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Robust Control for Nonlinear TCP Time-delay Dynamic Network Systems

***Abstract**. This paper presents a robust control scheme for nonlinear TCP time-delay dynamic network systems. By introducing the interference and time-delay to improve the nonlinear model, the objective of the work described here was to express various interference and time-delay factors in the actual network running environment. Using proper coordinate transformation, evaluation signal and energy storage function, the robust controller was guaranteed to satisfy nonlinear L2 gain performance index. The desired queue size was achieved, and the asymptotic stability was obtaineed for nonlinear TCP time-delay dynamic network systems. In the paper this method ameliorates the complex algorithms of nonlinear TCP time-delay dynamic network systems, simplifies derivation and achieves more easily robust control algorithms for nonlinear dynamic network systems. Finally, the simulation results confirm the feasibility and effectiveness*

***Streszczenie.** W artykule zaproponowano odporny system sterowania nieliniowym system sieciowym TCP z opóźnieniem czasowym. Zamodelowano róznego rodzaju interferencje i opóźnienia. Otrzymano pożądany rozmiar kolejki oraz stabilność systemu. Zaproponowana metoda stanowi uproszczenie możliwości sterowania nieliniowymi systemami sieciowymi. (**Odporne sterowanie nieliniowym dynamicznym systemem sieciowym TCP**)*

Keywords: Nonlinear TCP time-delay dynamic network systems; robust control; storage function; nonlinear L_2 gain performance index
Słowa kluczowe: systemy sieciowe, TVCP, sterowanie.

Introduction

With the development of Internet, communication networks are essential parts for a highly technical implementation. Since the number of users has grown rapidly, the networks quality of service cannot be guaranteed. Traffic congestion and networks time-delay are the major communication problems in today's Internet. Active queue management (AMQ) techniques are key congestion control schemes in routers, and are designed to reduce packet loss and the end-to-end delay, and to raise utilization rate of the network systems. The random early detection (RED) [1] algorithm, the earliest AQM scheme [2], attenuates the traffic load by monitoring the average queue length. Unfortunately, RED causes oscillations due to the parameter variation. Therefore, some modified RED schemes [3-4] have been proposed. However, these schemes cannot guarantee high utilization rate and low packet loss of the network systems.

Recently, control theory has been widely applied to congestion control in TCP Dynamic Network Systems. In [5], the authors designed a robust AQM controller for a class of TCP communication networks, the work [6] considered the robust H∞ control for network systems with time-delay and interference. However, in [5, 6], the system model was linearization, but the linearization technique is applied to convert the inherently nonlinear model into a linear systems, the linearization model is approximate description in the neighbourhood of an equilibrium point. While the error of approximate description exceeds the allowable range in the neighbourhood of an equilibrium point, the linearization model is not suitable for use. Therefore, the results of the above study have certain limitations. Due to physical limitations, a nonlinear of the network system usually exists in many practical engineering systems [7-9]. In [10, 11], an AQM controller using a variable structure control approach and nonlinear output feedback approach, which based on rate-queue length model of nonlinear TCP dynamic network systems, it was designed and their performance were validated. However, in the actual network running environment, owing to the time-delay and interference the state error arises. Therefore, the results of the above study also exists certain limitations. When systems exists interference and time-delay， the robust control method has been proposed [12-13]. Currently, few control methods were applicable to congestion and time-delay control for nonlinear TCP dynamic network systems, there still remain challenging.

In the paper, we deal with the problem of robust control of nonlinear TCP network systems. Here attention is focused on the design of controller which guarantees the robust stability as well as the prescribed L_2 performance, the systems exist interference and time-delay. The main contribution of this paper is as follows: First, the new TCP network systems model was constructed which base on rate-queue length and comprise interference and time-delay. Second, the complex algorithms of the design of controller were ameliorated for the nonlinear dynamic network systems. The upper of delay-time was gotten, which can be regarded as a specified constant. Third, the controller was solved by skilfully introducing some constant. A simulation example was given to show the effectiveness of the proposed approach.

The model of network systems

This study uses Jacobsion's algorithm based on rate-queue length [14,15] and Misra's algorithm based on window size and queue length to develop a new nonlinear fluid-flow dynamic model for time-delay TCP networks systems which base on rate-queue length [16,17]. It can be simplified as follows

$$(1)\qquad \begin{cases} \dot{r}(t) = \dfrac{M}{\tau^2(t)} - \dfrac{r(t)r(t-\tau(t))}{2M} p(t-\tau(t)) \\ \dot{q}(t) = r(t) - C_0 \end{cases}$$

where $r(t)$ is the transmission rate per unit time, $q(t)$ is the instantaneous queue length in packets, T_p denotes the propagation delay in seconds, $\tau(t) = q(t)/C_0 + T_p$ is the transmission round-trip time, C_0 is the link capacity (packets/second), M denotes the number of TCP sessions and $p(t)$ is the probability of a packet being marked, which is considered as the control input used to reduce the sending rate and regulate the bottleneck of the queue. All of the variables are assumed to be non-negative and are available.

Let us define the queue length error as $x_1(t) = q(t) - q_0$, where q_0 is the constant desired queue length, by denoting,

$x_2(t)=\dot{x}_1(t)$, $x_1(t)$, $x_2(t)$ is the state variables, $p(t)$ as the input, and systems (1) can be rewritten as follows

(2) $$\begin{cases}\dot{x}_1(t)=x_2(t)\\ \dot{x}_2(t)=\dfrac{M}{\tau^2(t)}-\dfrac{(x_2(t)+C_0)(x_2(t-\tau(t))+C_0)}{2M}u(t)\end{cases}$$

However, due to the actual network systems environment exist variety of electric and magnetic fields of interference, the model cannot be fully represented the actual state of the network systems. Therefore, the model of nonlinear TCP dynamic network systems was built by introducing the interference $\omega(t)$. So the systems (2) can be rewritten as the follows:

(3) $$\begin{cases}\dot{x}_1(t)=x_2(t)\\ \dot{x}_2(t)=-\dfrac{(x_2(t)+C_0)(x_2(t-\tau(t))+C_0)}{2M}(u(t)+\omega(t))\\ +\dfrac{M}{\tau^2(t)}\end{cases}$$

Select evaluation signal

(4) $$z(x)=\begin{bmatrix}\delta_1 x_1(t)\\ \delta_2(kx_1(t)+x_2(t))\end{bmatrix}$$

where δ_1 , δ_2 is randomly selected non-negative weight coefficient.

Nonlinear robust control and the L_2 performance

The objective of the work described here was to design the controller $u(t)$ to attenuate any various interference $\omega(t)$ and time-delay factors in the actual network running environment. The robust controller was designed which can achieve the desired queue size and guarantee the asymptotic stability for the systems (3).The state feedback is

(5) $$u(t)=\varphi(x)$$

The following two requirements need to be satisfied simultaneously.

[1] The closed-loop system (3) is asymptotical stability.

[2] Given disturbance attenuation coefficient $\gamma>0$, $z(t)$ satisfies

(6) $$\int_0^T\|z(x)\|^2dt\le\gamma^2\int_0^T\|\omega(t)\|^2dt$$

We give the lemma 1 that will be used in the proof of our main results.

Lemma 1 [18]: For the systems

$$\begin{cases}\dot{x}(t)=f(x)+g_1(x)u+g_2(x)\omega\\ z(x)=q(x)\end{cases}$$

and control low (5) constitute a closed-loop systems, if there is a positive definite storage function $V(x)$, satisfy the following inequality

(7) $$\dot{V}(x(t))\le\frac{1}{2}\left(\gamma^2\|\omega(t)\|^2-\|z(x)\|^2\right)$$

therefore, the closed-loop systems satisfy above-mentioning performance criteria [2], and if the closed-loop systems is zero-state detectable, when $z(t)\to 0$,it satisfies $x(t)\to 0$, then according to theory of LaSalle's invariant sets, while $x(t)=0$, the systems achieve asymptotical stability.

The systems (3) can be rewritten as follows

(8) $$\begin{cases}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}x_2(t)\\ \dfrac{M}{\tau^2(t)}-Q(t)(u(t)+\omega(t))\end{bmatrix}\\ z(x)=\begin{bmatrix}\delta_1x_1(t)\\ \delta_2(kx_1(t)+x_2(t))\end{bmatrix}\end{cases}$$

where

(9) $$Q(t)=\frac{(x_2(t)+C_0)(x_2(t-\tau(t))+C_0)}{2M}$$

Theorem1: Considering the closed-loop systems (3), if the controller $u(t)$ satisfies

$$u(t)=\frac{Q^{-1}(t)}{\theta_2}\left[(\theta_1-\theta_2k^2)x_1+\left(\theta_2k+\frac{1}{2}\delta_2\right)(kx_1+x_2)+\frac{M}{\tau^2(t)}\theta_2+\frac{\theta_2^2}{2\gamma^2}Q^2(t)(kx_1+x_2)\right]$$

So the time-delay systems which exist outside interference can achieve asymptotical stability.

Proof: Choose coordinate transformation as follows

$$\begin{cases}\hat{x}_1(t)=x_1(t)\\ \hat{x}_2(t)=kx_1(t)+x_2(t)\end{cases}$$

where k is the given positive constant. By coordinate transformation, the structure of the systems (8) can be changed as follows:

(10) $$\begin{cases}\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\end{bmatrix}=\begin{bmatrix}x_2(t)\\ k\dot{x}_1(t)+\dot{x}_2(t)\end{bmatrix}\\ =\begin{bmatrix}\hat{x}_2-k\hat{x}_1\\ k\hat{x}_2-k^2\hat{x}_1+\dfrac{M}{\tau^2}-Q(t)(u(t)+\omega(t))\end{bmatrix}\\ \hat{z}(\hat{x})=\begin{bmatrix}\delta_1\hat{x}_1\\ \delta_2\hat{x}_2\end{bmatrix}\end{cases}$$

where

$$Q(t)=\frac{(x_2(t)+C_0)(x_2(t-\tau(t))+C_0)}{2M}$$

When $\hat{z}(\hat{x})\to 0$, $\hat{x}\to 0$ clearly, the closed-loop systems is zero state detectable. Choose energy storage function $V(\hat{x})$, $V(\hat{x})$ is the Lyapunov function of the systems

(11) $$V(\hat{x})=\frac{\theta_1}{2}\hat{x}_1^2+\frac{\theta_2}{2}\hat{x}_2^2$$

where $\theta_1>0$, $\theta_2>0$.

The function $\hat{H}(\hat{x})$ was introduced according to (7), it can be defined as follows:

(12) $$\hat{H}(\hat{x})=\dot{V}(\hat{x})+\frac{1}{2}\left(\|\hat{z}(\hat{x})\|^2-\gamma^2\|\omega(t)\|^2\right)$$

By applying (10), (11), we can get:

(13) $$\begin{aligned}\hat{H}(\hat{x})&=\theta_1\cdot\hat{x}_1\cdot\dot{\hat{x}}_1+\theta_2\cdot\hat{x}_2\cdot\dot{\hat{x}}_2+\frac{1}{2}\delta_1^2\hat{x}_1^2+\frac{1}{2}\delta_2^2\hat{x}_2^2-\frac{1}{2}\gamma^2\omega^2(t)\\ &=\theta_1\hat{x}_1\hat{x}_2-k\theta_1\hat{x}_1^2+k\theta_2\hat{x}_2^2-k^2\theta_2\hat{x}_1\hat{x}_2+\frac{M}{\tau^2}\theta_2\hat{x}_2-\theta_2\hat{x}_2Q(t)u(t)\\ &-\theta_2\hat{x}_2Q(t)\omega(t)+\frac{1}{2}\delta_1^2\hat{x}_1^2+\frac{1}{2}\delta_2^2\hat{x}_2^2-\frac{1}{2}\gamma^2\omega(t)^2\\ &=(\theta_1-k^2\theta_2)\hat{x}_1\hat{x}_2-k\theta_1\hat{x}_1^2+k\theta_{22}\hat{x}_2^2+\frac{M}{\tau^2}\theta_2\hat{x}_2+\frac{1}{2}\delta_1^2\hat{x}_1^2+\frac{1}{2}\delta_2^2\hat{x}_2^2\\ &-\theta_2\hat{x}_2Q(t)u(t)+\frac{\theta_2^2Q^2(t)}{2\gamma^2}\hat{x}_2^2-\frac{1}{2}\left(\gamma\omega(t)-\frac{\hat{x}_2\theta_2}{\gamma}Q(t)\right)^2\end{aligned}$$

Choosing $\frac{1}{2}\delta_1 = k\theta_1$, we can get:

(14) $$\hat{H}(\hat{x}) = \hat{x}_2\left[(\theta_1 - k^2\theta_2)\hat{x}_1 + k\theta_2\hat{x}_2 + \frac{M}{\tau^2}\theta_2 + \frac{1}{2}\delta_2^2\hat{x}_2 - \theta_2 Q(t)u(t) + \frac{\theta_2^2 Q^2(t)}{2\gamma^2}\hat{x}_2\right] - \frac{1}{2}\left(\gamma\omega(t) - \frac{\hat{x}_2\theta_2}{\gamma}Q(t)\right)^2$$

Through coordinate anti-transformation, we can get

(15) $$u(t) = \frac{Q^{-1}(t)}{\theta_2}\left[(\theta_1 - \theta_2 k^2)x_1 + \left(\theta_2 k + \frac{1}{2}\delta_2\right)(kx_1 + x_2) + \frac{M}{\tau^2(t)}\theta_2 + \frac{\theta_2^2}{2\gamma^2}Q^2(t)(kx_1 + x_2)\right]$$

Therefore, based on (15), the (14) can be rewritten as follows

(16) $$\hat{H}(\hat{x}) = -\frac{1}{2}\left(\gamma\omega(t) - \frac{\hat{x}_2\theta_2}{\gamma}Q(t)\right)^2 \le 0$$

So the systems satisfy

$$\dot{V}(x(t)) \le \frac{1}{2}\left(\gamma^2\|\omega(t)\|^2 - \|z(x)\|^2\right)$$

and the evaluation signal satisfies zero-state detectable, when $z(x) \to 0$, $x(t) \to 0$, according to Lemma 1, the systems achieve asymptotical stability.

The robust controller can be written as follows

$$u(t) = \frac{Q^{-1}(t)}{\theta_2}\left[(\theta_1 - \theta_2 k^2)x_1 + \left(\theta_2 k + \frac{1}{2}\delta_2\right)(kx_1 + x_2) + \frac{M}{\tau^2(t)}\theta_2 + \frac{\theta_2^2}{2\gamma^2}Q^2(t)(kx_1 + x_2)\right]$$

where

$$Q(t) = \frac{(x_2(t) + C_0)(x_2(t - \tau(t)) + C_0)}{2M}$$

Simulation examples

In this section, we will give a simulation example to demonstrate the effectiveness of the proposed method. In the network topology, it is assumed that $M = 100$ homogeneous TCP connections share a bottleneck link with a capacity of 10 Mbps, i.e. $C_0 = 1250$ (packets/second). The range of RTT variation is $0.15 \le \tau \le 0.25$. The parameters of the controller are, $\delta_2 = 0.0616$, $k = 100$, $\theta_1 = 1$, $\theta_2 = 1.6 \times 10^{-4}$, the maximum time-delay selects 0.25s.

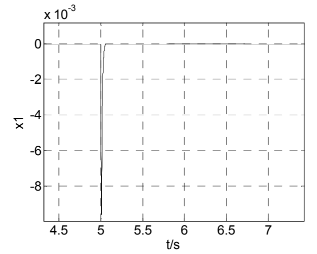

Fig.1 The curve of $x_1(t)$ using the robust control

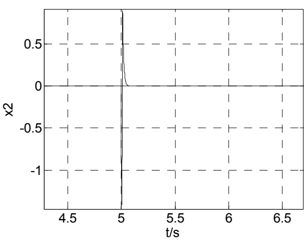

Fig.2 The curve of $x_2(t)$ using the robust control

We investigated the robustness of the robust $H\infty$ control schemes of linear systems, and VSC-AQM schemes of nonlinear systems, the responses of the queue length and the change rate of queue length are obtained by the robust $H\infty$ control and VSC-AQM schemes in Fig.3.-Fig.6.

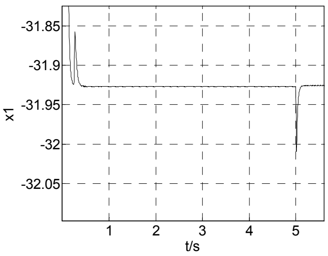

Fig.3 The curve of $x_1(t)$ using the robust $H\infty$ control

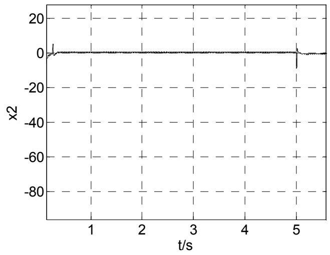

Fig.4 The curve of $x_2(t)$ using the robust $H\infty$ control

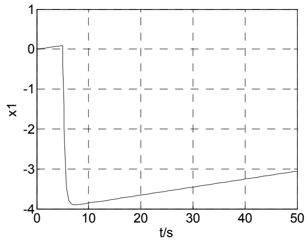

Fig.5 The curve of $x_1(t)$ using the VSC-AQM control

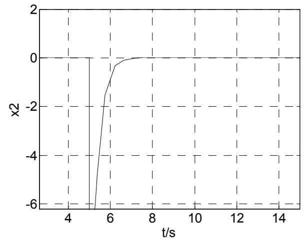

Fig.6 The curve of $x_2(t)$ using the VSC-AQM control

The state $x_1(t)$ is the error between the instantaneous queue length and the expected queue length, the state $x_2(t)$ is the rate of instantaneous change of error. It can be seen from the above curves, the systems under the non-linear robust controller can resolve the influence of the interference and the time-delay very well. However, when the systems suffer interference and time-delay, the controller using the VSC-AQM control and the robust $H\infty$ control cannot resolve the interference. The state of the systems deviates from the equilibrium state. The instantaneous queue length cannot effectively track the desired queue length.

Conclusions

The problem of robust control on nonlinear TCP time-delay network systems, which is subjected to interference, has been studied. Using proper coordinate transformation, evaluation signal and energy storage function to design a robustly stable controller which guarantees both the robust asymptotical stability and the nonlinear L_2 gain performance index. An illustrative example has been provided to show the effectiveness of proposed method.

Acknowledgements

This work is supported by the Doctor-start Foundation of Liaoning Province of China (No. 2008510).

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