Dong LI, Dongyun YI

Department of Mathematics and Systems Science, National University of Defense Technology, Changsha, China

Extended CPHD Filter for Combining Multi-Target Tracking with Sensor Alignment

Abstract. *An extended CPHD (Cardinalised Probability Hypothesis Density) filter for combining multi-target tracking with sensor alignment is proposed. The augmented state is established by appending the sensor biases to the single-target state. The cardinality distribution of the targets and the intensity of the augmented state are propagated by employing Gaussian mixtures. The target states and the sensor biases are jointly estimated. Simulation results show that the proposed filter successfully achieves the sensor alignment and outperforms the standard CPHD filter.*

Streszczenie: *Zaproponowano rozszerzony filtr CPHD do połączenia śledzenia wielu celów z wyrównaniem czujników. Wyrażenia dotyczące pojedynczego celu rozszerzono przez dodanie offsetu czujnika. Moc (kardynalność) rozkładu celów i intensywność rozszerzonego wyrażenia są zrealizowane przez zastosowanie przekształceń Gaussa. Przeprowadzono jednoczesna estymację wyrażeń celu i offsetu czujnika. Wyniki symulacji wykazują, że proponowane rozwiązanie satysfakcjonująco dokonuje wyrównania czujników i wyjściowych parametrów standardowego filtru CPHD.* ***Rozszerzony filtr CPHD do połączenia śledzenia wielu celów z wyrównaniem czujnika***

Keywords: multi-target tracking, sensor alignment, probability hypothesis density, Gaussian mixture.
Słowa kluczowe: Śledzenie wielu celów, Wyrównanie czujnika, Gęstość hipotez prawdopodobieństwa, Przekształcenie Gaussa

Introduction

Sensor alignment involving the estimation and correction of sensor biases plays a key role in multi-sensor tracking system. Most of the published methods of handling this problem, including two-stage estimation filter [1], maximum likelihood [2] etc., are only applicable when we know the association between the targets and the measurements. However, this association is usually unknown in the multi-target tracking (MTT) problem. Therefore it becomes a challenging problem to jointly achieve sensor alignment and MTT. The probability hypothesis density (PHD) filter [3, 4] has distinct advantage for MTT problem since it operates only on the single-target space and avoids the sophisticated data association. Hence, it provides an opportunity for us to combine the MTT with the sensor alignment. Lian et al. [5] have firstly considered the sensor alignment in the PHD filter. They proposed an extended PHD filter to jointly estimate the target states and sensor biases without data association. However, the PHD filter has the drawback that it has high variance in the estimates of the target number [6]. Mahler [7] proposed the cardinalised PHD (CPHD) filter to improve the accuracy of the target number estimations. To our based knowledge, combining MTT with sensor alignment based on CPHD approach has not been investigated.

The key contribution of this paper is an extended CPHD filter which combines the MTT with the sensor alignment. This filter augments the sensor biases into the single-target state, and jointly propagates the cardinality distribution of targets and the intensity function of the augmented state. The implementation of this filter is derived by using Gaussian mixtures. A simulation example is presented to demonstrate the performance of the proposed filter.

Augmented state model

Suppose that there are L biased sensors synchronously observing multi-target. Let $z_{k,1}^{[j]},\cdots,z_{k,M_k^{[j]}}^{[j]}$ denote $M_k^{[j]}$ measurements received by the sensor j at time k. Each measurement is originated either from a target or clutter. If the measurement $z_k^{[j]}$ is generated by the target with state x_k, then

(1) $$z_k^{[j]} = h_k^{[j]}(x_k) + b_k^{[j]} + \varepsilon_k^{[j]}$$

where $h_k^{[j]}(x_k)$ is the true measurement as a function of target state x_k, $b_k^{[j]}$ is the bias vector, and $\varepsilon_k^{[j]}$ is the measurement noise vector whose covariance is denoted by $R_k^{[j]}$. Let $b_k = \left[(b_k^{[1]})^T,\cdots,(b_k^{[L]})^T\right]^T \in \chi_b$ denote the bias vector of L sensors taking values in a state space χ_b. We assume that the biases are independent and that the system dynamics of each bias is Markovian. Then, the transition density of b_k can be written by

(2) $$f_{b,k|k-1}(b_k \mid b_{k-1}) = \prod_{j=1}^{L} f_{b,k|k-1}^{[j]}(b_k^{[j]} \mid b_{k-1}^{[j]})$$

where $f_{b,k|k-1}^{[j]}(b_k^{[j]} \mid b_{k-1}^{[j]})$ is the transition density of the bias of sensor j.

The basic idea of extended CPHD filter is to augment the sensor biases into the single-target state. Define the augmented state space $\tilde{\chi} = \chi \times \chi_b$, where χ denotes the state space for single-target state, and '×' denotes a Cartesian product. The augmented state defined on the augmented space is denoted by $\tilde{x}_k = \left[(x_k)^T,(b_k)^T\right]^T \in \tilde{\chi}$. Since the target state and sensor biases are independent, the transition density of augmented state can be written as

(3) $$f_{k|k-1}(x_k, b_k \mid x_{k-1}, b_{k-1}) = f_{x,k|k-1}(x_k \mid x_{k-1}) \times f_{b,k|k-1}(b_k \mid b_{k-1})$$

where $f_{x,k|k-1}(x_k|x_{k-1})$ denote the single-target transition density at time k given previous state x_{k-1}. The survival probability of augmented state is determined by

(4) $$p_{S,k}(x_k, b_k) = p_{S,k}(x_k)$$

where $p_{S,k}(x_k)$ is the target survival probability given the target state x_k.

According to (1), the measurement of sensor j generated by the augmented state $\tilde{x}_k$ can be written as

(5) $$z_k^{[j]} = H^{[j]}(x_k, b_k) + \varepsilon_k^{[j]}$$

where $H^{[j]}(x_k, b_k) = h_k^{[j]}(x_k) + b_k^{[j]}$.

Given the augmented state $\tilde{x}_k$, the detection probability of sensor j is determined by

(6) $$p_{D,k}^{[j]}(x_k, b_k) = p_{D,k}^{[j]}(x_k)$$

where $p_{D,k}^{[j]}(x_k)$ is target detection probability of sensor j given the target state x_k.

Recursion of extended CPHD filter

By substituting the augmented state model into conventional CPHD recursion [6], we obtain the recursion of extended CPHD filter. This filter jointly propagates the posterior cardinality distribution p_k and the posterior intensity v_k for the augmented state which now includes the unknown biases of sensors. The recursion of this filter consists of the following prediction and update steps.

Prediction step:

Given the posterior intensity v_{k-1} and posterior cardinality distribution p_{k-1} at time k-1, the predicted cardinality distribution $p_{k|k-1}$ and predicted intensity $v_{k|k-1}$ can be calculated as

(7) $$p_{k|k-1}(n)=\sum_{i=0}^{n}p_{\Gamma,k}(n-i)\times\sum_{l=i}^{\infty}\frac{l!\left\langle p_{S,k},v_{k-1}\right\rangle^{i}\left\langle 1-p_{S,k},v_{k-1}\right\rangle^{l-i}}{i!(l-i)!\left\langle 1,v_{k-1}\right\rangle^{l}}p_{k-1}(l)$$

(8) $$v_{k|k-1}(\boldsymbol{x},\boldsymbol{b})=\int p_{S,k}(\boldsymbol{\zeta})f_{x,k|k-1}(\boldsymbol{x}\mid\boldsymbol{\zeta})f_{b,k|k-1}(\boldsymbol{b}\mid\boldsymbol{\eta})\times v_{k-1}(\boldsymbol{\zeta})d\boldsymbol{\zeta}d\boldsymbol{\eta}+r_k(\boldsymbol{x},\boldsymbol{b})$$

where $p_{\Gamma,k}(\bullet)$ is the cardinality distribution of births at time k, $\left\langle\bullet,\bullet\right\rangle$ is the inner product defined between two real-valued functions α and β by $\left\langle\alpha,\beta\right\rangle=\int\alpha(\boldsymbol{x})\beta(\boldsymbol{x})d\boldsymbol{x}$ (or $\left\langle\alpha,\beta\right\rangle=\sum_{l=0}^{\infty}\alpha(l)\beta(l)$ when α and β are real sequences), and $r_k(\boldsymbol{x},\boldsymbol{b})$ is the intensity of target births given the augmented state at time k.

Update step:

Considering the multi-sensor case, we sequentially update the predicted intensity $v_{k|k-1}$ and predicted cardinality distribution $p_{k|k-1}$ using each sensor' measurements at time k.

Let $p_{k|k-1}^{[j]}$ and $v_{k|k-1}^{[j]}$ be the updated results of intensity function and cardinality distribution using the measurements of sensor j, $j=1,\cdots,L$. Denote the cardinality of the set Z by $|Z|$. Set

(9) $$p_{k|k-1}^{[0]}(n)=p_{k|k-1}(n)$$

(10) $$v_{k|k-1}^{[0]}(\boldsymbol{x},\boldsymbol{b})=v_{k|k-1}(\boldsymbol{x},\boldsymbol{b})$$

the update formulas for sensor j are

(11) $$p_{k|k-1}^{[j]}(n)=\frac{\Phi_k^0[v_{k|k-1}^{[j]},Z_k^{[j]}](n)}{\left\langle\Phi_k^0[v_{k|k-1}^{[j]},Z_k^{[j]}],p_{k|k-1}^{[j-1]}\right\rangle}p_{k|k-1}^{[j-1]}(n)\qquad j=1,\cdots,L$$

(12) $$v_{k|k-1}^{[j]}(\boldsymbol{x},\boldsymbol{b})=\frac{\left\langle\Phi_k^1[v_{k|k-1}^{[j-1]},Z_k^{[j]}],p_{k|k-1}^{[j-1]}\right\rangle}{\left\langle\Phi_k^0[v_{k|k-1}^{[j-1]},Z_k^{[j]}],p_{k|k-1}^{[j-1]}\right\rangle}\times[1-p_{D,k}^{[j]}(\boldsymbol{x})]v_{k|k-1}^{[j-1]}(\boldsymbol{x},\boldsymbol{b})+\sum_{z\in Z_k^{[j]}}\frac{\left\langle\Phi_k^1[v_{k|k-1}^{[j-1]},Z_k^{[j]}\setminus\{\boldsymbol{z}\}],p_{k|k-1}^{[j-1]}\right\rangle}{\left\langle\Phi_k^0[v_{k|k-1}^{[j-1]},Z_k^{[j]}],p_{k|k-1}^{[j-1]}\right\rangle}\times\psi_{k,z}^{[j]}(\boldsymbol{x},\boldsymbol{b})v_{k|k-1}^{[j-1]}(\boldsymbol{x},\boldsymbol{b})\quad j=1,\cdots,L$$

where

(13) $$\Phi_k^u[v_{k|k-1}^{[j-1]},Z_k^{[j]}](n)=\sum_{i=0}^{\min(|Z_k^{[j]}|,n)}(|Z_k^{[j]}|-i)!\times p_{K,k}^{[j]}(|Z_k^{[j]}|-i)\frac{n!\left\langle 1-p_{D,k}^{[j]},v_{k|k-1}^{[j-1]}\right\rangle^{n-(i+u)}}{(n-i-u)!\left\langle 1,v_{k|k-1}^{[j-1]}\right\rangle^{n}}\times e_i(\Omega(v_{k|k-1}^{[j-1]},Z_k^{[j]}))$$

(14) $$\psi_{k,z}^{[j]}(\boldsymbol{x},\boldsymbol{b})=g_k^{[j]}(\boldsymbol{z}\mid\boldsymbol{x},\boldsymbol{b})p_{D,k}^{[j]}(\boldsymbol{x})\left\langle 1,k_k^{[j]}\right\rangle\Big/k_k^{[j]}(\boldsymbol{z})$$

(15) $$\Omega(v_{k|k-1}^{[j-1]},Z_k^{[j]})=\left\{\left\langle v_{k|k-1}^{[j-1]},\psi_{k,z}^{[j]}\right\rangle:\boldsymbol{z}\in Z_k^{[j]}\right\}$$

$Z_k^{[j]}$ is the set of meaurements received by sensor j at time k, $p_{K,k}^{[j]}(\bullet)$ is the cardinality distribution of clutter measurements generated by sensor j at time k, $e_i(\bullet)$ is the elementary symmetric function [6] of order i, $k_k^{[j]}(\bullet)$ is the intensity of clutter measurements of sensor j, and $g_k^{[j]}(\bullet|\,\boldsymbol{x},\boldsymbol{b})$ is the single-target measurement likelihood of sensor j given the augmented state $(\boldsymbol{x},\boldsymbol{b})$.

After using the measurements of each sensor, the updated cardinality distribution p_k and updated intensity v_k at time k are determined by

(16) $$p_k(n)=p_{k|k-1}^{[L]}(n)$$

(17) $$v_k(\boldsymbol{x},\boldsymbol{b})=v_{k|k-1}^{[L]}(\boldsymbol{x},\boldsymbol{b})$$

The number of targets can be estimated using the maximum a posteriori (MAP) estimator $\hat{N}_k=\arg\max p_k(\bullet)$, and the estimates of target states $\{\hat{\boldsymbol{x}}_{k,1},\cdots,\hat{\boldsymbol{x}}_{k,\hat{N}_k}\}$ can be extracted by picking the $\hat{N}_k$ largest local maxima of $v_k(\boldsymbol{x},\boldsymbol{b})$. Since the sensor biases are the same for all targets, the estimates of biases at time k can be derived by

(18) $$\hat{\boldsymbol{b}}_k=\int_{\tilde{\chi}}\boldsymbol{b}v_k(\boldsymbol{x},\boldsymbol{b})d\boldsymbol{x}d\boldsymbol{b}\Big/\int_{\tilde{\chi}}v_k(\boldsymbol{x},\boldsymbol{b})d\boldsymbol{x}d\boldsymbol{b}$$

Implementation of extended CPHD filter

Consider the following assumptions:

Assumption 1. The system dynamics of single-target state and sensor biases follow Gaussian dynamical models. Then, the system dynamics of the augmented state also follows a Gaussian model. Let $\tilde{\boldsymbol{F}}_k(\bullet)$ and $\tilde{\boldsymbol{Q}}_{k-1}$ denote the transition function and the covariance of process noise for the augmented state.

Assumption 2. The measurement likelihood is a Gaussian density.

Assumption 3. The survival and detection probabilities are state independent.

Assumption 4. The birth intensity can be considered as a Gaussian mixture of the form

(19) $$r_k(\tilde{\boldsymbol{x}})=\sum_{i=1}^{J_{r,k}}w_{r,k}^{(i)}N(\tilde{\boldsymbol{x}};\tilde{\boldsymbol{m}}_{r,k}^{(i)},\tilde{\boldsymbol{P}}_{r,k}^{(i)})=\sum_{i=1}^{J_{r,k}}w_{r,k}^{(i)}N(\boldsymbol{x};\boldsymbol{m}_{r,k}^{(i)},\boldsymbol{P}_{r,k}^{(i)})N(\boldsymbol{b};\boldsymbol{m}_{rb,k}^{(i)},\boldsymbol{P}_{rb,k}^{(i)})$$

where $N(\bullet;\,\boldsymbol{m},\,\boldsymbol{P})$ denotes a Gaussian density with mean $\boldsymbol{m}$ and covariance $\boldsymbol{P}$, $N(\boldsymbol{x};\boldsymbol{m}_{r,k}^{(i)},\boldsymbol{P}_{r,k}^{(i)})$ and $N(\boldsymbol{b};\boldsymbol{m}_{rb,k}^{(i)},\boldsymbol{P}_{rb,k}^{(i)})$ are corresponding to the target state and the sensor biases.

Under these assumptions, the implementations of prediction and update steps of extended CPHD filter are shown below.

Prediction step:

At time k-1, the posterior density v_{k-1} and posterior cardinality distribution p_{k-1} are given, and v_{k-1} is a Gaussian mixture of the form

(20) $$v_{k-1}(\tilde{\boldsymbol{x}})=\sum_{i=1}^{J_{k-1}}w_{k-1}^{(i)}N(\tilde{\boldsymbol{x}};\tilde{\boldsymbol{m}}_{k-1}^{(i)},\tilde{\boldsymbol{P}}_{k-1}^{(i)})$$

According to (7), the predicted cardinality distribution $p_{k|k-1}$ becomes

(21) $$p_{k|k-1}(n)=\sum_{i=0}^{n}p_{\Gamma,k}(n-i)\sum_{l=i}^{\infty}\frac{l!}{i!(l-i)!}\times p_{k-1}(l)p_{S,k}^{i}(1-p_{S,k})^{l-i}$$

Considering the nonlinearity of transition function $\tilde{\boldsymbol{F}}(\bullet)$, we use the unscented transformation [8] with mean $\tilde{\boldsymbol{m}}_{k-1}^{(i)}$ and covariance $\tilde{\boldsymbol{P}}_{k-1}^{(i)}$ to generate a set of sigma points and weights, denoted by $\{\boldsymbol{\chi}_{k-1,p}^{(i)},w_{m,p},w_{c,p}\},\ p=1,\cdots,M$, where M is the number of sigma points. Then, the sigma points are propagated through the transition function as follows

(22) $$\tilde{\boldsymbol{x}}_{k|k-1,p}^{(i)}=\tilde{\boldsymbol{F}}(\boldsymbol{\chi}_{k-1,p}^{(i)})$$

According to (8), the predicted intensity $v_{k|k-1}$ can be approximated by a Gaussian mixture of the form

(23) $$v_{k|k-1}(\tilde{\boldsymbol{x}})=p_{S,k}\sum_{i=1}^{J_{k-1}}w_{k-1}^{(i)}N(\tilde{\boldsymbol{x}};\tilde{\boldsymbol{m}}_{S,k|k-1}^{(i)},\tilde{\boldsymbol{P}}_{S,k|k-1}^{(i)})+r_k(\tilde{\boldsymbol{x}})$$

where

(24) $$\tilde{\boldsymbol{m}}_{S,k|k-1}^{(i)}=\sum_{p=1}^{M}w_{m,p}\tilde{\boldsymbol{x}}_{k|k-1,p}^{(i)}$$

(25) $$\tilde{\boldsymbol{P}}_{S,k|k-1}^{(i)}=\sum_{p=1}^{M}w_{c,p}(\tilde{\boldsymbol{x}}_{k|k-1,p}^{(i)}-\tilde{\boldsymbol{m}}_{S,k|k-1}^{(i)})\times(\tilde{\boldsymbol{x}}_{k|k-1,p}^{(i)}-\tilde{\boldsymbol{m}}_{S,k|k-1}^{(i)})^T+\tilde{\boldsymbol{Q}}_{k-1}$$

Update step:

At time k, the predicted PHD $v_{k|k-1}$ and predicted cardinality distribution $p_{k|k-1}$ are given, and $v_{k|k-1}$ is a Gaussian mixture of the form

(26) $$v_{k|k-1}(\tilde{\boldsymbol{x}})=\sum_{i=1}^{J_{k|k-1}}w_{k|k-1}^{(i)}N(\tilde{\boldsymbol{x}};\tilde{\boldsymbol{m}}_{k|k-1}^{(i)},\tilde{\boldsymbol{P}}_{k|k-1}^{(i)})$$

The predicted intensity $v_{k|k-1}$ and predicted cardinality distribution $p_{k|k-1}$ are sequentially updated using each sensor' measurements.

Set

(27) $$p_{k|k-1}^{[0]}(n)=p_{k|k-1}(n)$$

(28) $$v_{k|k-1}^{[0]}(\tilde{\boldsymbol{x}})=v_{k|k-1}(\tilde{\boldsymbol{x}})$$

So, $v_{k|k-1}^{[0]}(\tilde{\boldsymbol{x}})$ is a Gaussian mixture written by

(29) $$v_{k|k-1}^{[0]}(\tilde{\boldsymbol{x}})=\sum_{i=1}^{J_{k|k-1}^{[0]}}w_{k|k-1}^{(i)[0]}N(\tilde{\boldsymbol{x}};\tilde{\boldsymbol{m}}_{k|k-1}^{(i)[0]},\tilde{\boldsymbol{P}}_{k|k-1}^{(i)[0]})$$

According to the recursion (12), the updated intensity $v_{k|k-1}^{[j]}(\tilde{\boldsymbol{x}})$ using the measurements of sensor j is also a Gaussian mixture denoted by

(30) $$v_{k|k-1}^{[j]}(\tilde{\boldsymbol{x}})=\sum_{i=1}^{J_{k|k-1}^{[j]}}w_{k|k-1}^{(i)[j]}N(\tilde{\boldsymbol{x}};\tilde{\boldsymbol{m}}_{k|k-1}^{(i)[j]},\tilde{\boldsymbol{P}}_{k|k-1}^{(i)[j]})\quad j=1,\cdots,L$$

Considering the nonlinearity of measurement function $\boldsymbol{H}^{[j]}(\bullet)$, a set of sigma points and weights denoted by $\{\boldsymbol{\chi}_{k|k-1,p}^{(i)[j-1]},w_{m,p},w_{c,p}\},\ p=1,\cdots,M,$ are generated by unscented transformation with mean $\tilde{\boldsymbol{m}}_{k|k-1}^{(i)[j-1]}$ and covariance $\tilde{\boldsymbol{P}}_{k|k-1}^{(i)[j-1]}$. Then, the predicted measurement for the sigma point $\boldsymbol{\chi}_{k|k-1,p}^{(i)[j-1]}$ is

(31) $$\boldsymbol{z}_{k|k-1,p}^{(i)[j]}=\boldsymbol{H}^{[j]}(\boldsymbol{\chi}_{k|k-1,p}^{(i)[j-1]})$$

The update formulae (corresponding to (11) and (12)) using measurements of sensor j become

(32) $$p_{k|k-1}^{[j]}(n)=\frac{\Theta_k^0[\boldsymbol{w}_{k|k-1}^{[j-1]},Z_k^{[j]}](n)}{\left\langle\Theta_k^0[\boldsymbol{w}_{k|k-1}^{[j-1]},Z_k^{[j]}],p_{k|k-1}^{[j-1]}\right\rangle}p_{k|k-1}^{[j-1]}(n)$$

(33) $$v_{k|k-1}^{[j]}(\tilde{\boldsymbol{x}})=\frac{\left\langle\Theta_k^1[\boldsymbol{w}_{k|k-1}^{[j-1]},Z_k^{[j]}],p_{k|k-1}^{[j-1]}\right\rangle}{\left\langle\Theta_k^0[\boldsymbol{w}_{k|k-1}^{[j-1]},Z_k^{[j]}],p_{k|k-1}^{[j-1]}\right\rangle}\times(1-p_{D,k}^{[j]})v_{k|k-1}^{[j-1]}(\tilde{\boldsymbol{x}})+\sum_{z\in Z_k^{[j]}}\sum_{i=1}^{J_{k|k-1}^{[j-1]}}w_k^{(i)[j]}(\boldsymbol{z})N(\tilde{\boldsymbol{x}};\tilde{\boldsymbol{m}}_k^{(i)[j]}(\boldsymbol{z}),\tilde{\boldsymbol{P}}_k^{(i)[j]})$$

where

(34) $$\boldsymbol{w}_{k|k-1}^{[j-1]}=\left[w_{k|k-1}^{(1)[j-1]},\cdots,w_{k|k-1}^{(J_{k|k-1}^{[j-1]})[j-1]}\right]^T$$

(35) $$\Theta_k^u[\boldsymbol{w}_{k|k-1}^{[j-1]},Z_k^{[j]}](n)=\sum_{i=0}^{\min(|Z_k^{[j]}|,n)}(|Z_k^{[j]}|-i)!\times p_{K,k}^{[j]}(|Z_k^{[j]}|-i)\frac{n!(1-p_{D,k})^{n-(i+u)}}{(n-i-u)!\left\langle 1,\boldsymbol{w}_{k|k-1}^{[j-1]}\right\rangle^{i+u}}\times e_i(\Delta(\boldsymbol{w}_{k|k-1}^{[j-1]},Z_k^{[j]}))$$

(36) $$\Delta(\boldsymbol{w}_{k|k-1}^{[j-1]},Z_k^{[j]})=\left\{\frac{\left\langle 1,k_k^{[j]}\right\rangle}{k_k^{[j]}(\boldsymbol{z})}p_{D,k}^{[j]}(\boldsymbol{w}_{k|k-1}^{[j-1]})^T\boldsymbol{q}_k^{[j]}(\boldsymbol{z}):\boldsymbol{z}\in Z_k^{[j]}\right\}$$

(37) $$\boldsymbol{q}_k^{[j]}(\boldsymbol{z})=\left[q_k^{(1)[j]}(\boldsymbol{z}),\cdots,q_k^{(J_{k|k-1}^{[j-1]})[j]}(\boldsymbol{z})\right]^T$$

(38) $$q_k^{(i)[j]}(\boldsymbol{z})=N(\boldsymbol{z};\boldsymbol{\eta}_{k|k-1}^{(i)[j]},\boldsymbol{S}_{k|k-1}^{(i)[j]})$$

(39) $$\boldsymbol{\eta}_{k|k-1}^{(i)[j]}=\sum_{p=1}^{M}w_{m,p}\boldsymbol{z}_{k|k-1,p}^{(i)[j]}$$

(40) $$\boldsymbol{S}_{k|k-1}^{(i)[j]}=\sum_{p=1}^{M}w_{c,p}(\boldsymbol{z}_{k|k-1,p}^{(i)[j]}-\boldsymbol{\eta}_{k|k-1}^{(i)[j]})(\boldsymbol{z}_{k|k-1,p}^{(i)[j]}-\boldsymbol{\eta}_{k|k-1}^{(i)[j]})^T+\boldsymbol{R}_k^{[j]}$$

(41) $$w_k^{(i)[j]}(\boldsymbol{z})=p_{D,k}^{[j]}w_{k|k-1}^{(i)[j-1]}q_k^{(i)[j]}(\boldsymbol{z})\times\frac{\left\langle\Theta_k^1[\boldsymbol{w}_{k|k-1}^{[j-1]},Z_k^{[j]}\setminus\{\boldsymbol{z}\}],p_{k|k-1}\right\rangle\left\langle 1,\kappa_k^{[j]}\right\rangle}{\left\langle\Theta_k^0[\boldsymbol{w}_{k|k-1}^{[j-1]},Z_k^{[j]}],p_{k|k-1}\right\rangle\kappa_k^{[j]}(\boldsymbol{z})}$$

(42) $$\boldsymbol{L}_{k|k-1}^{(i)[j]}=\sum_{p=1}^{M}w_{c,p}(\boldsymbol{\chi}_{k|k-1,p}^{(i)[j-1]}-\tilde{\boldsymbol{m}}_{k|k-1}^{(i)[j-1]})\times(\boldsymbol{z}_{k|k-1,p}^{(i)[j]}-\boldsymbol{\eta}_{k|k-1}^{(i)[j]})^T$$

(43) $$\tilde{\boldsymbol{m}}_k^{(i)[j]}(\boldsymbol{z})=\tilde{\boldsymbol{m}}_{k|k-1}^{(i)[j-1]}+\boldsymbol{L}_{k|k-1}^{(i)[j]}(\boldsymbol{S}_{k|k-1}^{(i)[j]})^{-1}(\boldsymbol{z}-\boldsymbol{\eta}_{k|k-1}^{(i)[j]})$$

(44) $$\boldsymbol{P}_k^{(i)[j]}=\boldsymbol{P}_{k|k-1}^{(i)[j-1]}-\boldsymbol{L}_{k|k-1}^{(i)[j]}((\boldsymbol{S}_{k|k-1}^{(i)[j]})^{-1})^T(\boldsymbol{L}_{k|k-1}^{(i)[j]})^T$$

We obtain the estimate of target number $\hat{N}_k$ by using MAP estimator. The estimates of multi-target states and $\hat{N}_k$ sensor biases $\{\hat{\boldsymbol{b}}_{k,1},\cdots,\hat{\boldsymbol{b}}_{k,\hat{N}_k}\}$ can be extracted by picking the means of $\hat{N}_k$ Gaussian terms of posterior density v_k with the largest weights $\{w_{k,1},\cdots,w_{k,\hat{N}_k}\}$. According to (18), the estimates of sensor biases can be determined by

(45) $$\hat{\boldsymbol{b}}_k=\sum_{i=1}^{\hat{N}_k}w_{k,i}\hat{\boldsymbol{b}}_{k,i}\Big/\sum_{i=1}^{\hat{N}_k}w_{k,i}$$

Simulation results

Consider a two-dimensional tracking scenario with four targets observed by three synchronous sensors as shown in Figure 1. These targets have various birth and death time. The state of each target is a vector of position and velocity $\boldsymbol{x}_k = [x_k, y_k, \dot{x}_k, \dot{y}_k]^T$, and follows a linear Gaussian dynamical model given by

(46) $$f_{x,k|k-1}(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}) = N(\boldsymbol{x}_k; \boldsymbol{F}_k \boldsymbol{x}_{k-1}, \boldsymbol{Q}_{k-1})$$

where

(47) $$\boldsymbol{F}_k = \begin{bmatrix} \boldsymbol{I}_2 & \Delta t \boldsymbol{I}_2 \\ \boldsymbol{0}_2 & \boldsymbol{I}_2 \end{bmatrix} \qquad \boldsymbol{Q}_{k-1} = \sigma_w^2 \begin{bmatrix} \frac{\Delta t^4}{4}\boldsymbol{I}_2 & \frac{\Delta t^3}{2}\boldsymbol{I}_2 \\ \frac{\Delta t^3}{2}\boldsymbol{I}_2 & \Delta t^2 \boldsymbol{I}_2 \end{bmatrix}$$

$\boldsymbol{I}_n$ and $\boldsymbol{0}_n$ denote the $n\times n$ identity and zero matrices, Δt=1s is the sampling period, and σ_w=0.01m/s^2 is the standard deviation of the process noise. The probability of target survival is fixed to $P_{S,k}$=0.99.

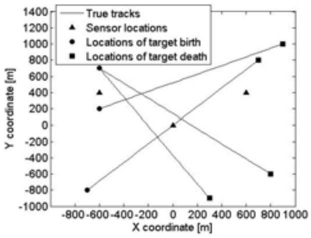


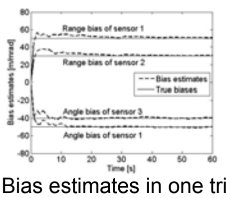
Fig.1. True target trajectories and sensor locations

Sensor 1 located at (600,400)m generates range and bearing measurements. Sensor 2 located at (0,0)m generates range measurements. Sensor 3 located at (-600,400)m generates bearing measurements. The measurement noise of these sensors is Gaussian white noise with the covariances $\boldsymbol{R}_k^{[1]} = \mathrm{diag}([12.5\mathrm{m}, 12.5\mathrm{mrad}]^2)$, $\boldsymbol{R}_k^{[2]} = [10\mathrm{m}]^2$ and $\boldsymbol{R}_k^{[3]} = [10\mathrm{mrad}]^2$. For many real-world problems, the bias usually does not drift against time. The Gaussian dynamic model of biases $\boldsymbol{b}_k$ is given by

(48) $$f_{b,k|k-1}(\boldsymbol{b}_k \mid \boldsymbol{b}_{k-1}) = \prod_{j=1}^{3} N(\boldsymbol{b}_k^{[j]}; \boldsymbol{b}_{k-1}^{[j]}, \boldsymbol{Q}_{b,k-1}^{[j]})$$

The means of $\boldsymbol{b}_k^{[1]}$, $\boldsymbol{b}_k^{[2]}$ and $\boldsymbol{b}_k^{[3]}$ are [50m,-50mrad], 30m and -40mrad. $\boldsymbol{Q}_{b,k-1}^{[1]} = \mathrm{diag}([0.025\mathrm{m}, 0.025\mathrm{mrad}]^2)$, $\boldsymbol{Q}_{b,k-1}^{[2]} = [0.025\mathrm{m}]^2$, and $\boldsymbol{Q}_{b,k-1}^{[3]} = [0.025\mathrm{mrad}]^2$. The detection probability of each sensor is 0.98. The clutter of each sensor is modeled as a Poisson random finite set with mean rate of 20 points per scan and uniform distribution over the surveillance region.

Figure 2 shows the bias estimates at each time step in one trial. We can see that all the estimates of biases converge to the true biases as time progressing.



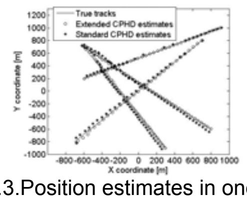
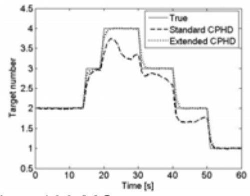


Fig.2.Bias estimates in one trial Fig.3.Position estimates in one trial

The estimates of target positions by standard CPHD (without sensor alignment) and extended CPHD filters in one trial are shown in Figure 3. It can be seen that the estimates of standard CPHD have noticeable deviation from the true tracks and several estimates are lost. On the other hand, the estimates of extended CPHD approximate the true tracks after several time steps.

Figure 4 shows 100 (Monte Carlo) MC runs average of the estimated target number. During the most of the period, the target number estimates of standard CPHD filter are smaller than the true number, whereas the extended CPHD filter' estimates are closed to the true number. It can be seen that the estimates of target number for extended CPHD filter does not immediately respond to the changes of target number. However, during the time intervals when the number of targets is steady, the extended CPHD filter gives unbiased estimation. The optimal sub-pattern assignment (OSPA) [9] metric is used for performance evaluation. Figure 5 shows 100 MC runs average of OSPA with parameters p=2 and c=100m on estimated position. It can be seen that the extended CPHD filter performs much better than standard CPHD filter. The OSPA of extended CPHD filter exhibits high peaks due to its delayed response to the changes of target number.



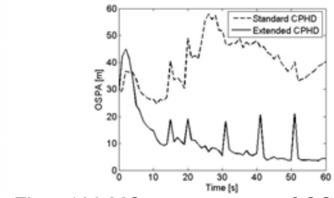


Fig.4.100 MC runs average of target number estimates Fig.5.100 MC runs average of OSPA

Conclusions

An extended CPHD filter for combining the MTT with the sensor alignment is proposed. This filter can jointly estimate the number and the states of the targets and the sensor biases. Simulation results show that the proposed filter successfully achieves on-line sensor alignment in the MTT problem and outperforms the standard CPHD filter in terms of the accuracy of the estimations for the target number and the target states.

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***Authors**: M.Sc. Dong Li, Prof. Dongyun Yi, Department of Mathematics and Systems Science, National University of Defense Technology, Changsha 410073, China, E-mail: nudtlidong@163.com, dongyun.yi@gmail.com.*