying inductance respecting an arrangement of turns and the inductive coupling between them.

Transformer winding model

The created model consists from the surge voltage source with the inner resistance and the transformer winding. At the end of the transformer winding is connected a resistor with variable value of resistivity.

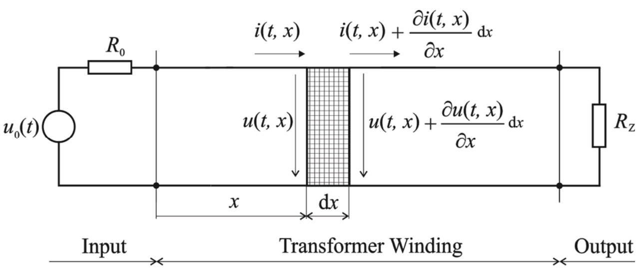

Fig.1. Model based on circuit with distributed parameters

The model and the numerical solution of very fast transient phenomena in a transformer winding were presented in our previous works. There some types of basic elements of the winding model were discussed. Now, the element depicted in Fig. 1 is used. It embraces the most important parameters of the transformer winding.

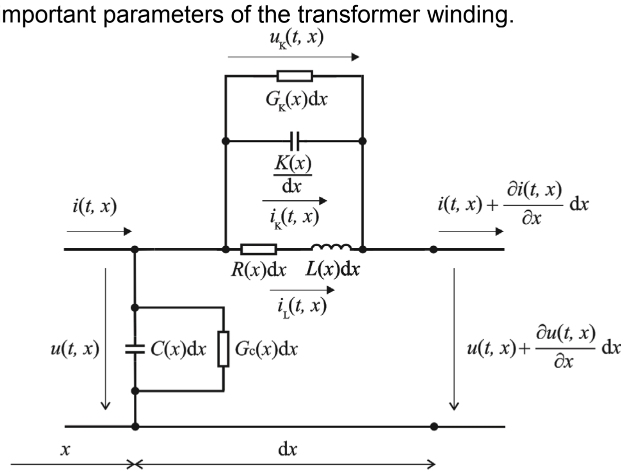

Fig.2. Basic element for transformer winding model

The circuit with space-varying parameters can be described by following partial differential equations of hyperbolic type

$$
\begin{aligned}
-\frac{\partial i_{\mathrm{K}}(t,x)}{\partial x} &= G_{\mathrm{K}}(x)\frac{\partial u_K(t,x)}{\partial x} + \mathrm{C}(x)\frac{\partial u(t,x)}{\partial t} + \frac{\partial i_{\mathrm{L}}(t,x)}{\partial x} + G_{\mathrm{C}}(x)u(t) \\
-\frac{\partial i_{\mathrm{L}}(t,x)}{\partial t} &= \frac{1}{L(x)}\frac{\partial u(t,x)}{\partial x} + \frac{R(x)}{L(x)} i_{\mathrm{L}}(t,x) \\
-\frac{\partial u_K(t,x)}{\partial t} &= -\frac{i_{\mathrm{K}}(t,x)}{K(x)} \\
-\frac{\partial u(t,x)}{\partial x} &= u_K(t,x)
\end{aligned}
\tag{1}
$$

This system of equations is solved numerically. Boundary conditions express the relations between the voltage and current at the input and output of the transformer winding. It is given by eq. (2).

$$
\begin{aligned}
t = 0;&\quad u(0,x) = 0,\ i_{\mathrm{K}}(0,x) = 0,\ i_{\mathrm{L}}(0,x) = 0 \\
x = 0;&\quad u(t,0) = u_0(t) - R_0\,(i_{\mathrm{K}}(t,0) + i_{\mathrm{L}}(t,0)) \\
x = l;&\quad u(t,l) = R_Z\, i(t,l)
\end{aligned}
\tag{2}
$$

Values of the inner resistor and resistor R_Z simulate typical configuration of the transformer winding connection.

Numerical solution

The numerical solution was done by the finite difference method. The time-space mesh is shown in Fig. 3.

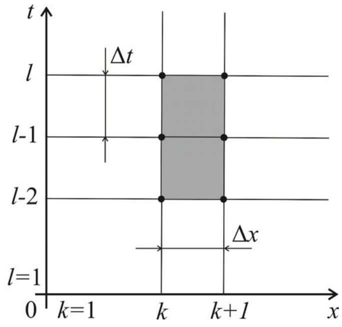

Fig.3. Time-space mesh

The Wendroff´s difference formula and formula according to eq. (3) were used to approximate the voltage and the current at every point of the time-space mesh.

(3) $$\left.\frac{\partial v^2(t,x)}{\partial x \partial t}\right|_k^l \cong \frac{1}{2\Delta t \Delta x}\left(v_{k+1}^l - v_k^l - v_{k+1}^{l\text{-}2} + v_k^{l\text{-}2}\right)$$

Applying every approximations on equation system (1) at every space nodes we obtain the recurrent formula in a matrix form

(4) $$\boldsymbol{A} \cdot \boldsymbol{v}^{(l)} = \boldsymbol{B}_1 \cdot \boldsymbol{v}^{(l-1)} + \boldsymbol{B}_2 \cdot \boldsymbol{v}^{(l-2)} + \boldsymbol{D}$$

This formula can be solved numerically. We used the professional program Matlab.

Winding parameters

Parameters should respect the arrangement of a given winding. At first, we supposed only a one-layer winding. In this case the turn-to-turn capacitance and the turn-to-earth capacitance can be considered as non-space-varying. We focus on evaluation of the turn inductance respecting the inductive coupling between turns in various distances. The configuration is depicted in Fig. 4. The inductive coupling between two turns *i* and *j* is given by the Neumann´s formula which can be evaluated analytically or numerically. We used both ways.

(5) $$L_{ij} = \frac{\mu}{4\pi} \oint_{Ci} \oint_{Cj} \frac{\overrightarrow{\mathrm{d}l_i}\,\overrightarrow{\mathrm{d}l_j}}{r_{ij}}$$

The analytical solution of integral (5) needs to evaluate the elliptic integrals $E(k)$ and $K(k)$. The self-inductance L_{ii} was calculated numerically via magnetic flux coupled with the *i*-th turn.

Having both inductances L_{ii} and L_{ij}, $i, j = 1, 2...N$ for each turn the inductance of the *i*-th turn is given by the following formula

(6) $$L_i = L_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{N} L_{ij} \cdot$$

The inductance distribution function $L(x)$ is depicted in Fig. 5. With regard to the number of turns N and real length of the winding it is possible to norm this dependence on the norm unit length.

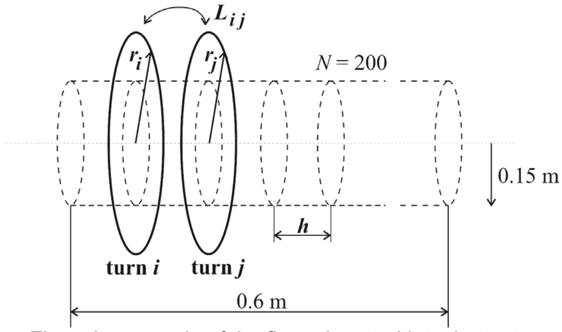

Fig.4. An example of the figure inserted into the text

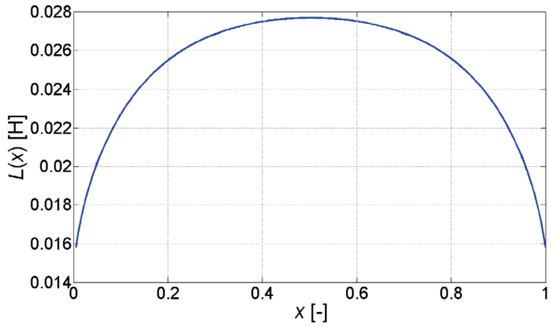

Fig.5. Space-varying inductance $L(x)$ for norm unit length

Illustrative examples

The suggested method was applied on illustrative examples. A transformer winding model with parameters R = 20.7 mΩ/m, C = 20.7 nF/m, K = 2.07 pF/m, G_C = 0.15 nS/m, G_K = 0.15 pS/m was considered. The evaluation was carried out for the norm unit length. The calculated distribution of the inductance in the winding is depicted in Fig. 5. The numerical analysis of eq. (1) was made for following boundary conditions: the transformer input was connected to the source of surge voltage pulse (0.1 μs/1 μs, 200 V), the inner source resistance R_0 = 2 Ω. The winding output was grounded. Fig. 6 shows the voltage distribution along the whole transformer winding.

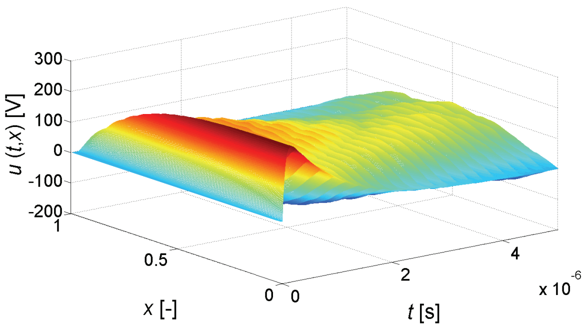

Fig.6. Voltage distribution in grounded transformer winding

The maximal voltage peak value about 21% higher than nominal one was detected at the inner point of the winding and reached U_{MAX} = 242 V. The evaluation was made for two cases, the first one for the winding with the varying inductance according to Fig. 5 and the second one with the non-varying inductance. The average value of the inductance L = 0.027 H/m was used. The comparison is presented in Fig. 7. From the curves of the time voltage distribution at point x = 0.1 (a cut of Fig. 6) is seen that the influence of the varying inductive coupling causes higher values of the voltages. The different voltage frequency has influence on the time response.

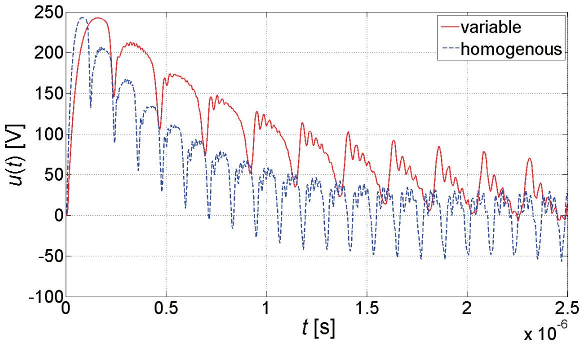

Fig.7. Voltage distribution for varying and non-varying inductance

The next illustrative example presents the voltage and current distributions for the following boundary conditions: the transformer input was connected to the source of surge voltage pulse (0.1 μs/1 μs, 200 V), the inner source resistance R_0 = 2 Ω. The winding output was unloaded. The voltage is depicted in Fig. 8 and the current in Fig. 9.

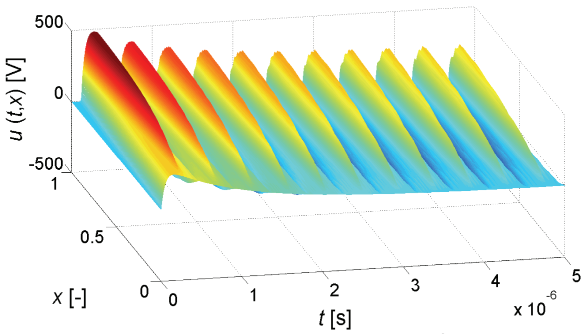

Fig.8. Voltage distribution in un-loaded transformer winding

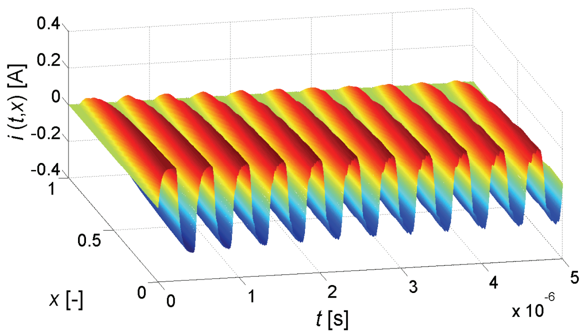

Fig.9. Current distribution in un-loaded transformer winding

The maximum value of the voltage in the transformer winding in this case is 143% higher than the nominal one. It can cause a destruction of the isolation system and aprotection has to be designed. The maximal value of the current in the transformer winding is I_{MAX} = 0.42 A. Because of the linear system presumption the model is appropriate to find out solution at a higher voltage level. The solution can by easily multiplied by a constant.

Conclusion

A computer model based on the transmission line approach and taking into account the space-varying inductance of the transformer coil has been presented. It allows respecting a different inductive coupling between turns along the winding. The suggested method for evaluation of the mutual turn-turn inductances is based on the Neumann´s formula. It can be solved analytically or numerically. We applied this algorithm on the one-layer coil but it is possible to extend the proposed method on more-layer coils according to an arrangement of a considered transformer. Obtained results were compared with analysis of the winding model with the non-varying inductances. It was proved that the influence of the varying inductance is not negligible.

Acknowledgements

Authors want to thank for the financial support from students grant SGS-2012-039.

REFERENCES

[1] Předota, A., Benešová Z., Voltage Surge Wave Propagation in Transformer Winding, *The 14th International IGTE Symposium on Numerical Field Calculation in Electrical Engineering*, Graz University of Technology, 2010, str. 90

[2] Chan, H.L.; Cheng, K.W.E., Sutanto, D., Calculation of inductances of high frequency air-core transformers with superconductor windings for DC-DC converters *Electric Power App., IEE Proceedings*, vol.150, no.4, pp. 447- 454, 8 July 2003.

[3] Shibuya, Y.; Fujita, S., High frequency model and transient response of transformer windings, *Transmission and Distribution Conference and Exhibition 2002*: Asia Pacific. IEEE/PES, vol.3, no., pp. 1839- 1844 vol.3, 6-10 Oct. 2002.

[4] Předota A.; Benešová Z., Fast Transient Overvoltage in Transformer Winding *Przeglad Elektrotechniczny (Electrical Review)*, ISSN 0033-2097, R. 87 NR 5/2011, str. 142-145

[5] Mombello, E.E. Impedances for the calculation of electromagnetic transients within transformers, *Power Delivery, IEEE Transactions*, vol.17, no.2, pp.479-488, Apr 2002

[6] Singh, A.; Marti, J.R.; Srivastava, K.D.Algorithms for Fast Simulation of Transformer Windings for Diagnostic Testing of Power Transformers, *Power Delivery,IEEE Transactions*, vol.25, no.3, pp.1564-1572, July 2010

Authors:

Ing. Antonín Předota, Department of Theory of Electrical Engineering, University of West Bohemia in Pilsen, Univerzitní 8, 306 14 Plzeň, The Czech Republic, E-mail: antonin@predota.eu.

Prof. Ing. Zdeňka Benešová,CSc., Department of Theory of Electrical Engineering, University of West Bohemia in Pilsen, Univerzitní 8, 306 14 Plzeň, The Czech Republic, E-mail: bene@kte.zcu.cz.

Ing. Lukáš Koudela, Department of Theory of Electrical Engineering, University of West Bohemia in Pilsen, Univerzitní 8, 306 14 Plzeň, The Czech Republic, E-mail: koudela@kte.zcu.cz.