Bogusław GRZESIK, Piotr SIEWNIAK

Silesian University of Technology, Department of Power Electronics, Electrical Drives and Robotics

Lyapunov Stability Analysis of DC-DC Power Electronic Converters: A Brief Overview

Abstract. *The brief overview of problems on Lyapunov stability analysis of DC-DC power electronic converters (PECs) is presented in this article. Problems of the PECs global and local stability analysis based on both continuous-time and discrete-time PECs models are discussed here. Special attention is addressed to the PECs stability analysis using direct Lyapunov method.*

Streszczenie. *W artykule przedstawiono zwięzły przegląd tematyki analizy stabilności Lapunowa przekształtników energoelektronicznych DC-DC. Omówiono tutaj problematykę analizy stabilności globalnej i lokalnej PECs na podstawie modeli PECs czasu ciągłego i czasu dyskretnego. Szczególną uwagę zwrócono na analizę stabilności PECs za pomocą bezpośredniej metody Lapunowa.* (***Analiza stabilności Lapunowa przekształtników energoelektronicznych DC-DC: zwięzły przegląd tematyki)***.

Keywords: DC-DC power electronic converter, Lyapunov stability, review article.
Słowa kluczowe: przekształtnik energoelektroniczny DC-DC, stabilność w sensie Lapunowa, artykuł przeglądowy.

Introduction

Stability of power electronic converters is one of the most important problems related to their dynamics. In this article, a brief overview of Lyapunov stability of DC-DC power electronic converters (PECs) [1] is presented.

The PECs belong to a class of dynamical systems where the switching occurs. The switching is linked with the PECs periodic behaviour. If the switching frequency is finite, then the PEC equilibrium state is steady-state periodic orbit (limit cycle), but not equilibrium point (points).

The PEC is called an orbital asymptotic stable in the Lyapunov sense (Lyapunov stable) [2], [3], if its trajectory deviated from the steady-state periodic orbit returns back to it after a certain time. Otherwise, the PEC is unstable. The PEC stability means the convergence of its trajectories to the orbit. If the PEC is stable for any initial conditions, then it is globally stable (stable in the large). Apart from that, the PEC can be stable in a bounded region in state space. This region is called the region (basin, domain) of attraction [3], [4]. If the PEC is stable only for sufficiently small deviations, then it is locally stable.

If the PEC is not stable, then it is unstable. So, the PEC instability and its character are interesting like the stability. The analysis of the PECs instability is a complementary problem in relation to the analysis of the PECs stability. It is possible to distinguish several types of instabilities. In [5], three types of the instabilities are considered: unboundedness, chattering and chaos. The PECs instabilities are divided into slow-scale and fast-scale [6].

The rest of this paper is devoted to the details of the analysis of the PECs global and local stability as well as their instability.

The motivation of this paper comes from the need to supplement the power electronics literature for publication containing a brief overview of problems on the PECs stability and instability analysis. In particular, this paper should be complementary to the article [7] which in addition to other problems provides a brief overview on the PECs local stability analysis in discrete-time domain.

The purpose of this paper is a brief overview of the literature on the analysis of the PECs stability and instability. A special attention is addressed to the PECs stability analysis using direct Lyapunov method [4].

Structure of this paper is as follows. A brief review of the literature on the PECs global stability analysis is presented after the introduction. The next section provides the review of literature on the PECs local stability analysis. Then, the analysis of the PECs instabilities is discussed. The last section contains summary and conclusions.

Analysis of PECs global stability

The analysis of the PECs global stability is based on the large-signal PECs models [8], often piecewise-linear models [9], [10]. They can be continuous-time models, as e.g. in [11], [12], as well as discrete-time, as e.g. in [13], [14], and [15].

In all of the listed papers, the analysis of the PECs stability is carried out using the direct Lyapunov method [4]. Thus, the method takes a very important place in power electronic literature on the PECs stability analysis. In engineering practice, it is desirable that given dynamical system, e.g. PEC, has been exponentially stable [4]. This ensures exponential convergence of its trajectories to the equilibrium state (steady-state periodic orbit). This PEC stability can be analysed using the direct method [4].

In general, the direct Lyapunov method consists of the search for the Lyapunov function (LF) [4]. This function must satisfy specific criteria [4]. In the classical stability theory [4], stability conditions defined by LF are only sufficient conditions (and not necessary and sufficient) [4]. In other words, the stability implies the existence of Lyapunov function, but not vice versa. This means that the given dynamical system can be stable, but determination of LF failed. The failure in the search for LF for the system does not mean that it is unstable.

The search for LF is usually very complicated, because there is no general rule for constructing these functions [4]. Often, search for suitable LF is an experiment. In the general case, the construction of LF is to propose LF candidate, and then check the conditions that it must fulfill [4]. If these conditions are fulfilled, then given system is stable. Otherwise, another LF candidate should be proposed. One of the best LF candidates is a function with a quadratic form [4]. Principles of construction of such a function, as well as other Lyapunov functions, are presented e.g. in [4].

In power electronics, the methods and tools dedicated to hybrid dynamical systems stability analysis [16], [17] have a special importance. This is due to the fact that the PECs are hybrid systems [18]. These methods and tools are presented in detail in the cited papers: [16] and [17]. Therefore, these papers deserve a brief comment.

In [16], Branicky presents the multiple Lyapunov function. It is a very effective tool for stability analysis of the switched and hybrid systems [16] including the PECs. The multiple

LF consists of member functions called Lyapunov-like functions. Each of Lyapunov-like functions corresponds to a specific system dynamics in a certain region in state space. All the Lyapunov-like functions as well as the entire multiple Lyapunov function should satisfy certain criteria [16], [19].

In [17], Johansson and Rantzer discuss the analysis of quadratic and piecewise-quadratic stability of nonlinear and hybrid systems. This analysis is based on (common) quadratic and/or (multiple) piecewise-quadratic Lyapunov functions. Stability conditions based on multiple LF are less conservative from the ones which are formulated using common LF. This is the biggest advantage of the application of multiple LFs. However, the number of expressions (equations and/or inequalities) needed to determine the multiple LF is large. It is several times higher than that which is necessary to define common LF.

Interpretation of different Lyapunov functions is presented in fig. 1. In particular, common quadratic Lyapunov function is shown in fig. 1.a. It is valid in each region R_1, ..., R_3 in state space. The discontinuous piecewise-quadratic Lyapunov function is shown in fig. 1.b. The member Lyapunov-like functions: V_1, V_2, and V_3, are quadratic. This ensures that they are descending. Each of them corresponds to different region in state space: R_1, ..., R_3. These functions should create non-increasing sequences at the switching times, e.g. for V_1: $V_1(t_0) \geq V_1(t_3) \geq V_1(t_6)$. Multiple Lyapunov function shown in fig. 1.c is weakened with respect to the piecewise-quadratic function in fig. 1.b. The Lyapunov-like functions are polynomial, not quadratic. They need not be decreasing, but as previously, they should create non-increasing sequences at switching times [19].

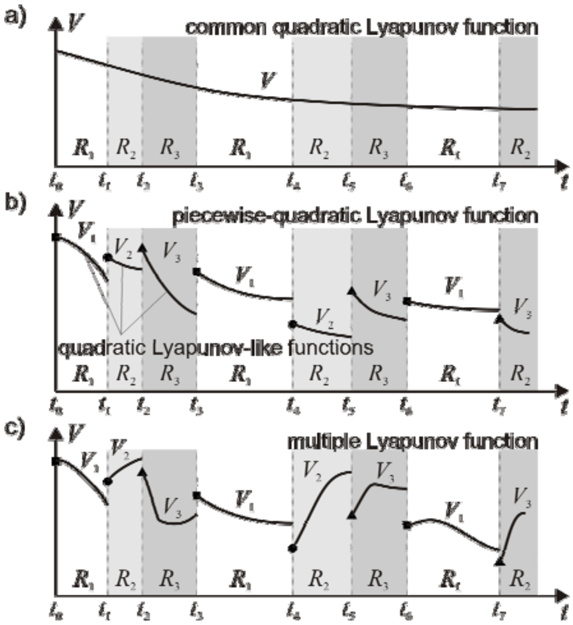

Fig. 1. Interpretation of Lyapunov functions: a) common quadratic LF [17], b) discontinuous piecewise-quadratic LF [16], [17], c) multiple LF [19]

In all the papers listed at the beginning of this section, i.e. [11], [12], [13], [14], and [15], the (multiple) piecewise-quadratic Lyapunov functions are used. In [11] and [12] their domain is continuous time, but in [13], [14], and [15] – discrete time. The common quadratic Lyapunov function applies only in [15].

The Lyapunov function search can be greatly simplified for the PECs because they are hybrid systems [18]. Then, the above mentioned methods and tools dedicated to these systems are used. A very comfortable solution is to present the PECs stability conditions as optimization problems [20] in terms of linear matrix inequalities (LMIs) [20]. They are semi-definite programming problems [20]. In power electronics, they are expressed only by quadratic and/or piecewise-quadratic Lyapunov functions. This way is used in many papers, e.g. in [11], [12], [13], [14], and [15]. These optimization problems can be solved using appropriate computer techniques and tools, e.g. *Matlab* package [21]. The package *Robust Control Toolbox* of *Matlab* contains *LMILab* solver [22]. It is destined to solve semidefinite programming problems. To solve such problems one can also apply other external solvers, e.g. *SeDuMi* [23]. A very comfortable and useful framework is *Yalmip* [24].

In addition, the optimization methods can be also used for other purposes. An example would be the estimation of the rate of trajectory convergence to the steady-state periodic orbit as shown e.g. in [13] and [14]. Apart from that, the LMIs are often applied in design of the PECs control. It is the so-called LMI-based control.

The recommended method of analysis of the PECs global stability is presented in [15]. It is the direct Lyapunov method based on the results for piecewise-linear systems introduced in [17]. The basis of this analysis is the sampled-data PEC model similar to that presented in [6]. Using the discrete-time framework is especially useful for the PECs where there is no single equilibrium point, and more. The PEC equilibrium state is the steady-state periodic orbit which is an attractor. It is represented by a (single) fixed point which is the big advantage of the discrete-time approach of the PECs modelling and stability analysis.

Illustrative algorithm of the PECs stability analysis based on the approach presented in [15] is shown in fig. 2.

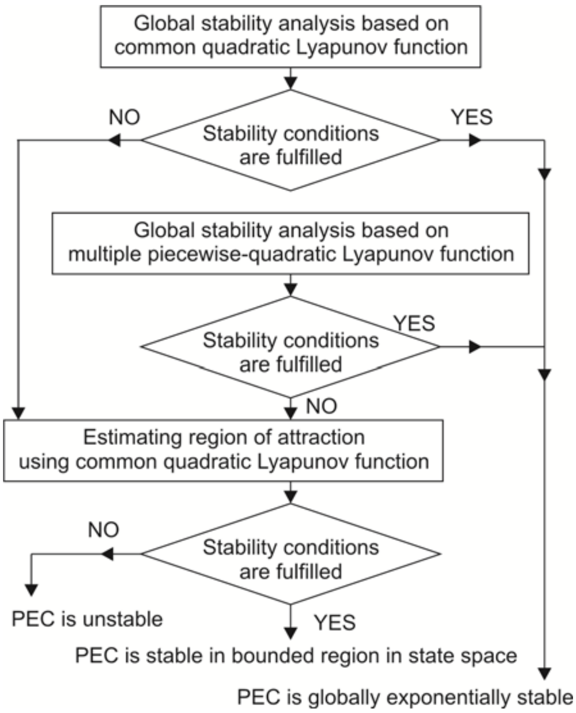

Fig. 2. The PECs stability analysis based on quadratic and piecewise-quadratic Lyapunov functions [15]

Conditions of the PEC (exponential) stability are formulated there using common quadratic or piecewise-quadratic Lyapunov functions. The conditions expressed by common LF are necessary and sufficient conditions of the PECs global stability. Thus, if they are not fulfilled, it means that the PEC is unstable. On the other hand, the PECs stability conditions described by piecewise-linear LF are only sufficient. If these conditions are not fulfilled, then additional analysis is necessary. One can try to estimate the region of attraction or use another Lyapunov function or method. The estimation of the region of attraction is based on common quadratic Lyapunov function. Hence, this region has an ellipsoid form. Systematic procedures to search for common and multiple Lyapunov functions are discussed there. They are based on optimizations problems in terms of parameterised LMIs. Division of state space into regions is achieved there using the PEC feedback control structure.

As an example, stability analysis of bidirectional boost converter is discussed there.

The other method of estimating region of attraction is presented in [12]. However, in contrast to [15], stability analysis of the PEC, i.e. buck-boost under PWM control, is carried out there using the continuous-time PEC model described by piecewise-linear differential inclusions.

Another completely different approach to the PECs global stability analysis is introduced in [25]. This analysis uses the averaging technique. However, it is not classical averaging (approximation) in state space, because it is based on the theory of absolute stability [4]. As an example, stability analysis of PWM buck converter is discussed there.

The next approach to the PECs stability analysis is shown in [26]. There is the classical Lyapunov theory, i.e. the direct Lyapunov method used as well. It examines a boundary control of buck converter with instantaneous constant-power loads. The PEC dynamics is analysed there in both switching states and various operating regions. There are established sufficient conditions for the PEC large-signal (global) stability. In addition, the region of attraction is derived.

The special attention deserves the approach to the PECs stability analysis presented in [27]. The current-mode controlled boost converter is modelled there as nonsmooth Takagi-Sugeno fuzzy system. This model enables the analysis of global exponential PEC stability and the fast-scale instabilities such as period doubling-bifurcations [1]. Stability conditions are formulated there also by means of piecewise-quadratic Lyapunov functions with LMIs. The main advantage of this approach is the possibility of design of switching fuzzy controller. The control scheme which has been shown there improves the fast-scale PEC stability [6] by extending stable period-one operations.

Analysis of PECs local stability

In engineering practice the analysis of the PECs local stability can be performed using both continuous-time and discrete-time small-signal PECs models [8].

Linearized averaged PECs models [1], [28], [29] are among the most widely used continuous-time PECs models. These models are linear time-invariant. Hence, the methods dedicated for linear dynamical systems can be applied for the PECs stability analysis. Namely, in [28], Tse, Lai, and Iu present the method based on eigenvalues of Jacobian matrix $\mathbf{J}$ for Ćuk converter with hysteresis current-mode control. The matrix $\mathbf{J}$ is calculated at equilibrium point $\mathbf{x}_{eq}$: $\mathbf{J}(\mathbf{x}_{eq})$. The next step consists of determining the eigenvalues λ_i $(i = 1, 2, ..., n)$ of $\mathbf{J}(\mathbf{x}_{eq})$ which are the roots of characteristic equation: $det[\lambda\cdot\mathbf{I} - \mathbf{J}(\mathbf{x}_{eq})] = 0$. If the real parts of all λ_i are negative: $Re(\lambda_i) < 0$ for all i, then the PEC is stable. Otherwise, the PEC is unstable.

The analysis of the PECs stability based on the linearized averaged models can also be carried out using any stability criterion for linear dynamical systems, e.g. Routh-Hurwitz criterion [3]. The application of this criterion is presented in [29] for buck converter with fixed frequency voltage-mode control. Moreover, a different approach to the PECs local stability analysis has been discussed there, namely, the ripple based index approach.

The analysis of the PEC local stability in the discrete-time domain can be performed using different methods, models and tools. Among them are some mentioned as: (*i*) methods based on eigenvalues of the Jacobian matrix of linearized Poincaré map [30], (*ii*) auxiliary state vector method [31], (*iii*) trajectory sensitivity approach [32], (*iv*) Filippov method [7].

In [30], Dranga, Buti, and Nagy present conditions of the PEC local stability using the Poincaré map: $\mathbf{x}_{k+1} = \mathbf{P}(\mathbf{x}_k)$ $(k = 1, 2, ...)$. Stability of the steady-state periodic orbit is determined by stability of a fixed point: $\mathbf{x}^*$: $\mathbf{x}^* = \mathbf{P}(\mathbf{x}^*)$. Stability of the fixed point $\mathbf{x}^*$ is defined by local behaviour of the Poincaré map in its small neighbourhood. If sufficiently small deviations around $\mathbf{x}^*$ tend to zero, then a fixed point $\mathbf{x}^*$ is asymptotic stable. Local dynamic behaviour of Poincaré map around $\mathbf{x}^*$ can be determined using the linearization: $\Delta\mathbf{x}_{k+1} = \mathbf{J}_k(\mathbf{x}^*)\cdot\Delta\mathbf{x}_k$, where: $\mathbf{J}_k(\mathbf{x}^*) = d\mathbf{P}(\mathbf{x})/d\mathbf{x}\ |_{\mathbf{x}^*}$ - the Jacobian matrix of the Poincaré map at $\mathbf{x}^*$. Stability of $\mathbf{x}^*$ is defined by the location of the eigenvalues of $\mathbf{J}_k(\mathbf{x}^*)$: λ_i, $i = 1, 2, ...$. The PEC is local asymptotic stable if all eigenvalues λ_i are situated inside the unit circle in complex plane: $|\lambda_i| < 1$. Otherwise, the PEC is unstable. As an illustrative example, stability analysis of dual-channel resonant buck converter with PWM control is presented there. It should be noted that there are readily available computer tools for determination of the Jacobian matrix at a fixed point, calculation of its eigenvalues and the analysis of loci of these eigenvalues in complex plane.

The article [31] is a development of mentioned above [30]. Theoretical principles of the auxiliary state vector approach are presented there. Using this method allows to simplify the calculation of Jacobian $\mathbf{J}_k$ at a fixed point $\mathbf{x}^*$: $\mathbf{J}_k(\mathbf{x}^*)$. This simplification implies no need to calculate the derivative of the Poincaré map at $\mathbf{x}^*$. As a result, it is simpler and faster way to obtain the $\mathbf{J}_k(\mathbf{x}^*)$ in relation to the classical approach [30]. Stability analysis of PWM resonant buck converter is presented there as an example.

The trajectory sensitivity analysis is discussed by Hiskens in [32]. In general, this method concerns nonsmooth hybrid systems where periodic motion appears [7]. Therefore, it can be used also for the PECs because they belong to the class of piecewise-smooth hybrid dynamical systems [7]. Stability analysis in [32] is based on hybrid model which is a differential-algebraic-discrete model. It allows to define the trajectory flow: $\mathbf{x}(t) = \phi_\mathbf{x}(\mathbf{x}_0, t)$, $\mathbf{x}_0 = \phi_\mathbf{x}^*(\mathbf{x}_0, t_0)$. Sensitivity of the flow to disturbances of initial conditions is defined using the trajectory sensitivity matrix: $\mathbf{\Phi}_\mathbf{x}$: $\mathbf{\Phi}_\mathbf{x}(\mathbf{x}^*, T) = \partial\phi_\mathbf{x}(\mathbf{x}_0, t)/\partial\mathbf{x}_0$, where: $\mathbf{x}^*$ - fixed point, T – period of state-error trajectory. Stability of the steady-state periodic orbit is determined by the Poincaré map: $\mathbf{x}_{k+1} = \mathbf{P}\cdot\mathbf{x}_k = \phi_\mathbf{x}(\mathbf{x}_k, \cdot)$ which is linearized later around a fixed point $\mathbf{x}^*$. The result is the linear map: $\Delta\mathbf{x}_{k+1} = D\mathbf{P}(\mathbf{x}^*)\cdot\Delta\mathbf{x}_k$, where $D\mathbf{P}(\mathbf{x}^*)$ is dependent on the trajectory sensitivity matrix $\mathbf{\Phi}_\mathbf{x}(\mathbf{x}^*, T)$. A single eigenvalue of $\mathbf{\Phi}_\mathbf{x}$ is always equal 1, and the remaining are equal to eigenvalues of linear map $D\mathbf{P}(\mathbf{x}^*)$, i.e. Floquet multipliers [3]. The Floquet multipliers of $D\mathbf{P}(\mathbf{x}^*)$ define stability of nonlinear map $\mathbf{P}(\mathbf{x}^*)$, i.e. stability of the PECs steady-state periodic orbit.

If the determination of the Poincaré map in closed form is not possible, then the Filippov method [7] can be applied. In [7], the local stability analysis of voltage-mode controlled buck converter using the Filippov method is described. The method is based on the calculation of the monodromy matrix [3]. It consists of components which are the state transition matrices defined before and after switching, and at switching instant along the switching hypersurface. This last matrix is called the saltation matrix. The PECs stability is defined by the Floquet multipliers of monodromy matrix. In general, the multipliers are identical as the eigenvalues of the Jacobian matrix of the Poincaré map at a fixed point. The basic influence on the Floquet multipliers, i.e. on the PECs stability, has the saltation matrix.

An efficient and convenient approach to the PECs stability analysis on the base of sampled-data PECs models is presented also in [2] and [33]. In [2], Fang presents the general sampled-data PECs modelling technique. Different sampled-data PECs models, i.e. exact block diagram model, large-signal nonlinear model, and small-signal linearized model are discussed there. The linearized sampled-data model is suitable for the PECs local stability analysis which has been presented there for buck under one-cycle control and buck under charge control. A similar modelling technique has been used in [33] by Song, Chen, and Liaw. The analysis of stability of period-one and period-two orbits of buck converter under PWM voltage-mode control is presented there. The analysis is based on the eigenvalues of the Jacobian matrix at a fixed point.

In practice, the PECs local stability analysis should be carried out including analysis of the PECs instability. This is due to the different properties and applicability of this analysis in continuous-time and discrete-time domain.

Analysis of PECs instability

Problems of the PECs instability are strictly related to different nonlinear phenomena occurring in the PECs. These phenomena are discussed in several articles, e.g. in [6], [34], [35] and in the book mentioned earlier [1].

As mentioned in the section 1, the PECs instabilities are divided into slow-scale and fast-scale [6]. Low-frequency oscillations (slow-scale instabilities) can occur in the PECs e.g. via Neimark-Sacker bifurcations [1]. They are shown in [29] using Routh-Hurwitz criterion [3]. Loss of the PEC stability via Hopf bifurcations [1] is presented e.g. in [28]. These instabilities are also slow-scale. In order to determine the PEC stability and instability, the eigenvalues of the Jacobian matrix at equilibrium point are used there. In both cases the analysis of the PECs instabilities is based on linearized averaged models.

As demonstrated in the cited papers, the slow-scale PECs instabilities can be effectively predicted and detected using the continuous-time linearized averaged PECs models. At the same time, e.g. in [6], [29], and [30], has been shown that these PECs models and the classical methods (e.g. Routh-Hurwitz criterion) are not suitable for the analysis of fast-scale nonlinear phenomena like period-doubling bifurcations [1]. Thus, the continuous-time linearized averaged PECs models are useful only if the analysis of the PECs dynamics in wide frequency range is not required. This follows from the fact that these models ignore the PECs dynamics in the switching events. In addition, the details of the PECs dynamics inside the PECs operating cycle are also ignored. Hence, the linearized averaged PECs models are not suitable for analysis of the high-frequency dynamic phenomena, i.e. the phenomena with a frequency close to the ramp signal. However, the application of the ripple based index approach presented in [29] enables the analysis of fast-scale nonlinear phenomena like period-doubling bifurcations. So, this approach complements the averaged PECs models.

On the other hand, the discrete-time (sampled-data) PECs models can be applied in a wide frequency range. Therefore, it is possible to use them for prediction and detection of both the slow-scale as well as the fast-scale PECs instabilities. E.g., in [6], Mazumder, Nayfeh, and Boroyevich present different PECs instabilities. In order to illustrate them, the bifurcation diagrams, time-waveforms, and loci of Floquet multipliers in relation to the unit circle in complex plane, are presented there. The loss of the PEC stability via Hopf bifurcations (slow-scale instabilities) and period-doubling bifurcations (fast-scale instabilities) are discussed there. Different ranges of the PEC operations, e.g. period-one, quasi-periodic, period-two and chaotic operations are emphasized there as well. The same problems are discussed in many other articles, e.g. in [7], [30], [31], and [33].

Several papers include a comparative analysis of different methods of analysis of the PECs stability and instability. Namely, in [29], the PEC stability analysis based on the averaged PEC model using Routh-Hurwitz criterion is compared to the analysis based on the ripple index approach. In [6], the PEC stability analysis based on the linearized averaged PEC model is compared to the analysis based on the Poincaré map. In particular, the buck converter under voltage-mode control is analysed there.

In [36], the multi-frequency high-order averaged PEC model and the low-order sampled-data PEC model are compared in relation to the local stability and instability analysis. The analysed PEC is boost converter with PWM control.

In summary, in order to predict and detect the PECs instabilities an appropriate model of the PECs dynamics should be used. The linearized discrete-time (sampled-data) PECs models can be applied in wide frequency range. Therefore, they can be used to predict and detect the PECs instabilities of both slow-scale as well as fast-scale. On the other hand, use of the continuous-time linearized averaged PECs models allows predicting and detecting only low-frequency instabilities. However, application of the ripple based index approach enables the analysis of fast-scale nonlinear phenomena like period-doubling bifurcations. So, this approach complements the averaged PECs models.

Summary and conclusions

The aim of this paper, which provides an overview of the problems on the PECs stability, has been achieved. The overview shows that the analysis of PECs stability can be carried out using various PECs models and methods. Both the PECs models and the methods of the PECs stability analysis have different scopes of applicability in relation to each other.

The recommended method of the PECs global stability analysis is the direct Lyapunov method. It is in fact the only method of the PECs global stability analysis. The direct Lyapunov method has many advantages. Firstly, it allows the analysis of the PECs stability in both continuous-time domain and discrete-time domain. Secondly, it is the only known tool enabling the estimation of the region of attraction. Thirdly, the analysis of the PECs stability based on the direct Lyapunov method can be carried out using universal computer techniques. In particular, convex optimization techniques with linear matrix inequalities are usually used. These optimization problems can be solved effectively using semidefinite programming software, e.g. *SeDuMi* with *Matlab* environment. Fourthly, general algorithm of the PECs stability analysis based on this method is analogous for each PEC, irrespective of its topology, control or operating mode (CCM/DCM) as well. Fifthly, the direct method has a transparent physical interpretation with respect to the measure of the PECs energy.

In the case of the analysis of the PECs local stability and instability, the recommended method is the method based on the linearized Poincaré map. This is due to several reasons. Firstly, the PECs model based on linearized map can be used in a wide frequency range.

Hence, it can be applied to predict and detect both slow-scale and fast-scale PECs instabilities. Secondly, the general algorithm of the PECs stability analysis is similar for different topologies and control of the PECs. Thirdly, there are ready available computer tools for determination of the Jacobian matrix at a fixed point, calculation of its eigenvalues and analysis of loci of these eigenvalues in complex plane. Fourthly, the results of stability analysis obtained using Poincaré map have a simple and transparent interpretation in relation to the loci of the eigenvalues of the Jacobian matrix inside unit circle in complex plane. The disadvantages in applying linearized maps include a more complex construction of these models compared to the linearized averaged PECs models.
In practice, the simplified discrete-time method of analysis of the PECs stability can also be used, namely, the auxiliary state vector method. This simplification is no need to calculate the derivative of the Poincaré map. On the other hand, if the Poincaré map in closed form cannot be obtained, then the Filippov method based on the monodromy matrix and the saltation matrix is recommended to use.

The direction of future works is the analysis of problems of the PECs practical stability analysis [4] in contrast to the Lyapunov stability analysis presented in this article.

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***Authors**: dr hab. inż. Bogusław Grzesik, mgr inż. Piotr Siewniak, Politechnika Śląska, Katedra Energoelektroniki, Napędu Elektrycznego i Robotyki, 41-400 Gliwice, ul. Krzywoustego 2, E-mail: boguslaw.grzesik@polsl.pl, siewniak@zse.edu.pl.*