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A Fast and Efficient 3D Medical Image Registration Method

Abstract. *A fast and efficient 3D medical image registration method based on adjusting divergence and curl of image displacement field is presented. Using the fact that an image registration problem can be formulated as an optimal control problem, corresponding optimization problem is reduced to solving several Poisson equations. These Poisson equations are solved by finite element (FEM) and multigrid methods (MG), separately. Computational examples indicate that both solution approaches produce similarly good registration quality but that the cost associated with the multigrid approach is, on the average, less than that for the FEM.*

Streszczenie. *W artykule przedstawiono metodę rejestracji obrazu medycznego 3D, opartą na dywergencji i rotacji przesunięć obrazu. W rozwiązaniu zastosowano równania Poissona, które analizowano niezależnie, metodą elementów skończonych oraz wielosiatkową. Obliczenia pokazują podobną jakość rejestracji obrazu dla obydwu metod, lecz metoda wielosiatkowa ma mniejsze koszty obliczeniowe.* ***(Szybka i efektywna metoda rejestracji obrazu medycznego 3D).***

Keywords: Optimization, image registration, finite element, multigrid method.
Słowa kluczowe: optymalizacja, rejestracja obrazu, element skończone, metoda wielosiatkowa.

Introduction

The goal of image registration is to align two or more images of the same scene obtained at different times, perspectives or sensors such as MRI, X-ray, CT, PET, SPECT and tomography. Given a *reference image* $\mathbf{R}(\mathbf{x})$ and a *template image* $\mathbf{T}(\mathbf{x})$ the main idea behind image registration paradigm is to find a *reasonable* transformation such that transformed image becomes *similar* to the reference image. Image registration has a broad range of applications such as object or motion tracking, detecting tumors, image fusion among many other (see, for example, [7, 8]). Image registration is a significant and challenging subject which usually involves high storage requirements, high CPU costs and mostly deals with noisy, distorted and occluded data. In literature many different types of image registration techniques (see, for instance, [1, 2, 4, 5, 6] and [9, 10]) were developed, some of these are landmark-based, principal axes-based, elastic, fluid, diffusion and curvature-based registration algorithms. Each of these algorithms was generated based on a specific application, disease or image modality. There is still not any general image registration technique which could be used in every sorts of data. Based on these facts finding fast and efficient image registration techniques are quite useful and still significantly important area of research.

Organization of the paper is as follows. In the first section we overview the methodology behind image registration paradigm. In Section 2 we present a new method for non-rigid registration of 3D medical images. We express the image registration problem as an optimization problem and use the sum of squared difference as the similarity metric. Using Lagrange multipliers method we obtain the optimality system consisting of state and costate equations as well as optimality conditions. From the obtained optimality system we obtain several Poisson different equations. In Section 3 we solve these Poisson equations with FEM and MG methods. Computational examples indicate that both of these numerical solution approaches produce similarly good registration quality but that the cost associated with the multigrid approach is, on the average, less than that for the FEM.

Optimal Control Approach for Image Registration

The state-of-the-art of the image registration problem can be expressed in the following way. Assume that both the template $\mathbf{T}$ and reference $\mathbf{R}$ images are defined on the same domain Ω. Then, the image registration problem can be formulated as the optimization problem

$$\min_{\boldsymbol{\phi}\in\boldsymbol{\Gamma}} \mathcal{J}[\mathbf{R},\mathbf{T};\boldsymbol{\phi}_u] \tag{1}$$

for the functional

$$\mathcal{J}[\mathbf{R},\mathbf{T};\boldsymbol{\phi}_u] = C_{sim}[\mathbf{R},\mathbf{T};\boldsymbol{\phi}_u] + \lambda C_{reg}[u], \tag{2}$$

where $C_{sim}[\mathbf{R},\mathbf{T};\boldsymbol{\phi}_u]$ denotes a similarity measure between the template image $\mathbf{T}$ and the reference image $\mathbf{R}$, $\boldsymbol{\phi}_u(\mathbf{x}) := \mathbf{x} + u(\mathbf{x})$ is the deformation field, u is displacement field, Γ is the set of all possible admissible transformations, $C_{reg}[u]$ is a regularization term and, λ is a regularization constant. Because reference and template images are obtained from different distances, angles, times and sometimes even by different individuals, a deformation field may occur between these images. A deformation field is a vector field that maps pixels (or coordinates) of reference image to the corresponding ones of the template image. One of the major goals of this paper is to compute the deformation field in a systematic way.

We choose the L^2-norm type similarity measure defined as

$$C_{sim}[\mathbf{R}(\mathbf{x}),\mathbf{T}(\mathbf{x});\boldsymbol{\phi}(\mathbf{x})] = \frac{1}{2}\int_\Omega \big(\mathbf{T}(\mathbf{x}+\mathbf{u}(\mathbf{x}))-\mathbf{R}(\mathbf{x})\big)^2\, d\mathbf{x}. \tag{3}$$

This similarity measure is often referred to as the “sum of squared differences” (SSD) measure. Note that other similarity measures can be selected depending on the problem. We choose the similarity measure (3) due to its well-known effectiveness, for the convenience in computations and for easily adapting the regularization terms in numerical solutions.

While $u = (u_1, u_2, u_3)$ denotes the image displacement field, we minimize (3) subject to the constraints

$$\begin{aligned}
div\, u &= \nabla\cdot u = u_{1_{x_1}} + u_{2_{x_2}} + u_{3_{x_3}} := f^1 - 1,\\
curl\, u &= (curl_{x_1}u,\ curl_{x_2}u,\ curl_{x_3}u),\\
&= (u_{3_{x_2}} - u_{2_{x_3}}, u_{1_{x_3}} - u_{3_{x_1}}, u_{2_{x_1}} - u_{1_{x_2}})\\
&:= (f^2, f^3, f^4)\\
u &= 0 \text{ on } \partial\Omega.
\end{aligned}$$

Let us do notice that because we are able to control the translation and rotation of image pixels by means of $div\, u$ and $curl\, u$, we are using these constraints in the corresponding optimization problem. In order to solve this constrained optimization problem, we express it as an unconstrained optimization problem using the Lagrange multipliers method where $v = (v_1, v_2, v_3, v_4)$ are Lagrange multipliers.

Specializing an abstract theorem concerning the existence of Lagrange multipliers for minimizations on Banach space [3], we present the following theorem:

Theorem 0.1 *Let V_1 and V_2 be two Hilbert spaces, $\mathcal{F}$ a functional on V_1, and $\mathcal{G}$ a mapping from V_1 to V_2. Assume $\hat{u}$ is a solution of the following constrained minimization problem: Find $u \in V_1$ that minimizes $\mathcal{F}(u)$ subject to $\mathcal{G}(u) = 0$. Assume further that the following conditions are satisfied:*
(i) $\mathcal{F} : Nbhd(\widehat{u}) \subset V_1 \to \mathbf{R}$ is Frechet-differentiable at $\widehat{u}$;
(ii) $\mathcal{G}$ is continuously Frechet-differentiable at $\widehat{u}$;
(iii) $\mathcal{G}'(\widehat{u}) : V_1 \to V_2$ is onto.
Then, there exists a $\mu \in (V_2)^*$ such that

$$\mathcal{F}'(\widehat{u})v - \langle \mu, \mathcal{G}'(\widehat{u})v \rangle = 0, \qquad \forall v \in V_1.$$

Proof: See [3], Theorem 43.19. Here, $\langle \cdot, \cdot \rangle$ denotes the duality pairing between V_2 and $(V_2)^*$ and $\mathcal{F}'(\widehat{u})v$ and $\mathcal{G}'(\widehat{u})v$ denote the actions of $\mathcal{F}'(\widehat{u})$ as an operator mapping $v \in V_1$ into $\mathbf{R}$ and $\mathcal{G}'(\widehat{u})$ as an operator mapping $v \in V_1$ into V_2, respectively. We will fit our optimization problem into the above abstract framework.

By using Lagrange multipliers optimization method, we can express the aforementioned optimization problem as (4) in page 3 below.
Minimizer of this Lagrange functional results an optimality system which consists of state equations, co-state equations, and optimality conditions. Next we obtain the corresponding optimality system.
State Equations: The state equations are obtained from $L_{v_1} = 0,\quad L_{v_2} = 0,\quad L_{v_3} = 0$ and $L_{v_4} = 0$, where L_{v_j} denotes the Frèchet derivative of L.

$$\begin{aligned} L_{v_1} &= \frac{d}{d\varepsilon}\Big|_{\varepsilon=0} L[v_1 + \varepsilon\delta v_1] \\ &= \frac{d}{d\varepsilon}\Big|_{\varepsilon=0} \int_\Omega (v_1 + \varepsilon\delta v_1)(div\, u - f^1) \\ &= \int_\Omega \delta v_1 (div\, u - f^1) = 0 \quad \text{for every} \quad \delta v_1. \end{aligned}$$

Then,

$$div\, u(\mathbf{x}) = f^1(\mathbf{x}).$$

In the similar way, we obtain the other state equations as follows:

$$\begin{aligned} L_{v_1} = 0 &\quad\Rightarrow\quad div\, u = f^1, \\ L_{v_2} = 0 &\quad\Rightarrow\quad curl_{x_1} u = f^2, \\ L_{v_3} = 0 &\quad\Rightarrow\quad curl_{x_2} u = f^3, \\ L_{v_4} = 0 &\quad\Rightarrow\quad curl_{x_3} u = f^4. \end{aligned}$$

Costate equations: The costate equations are obtained from the equations $L_{u_1} = 0,\ L_{u_2} = 0$ and $L_{u_3} = 0$.

$$\begin{aligned} L_{u_1} &= \frac{d}{d\varepsilon}\Big|_{\varepsilon=0} \Big[\frac{1}{2}\int_\Omega [T(x_1 + u_1(\mathbf{x}) + \varepsilon\delta u_1(\mathbf{x}), \\ x_2 &+ u_2(\mathbf{x}), x_3 + u_3(\mathbf{x})) - R(\mathbf{x})]^2 \\ &+ \int_\Omega v_1(div\,(u_1 + \varepsilon\delta u_1, u_2, u_3) - f^1) \\ &+ \int_\Omega v_3(curl_{x_2}\,(u_1 + \varepsilon\delta u_1, u_2, u_3) - f^3) \\ &+ \int_\Omega v_4(curl_{x_3}\,(u_1 + \varepsilon\delta u_1, u_2, u_3) - f^4)\Big] \\ &= \int_\Omega (T(\mathbf{x} + u(\mathbf{x})) - R(\mathbf{x}))\, T_{\phi_1}\, \delta u_1 + \int_\Omega v_1(\delta u_1)_{x_1} \\ &+ \int_\Omega v_3(\delta u_1)_{x_3} + \int_\Omega v_4(-\delta u_1)_{x_2} \\ &= \int_\Omega (T(\mathbf{x} + u(\mathbf{x})) - R(\mathbf{x}))\, T_{\phi_1}\, \delta u_1 \\ &+ \int_\Omega (v_1, -v_4, v_3) \cdot \nabla\delta u_1 \\ &= \int_\Omega [(T - R)\, T_{\phi_1}\, \delta u_1 - \nabla \cdot (v_1, -v_4, v_3)\delta u_1] \\ &= \int_\Omega [(T - R)\, T_{\phi_1} - \nabla \cdot (v_1, -v_4, v_3)]\, \delta u_1 = 0 \end{aligned}$$

for every δu_1, which gives us the first costate equation

$$\nabla \cdot (v_1, -v_4, v_3) = (T - R)T_{\phi_1}.$$

Hence, the costate equations are given by

$$\begin{aligned} L_{u_1} = 0 &\quad\Rightarrow\quad (T - R)\, T_{\phi_1} = \nabla \cdot (v_1, -v_4, v_3), \\ L_{u_2} = 0 &\quad\Rightarrow\quad (T - R)\, T_{\phi_2} = \nabla \cdot (v_4, v_1, -v_2), \\ L_{u_3} = 0 &\quad\Rightarrow\quad (T - R)\, T_{\phi_3} = \nabla \cdot (-v_3, v_2, v_1). \end{aligned}$$

Optimality conditions: The optimality conditions are obtained from the equations $L_{f^1} = 0,\ L_{f^2} = 0,\ L_{f^3} = 0$ and $L_{f^4} = 0$.

$$\begin{aligned} L_{f^1} &= \frac{d}{d\varepsilon}\Big|_{\varepsilon=0} \Big[\frac{w_1}{2}\int_\Omega (f^1 + \varepsilon\delta f^1)^2 \\ &+ \int_\Omega v_1(div\, u - (f^1 + \varepsilon\delta f^1))\Big] \\ &= \int_\Omega (w_1 f^1 - v_1)\delta f^1 = 0 \text{ for every } \delta f^1, \end{aligned}$$

which gives us the first optimality condition

$$w_1 f^1 = v_1.$$

Hence, the optimality conditions are given by

$$\begin{aligned} L_{f^1} = 0 &\quad\Rightarrow\quad w_1 f^1 = v_1, \\ L_{f^2} = 0 &\quad\Rightarrow\quad w_2 f^2 = v_2, \\ L_{f^3} = 0 &\quad\Rightarrow\quad w_3 f^3 = v_3, \\ L_{f^4} = 0 &\quad\Rightarrow\quad w_4 f^4 = v_4,. \end{aligned}$$

Decoupling: We will reduce this system of equations to a set of Poisson equations in the following way: Define $F_1 := f^1 - 1,\quad F_2 := f^2,\quad F_3 := f^3,\quad F_4 := f^4$. Then, the

$$L[u;v;f] = \frac{1}{2}\int_\Omega [T(\phi(\mathbf{x})) - R(\mathbf{x})]^2\, d\mathbf{x} + \frac{w_1}{2}\int_\Omega (f^1)^2(\mathbf{x})d\mathbf{x} + \frac{w_2}{2}\int_\Omega (f^2)^2(\mathbf{x})d\mathbf{x} + \frac{w_3}{2}\int_\Omega (f^3)^2(\mathbf{x})d\mathbf{x} \quad + $$

$$(4) \qquad \frac{w_4}{2}\int_\Omega (f^4)^2(\mathbf{x})d\mathbf{x} \int_\Omega v_1(\mathbf{x})(div\, u(\mathbf{x}) - f^1(\mathbf{x}))d\mathbf{x} + \int_\Omega v_2(\mathbf{x})(curl_{x_1}\, u(\mathbf{x}) - f^2(\mathbf{x}))d\mathbf{x} \quad +$$

$$\int_\Omega v_3(\mathbf{x})(curl_{x_2}\, u(\mathbf{x}) - f^3(\mathbf{x}))d\mathbf{x} + \int_\Omega v_4(\mathbf{x})(curl_{x_3}\, u(\mathbf{x}) - f^4(\mathbf{x}))d\mathbf{x}.$$

state equations imply that

$$\begin{aligned} \Delta u_1 &= F_{1_{x_1}} + F_{3_{x_3}} - F_{4_{x_2}}, \\ \Delta u_2 &= F_{1_{x_2}} - F_{2_{x_3}} + F_{4_{x_1}}, \\ \Delta u_3 &= F_{1_{x_3}} + F_{2_{x_3}} - F_{3_{x_1}},. \end{aligned}$$

For simplicity, letting $w_2 = w_3 = w_4$, and defining $G := (G_1, G_2, G_3)$ as $G_i := (T - R)\, T_{\phi_i}$, $i = 1, 2, 3$, and using the fact that div curl u$= 0$, we obtain

$$\begin{aligned} \Delta v_1 &= G_{1_{x_1}} + G_{2_{x_2}} + G_{3_{x_3}}, \\ \Delta v_2 &= -G_{2_{x_3}} + G_{3_{x_2}}, \\ \Delta v_3 &= G_{1_{x_3}} - G_{3_{x_1}}, \\ \Delta v_4 &= -G_{1_{x_2}} + G_{2_{x_1}},. \end{aligned}$$

This optimality system is numerically solved by a simple iterative scheme in a decoupled manner that can be described as follows:

- Suppose that at the k^{th} step, we have found $(f^1)^k$, (where f^1 =div u) and $((f^2)^k, (f^3)^k, (f^4)^k)$, where (curl u$= (f^2, f^3, f^4)$).
- Obtain $u^k = (u_1^k, u_2^k, u_3^k)$ from the decoupled state equations.
- Obtain $v_1^k, v_2^k, v_3^k, v_4^k$ from the costate equations.
- Next get new controls $((f^1)^{k+1}, (f^2)^{k+1}, (f^3)^{k+1}, (f^4)^{k+1})$ from the optimality conditions.
- Normalize controls and repeat the same process until the error condition is satisfied or a present number of iterations is achieved.

Computational Results

Example: In this example, we demonstrate the registration of a three-dimensional MRI image. The reference and the template images are given as $65 \times 65 \times 33$ data. We set $\omega = 200$ and $w_i = 120$ for $i = 1, 2, 3$ and, after 200 iterations, $C_{sim} = 3.32$ with MG method. The template, reference, and registered images are shown in Figure 1.

Table 1. The similarity measure C_{sim} and the cost in computational time.

Iterations	MG		FEM	
	C_{sim}	cost	C_{sim}	cost
1	1674.6	1 sec	1674.6	2 sec
2	908.5	1.1 sec	1102.5	2 sec
5	160.7	2 sec	209.9	4 sec
20	40.7	7 sec	63.5	16 sec
40	15.8	15 sec	26.5	34 sec
60	10.7	20 sec	16.5	55 sec
120	6.4	38 sec	12.5	2 mins
200	3.9	55 sec	7.5	3 mins

Comparison with some other related methods

In this paper we present a $3-$D medical image registration method based on adjusting divergence and curl of image displacement field. In this section first we briefly overview some related methods and secondly compare the present method with those related methods.

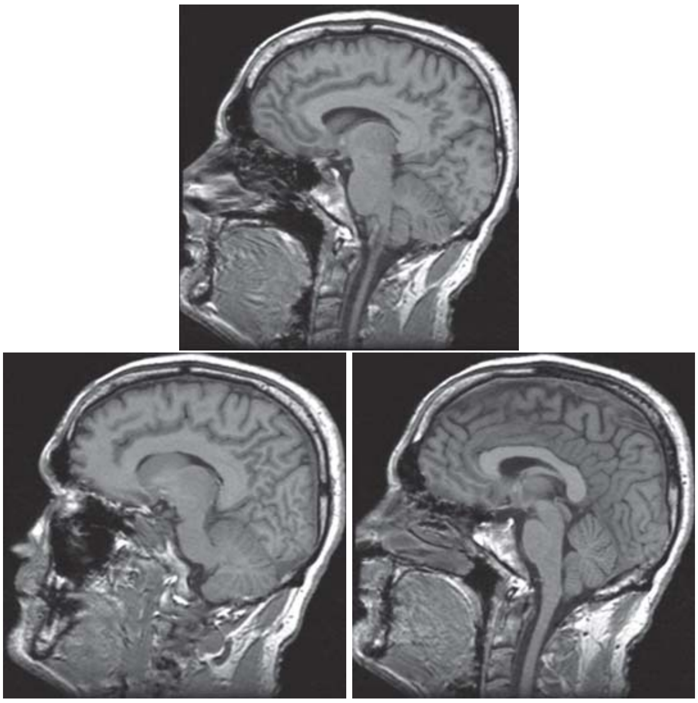

Fig. 1. The template image (top), the reference image (bottom left), and registered image (bottom right).

In a recent paper [11] the authors presented a method for integration of $3-$D medical data by utilizing the advantages of $3-$D multiresolution analysis and techniques of variational calculus. They first expressed the data integration problem as a variational optimal control problem where the displacement field was written in terms of wavelet expansions and secondly they wrote the components of the displacement field in terms of wavelet coefficients. The authors solved this optimization problem with a blockwise descent algorithm and demonstrated the application of the method by the registering $3-$D brain MR images in the size of $257 \times 257 \times 65$. Duration of the medical data integration process was about 2 minutes and the registered image seems has features of both reference and template image. Detailed information about this method can be seen at [11].

In another paper [12] the authors introduces several mathematical image registration models employing some curvature driven diffusion based techniques, in particular, Perona–Malik, anisotropic diffusion, mean curvature motion (MCM), affine invariant MCM (AIMCM). Adopting the steepest-descent marching with an artificial time t, Euler-Lagrange (EL) equations with homogeneous Neumann boundary conditions are obtained. These EL equations are approximately solved by the explicit Petrov-Galerkin scheme. The method is applied to the registration of brain MR images of size 257×257. Computational results indicate that all these regularization terms produce similarly good registration quality but that the cost associated with the AIMCM approach is, on the average, less than that for the others. Duration of the registration with each model was around 1 to 3 minutes depending on the diffusion term and the quality of the registered images was quite good as well.

An image registration method might be described as efficient if the quality of the registered images is *good*, duration of the registration process is short and the amount of the similarity measure is small. The quality of the matched images using aforementioned techniques is almost the same and the information about duration of registration and the amount of the similarity measure is given by .

Table 2. The similarity measure C_{sim} and the cost in computational time.

Iterations	Ours with MG		The method at [11]	
	C_{sim}	cost	C_{sim}	cost
1	1674.6	1 sec	2400	2 sec
2	908.5	1.1 sec	2100	2 sec
15	55.1	7 sec	1200	4 sec
40	15.8	15 sec	400	19 sec
120	6.4	38 sec	160	44 sec
200	3.9	55 sec	22	1 mins

Table 3. The similarity measure C_{sim} and the cost in computational time.

Iterations	AIMCM		PM	
	C_{sim}	cost	C_{sim}	cost
1	1674.6	1 sec	1674.6	2 sec
2	908.5	1.1 sec	1102.5	2 sec
15	160.7	2 sec	209.9	4 sec
30	40.7	7 sec	63.5	16 sec
80	15.8	15 sec	26.5	34 sec
160	10.7	20 sec	16.5	55 sec
240	6.4	38 sec	12.5	2 mins
400	3.9	55 sec	7.5	3 mins

As the and the medical images indicate the present method is a highly efficient medical image registration method.

Concluding remarks

Nonrigid image registration is a significant branch of image processing. It has broad application in medical and nonmedical imaging. For example, it can be used in analyzing local anatomical variations that exist between images acquired from different individuals or atlases. It can serve as a powerful tool for combining information from multiple sources, monitoring changes in an individual, detecting tumors and locating disease, motion correction, image fusion, and many more.

In this paper, we presented a systematic method for the nonrigid registration of 3D images. The Lagrange multiplier method was used to describe the constrained optimization problem as an unconstrained optimization problem. We solved the resulting coupled system of Poisson equations by the finite element and multigrid methods. A computational example was given to test the algorithm. Preliminary experiments show promising results and great potential for future extensions. We tested the algorithm using some other medical images and some 3D synthetic images. Because the computational results are quite similar to the one presented in this paper, we skip to introducing them inhere for the sake of brevity. The registration of example images currently took between less than 1 minute to about 3 minutes of CPU time via MG and FEM, respectively, which makes routine application in a clinical environment possible. In the future work we plan to apply the present method to the registration of image which includes certain level of noise and compare the method with some other well-known methods in the literature. Finally, we list some advantages of present method:

- It is based on a solid mathematical foundation. In particular, it accounts for local volume changes through the divergence of the transformation and accounts for local rotations through the curl of the transformation.
- The method is based on a linear differential system; its numerical implementation is fast and quite efficient.
- The method is general in the sense that it may be used in any optimization problem that involves motion estimation. Thus, it has the potential to be the numerical kernel for a wide range of applications.

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