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Time Varying Impulse Response Representing First Order PMD Effects of a Single Mode Fibre Optic Link with Polarization Scramblers

***Abstract**. Optical fibre line with polarization scramblers is modelled as a time-variant linear system. The formula for time varying impulse response of such a system is derived and relations to PMD and to parameters of scramblers control are shown. One possible application of the time varying impulse response is considered and illustrated with results of simulations.*

Streszczenie.** Linia światłowodowa ze skramblerami polaryzacji została zamodelowana jako zmienny w czasie system liniowy. Wyprowadzono zależności na zmienną w czasie odpowiedź impulsową i powiązano jej parametry z PMD linii oraz z parametrami sygnałów sterujących skramblerami. Przedstawiono i zilustrowano wynikami symulacji jedno z możliwych zastosowań odpowiedzi impulsowej. – **Zmienna w czasie odpowiedź impulsowa jednomodowej linii światłowodowej ze skramblerami polaryzacji reprezentująca pierwszorzędowe efekty PMD

Keywords: Polarization mode dispersion, single mode fibre, polarization scrambling.
Słowa kluczowe: dyspersja polaryzacyjna, światłowód jednomodowy, skrambling polaryzacji.

Introduction

Optical fibers suffer from polarization mode dispersion (PMD) which affects capability of links using using On-Off-Keying (OOK) signalling to achieve transmission capacity exceeding 10Gbps per wavelenght channel [1]. A number of methods were developed to combat the effects of PMD [2]. One case of practical interest is the link with burried or undersea cables in which temporal variations of PMD are relatively slow i.e. PMD can be static (“frozen”) for days or even months [3]. Slow changes of PMD result in long lasting link outages. For a link of such type the technique using polarization scrambling distributed along the link span in combination with proper Forward Error Correction (FEC) was shown a method of choice [2]. In this method “frozen” PMD becomes time-varying which potentially shortens time intervals of unacceptably high signal deterioration. Short error bursts can be corrected with the use of an appropriate FEC which potentially converts an inoperable link into a fully functional low bit error rate (BER) one.

Theoretically, this technique is effective provided each polarization scrambler uniformly distributes it’s output states of polarization (SOP) on the Poincare sphere within each data transmission frame [4]. This would require scrambling with unrealistic infinite speed. In practical realizations a question arises on how to control the scramblers with real world signals to satisfy the conditions that are imposed by the ability of the FEC to correct error bursts. One way of analysing this aspect can be to inspect the time varying impulse response (TVIR) of the polarization scrambled fibre optic link. In this paper such idea is developed and prospects for its use are shown through examples. Analytic model which describes the TVIR of a fibre optic polarization scrambled line as a function of parameters of fibre segments, of control signals, etc. can be useful in design of links with PMD affected fibres as well as computer simulations.

Linear time-variant model of optical fibre line with polarization scramblers

Generally, silica fibres are nonlinear media however, under certain conditions assumption of approximate linearity holds [5] which allows to model such a fibre as a linear system. Such an assumption will be made for the following text.

A single mode fibre supports two polarization modes. The input and output of the system which is the model of a single mode fibre can be considered 2-dimensional vectors with components describing the polarization orthogonal constituents of light. Optical signals propagating in fibre optic telecommunication links can be regarded modulated monochromatic polarized waves [6]. Such a wave can be described by 2-dimensional complex envelope vector $\hat{\mathbf{s}}(t)$ defined by:

$$(1)\qquad \hat{\mathbf{s}}(t)=\begin{bmatrix}\hat{s}_x(t)\\ \hat{s}_y(t)\end{bmatrix}=\begin{bmatrix}a_x(t)e^{\varphi_x}\\ a_y(t)e^{\varphi_y}\end{bmatrix},$$

which is related to the wave $\mathbf{s}(t)$ by:

$$(2)\qquad \mathbf{s}(t)=\begin{bmatrix}s_x(t)\\ s_y(t)\end{bmatrix}=\mathrm{Re}\left\{\hat{\mathbf{s}}(t)e^{j\omega_0 t}\right\}.$$

Here, $s_x(t)$ and $s_y(t)$ are polarization components of the wave $\mathbf{s}(t)$ while pairs $(a_x(t),\varphi_x(t))$ and $(a_y(t),\varphi_y(t))$ represent their envelopes and phases. The ω_0 is the angular frequency of the optical carrier.

The paper analyses the case in which polarization scramblers are inserted at discrete points of a fibre link in order to animate PMD. Time varying polarization mixing occurs in fibres in the effect of scrambling. Thus, TVIR of such a link shows temporal variations and the link can be regarded a time variant linear system. Its’ output signal is related to the input signal via [7]:

$$(3)\qquad \mathbf{w}(t)=\int_{-\infty}^{\infty}\mathbf{h}(t,t-t_2)\mathbf{v}(t_2)dt_2=(\mathbf{h}\bullet\mathbf{v})(t).$$

In (3) $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are 2-dimensional signals of the form described by (2) which correspond to the input and output waves, respectively. The $\mathbf{h}(t,t_2)$ is the time varying impulse response (TVIR) of the system:

$$(4)\qquad \mathbf{h}(t,t_2)=\begin{bmatrix}h_{xx}(t,t_2) & h_{yx}(t,t_2)\\ h_{yx}(t,t_2) & h_{yy}(t,t_2)\end{bmatrix}.$$

The scalar components $h_{ij}(t,t_2)$ represent in time domain the transfer characteristics of the system with respect to the two polarization modes. The t parameter is the “running time” characterizing the time flow in the system while the t_2 parameter is the “lag”, the delay with respect to the running time.

In general, the components of $\mathbf{h}(t,t_2)$ take account for all factors affecting propagation of polarization modes: polarization mode dispersion, other types of dispersion (polarization independent), polarization dependent loss (PDL), and time varying polarization mixing due to scrambling.

Considering that the input and output signals of the system of interest are modulated sinusoids it is more convenient to express the input-output relation in terms of their complex envelopes. The relation equivalent to (3) is given by [7]:

$$(5)\qquad \hat{\mathbf{w}}(t) = \int_{-\infty}^{\infty} \hat{\mathbf{h}}(t, t-t_2)\hat{\mathbf{v}}(t_2)dt_2 = (\hat{\mathbf{h}} \bullet \hat{\mathbf{v}})(t),$$

where: $\hat{\mathbf{v}}(t)$ and $\hat{\mathbf{w}}(t)$ are the complex envelopes (as defined by (1)) of the input and output waves, respectively. The $\hat{\mathbf{h}}(t,t_2)$ is the "baseband" TVIR of the system defined as follows:

$$(6)\qquad \hat{\mathbf{h}}(t,t_2) = \mathbf{h}(t,t_2)\exp(-j\omega_0 t_2)\ .$$

It will be further referred to as the complex time variant impulse response (CTVIR). One shall note that with the definition (6) the function $\mathbf{h}$=$\mathbf{h}(t{=}const,t_2)$ need not be bandpass which is the case of any PMD model.

For a system that is a concatenation of a number of time variant linear systems the effective CTVIR shall be a linear transformation of the CTVIRs of the component systems. Let's limit, for the moment, to a cascade of only two systems, each with the corresponding CTVIR $\hat{\mathbf{h}}_1(t,t_2)$ and $\hat{\mathbf{h}}_2(t,t_2)$ (the systems are indexed starting from the input in the direction to the output of the cascade). Acording to (5) the resultant input-output relation of the cascade of the two systems is given by:

$$(7)\qquad \hat{\mathbf{w}}(t) = (\hat{\mathbf{h}}_2 \bullet (\hat{\mathbf{h}}_1 \bullet \hat{\mathbf{v}}))(t)\ .$$

The CTVIR $\hat{\mathbf{h}}_e(t,t_2)$ of the compound system can be derived from (7) after $\delta(t-t_2)=[\delta(t-t_2),\delta(t-t_2)]^T$ is put in place of $\hat{\mathbf{v}}(t)$. This leads to the following formula:

$$(8)\quad \hat{\mathbf{h}}_e(t,t_2) = (\hat{\mathbf{h}}_2 \circ \hat{\mathbf{h}}_1)(t,t_2) = \int_{-\infty}^{\infty} \hat{\mathbf{h}}_2(t,\xi)\hat{\mathbf{h}}_1(t-\xi, t_2-\xi)d\xi\ .$$

The operator $\circ$ appearing in (8) denotes the time-variant convolution generalized to two-dimensional signals which is formally defined by the last expression in (8).

Generalizing the result in (8) to the case of a cascade of any *M* components one obtains the effective CTVIR:

$$(9)\qquad \hat{\mathbf{h}}_e(t,t_2) = (\hat{\mathbf{h}}_M \circ \hat{\mathbf{h}}_{M-1} \circ \ldots \circ \hat{\mathbf{h}}_1)(t,t_2),$$

where $\hat{\mathbf{h}}_M(t,t_2)$, $\hat{\mathbf{h}}_{M-1}(t,t_2)$,..., $\hat{\mathbf{h}}_1(t,t_2)$ are the CTVIRs of components indexed from the cascade's input to its' output.

Complex time-variant impulse response of a fibre optic line with polarization scramblers

A. Complex time-varying impulse response of a cascade of DGD components

Let's start the derivation from an example being a cascade of time-invariant components, each with being a waveplate with fixed DGD. The well known 2×2 Jones matrix representing a frequency domain transfer function of a single component of this type can be rearranged to the form:

$$(10)\qquad \mathbf{U}(\omega) = e^{-j0.5\omega\tau}\mathbf{U}_0 + e^{j0.5\omega\tau}\mathbf{U}_1,$$

where: $\mathbf{U}_0$ and $\mathbf{U}_1$ are frequency invariant matrices and τ is the DGD of the component. PDL is excluded in the forthcoming analysis. Under these assumptions the $\mathbf{U}_0$ and $\mathbf{U}_1$ are given by:

$$(11)\qquad \mathbf{U}_0 = \frac{1}{2}\begin{bmatrix} 1+\cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & 1-\cos 2\alpha \end{bmatrix}, \quad \mathbf{U}_1 = \frac{1}{2}\begin{bmatrix} 1-\cos 2\alpha & -\sin 2\alpha \\ -\sin 2\alpha & 1+\cos 2\alpha \end{bmatrix}$$

where: α is a rotation angle of the component's birefringence axes with respect to the *X0Y* axes. The corresponding time-invariant impulse response is as follows:

$$(12)\qquad \mathbf{h}(t) = \delta\left(t-\frac{\tau}{2}\right)\mathbf{U}_0 + \delta\left(t+\frac{\tau}{2}\right)\mathbf{U}_1\ .$$

The complex impulse response is obtained through multiplying (12) by $\exp(-j\omega_0 t)$ which yields:

$$(13)\qquad \hat{\mathbf{h}}(t) = \delta\left(t-\frac{\tau}{2}\right)\mathbf{U}_0 e^{-j\omega_0\frac{\tau}{2}} + \delta\left(t+\frac{\tau}{2}\right)\mathbf{U}_1 e^{j\omega_0\frac{\tau}{2}}\ .$$

For a cascade of N fixed DGD components we get from (9) and (13):

$$(14)\qquad \hat{\mathbf{h}}_e(t) = \sum_{i=0}^{2^N-1}\delta\left(t-\frac{\Delta\tau_i}{2}\right)\mathbf{U}_i\exp\left(-j\omega_0\frac{\Delta\tau_i}{2}\right),$$

where:

$$(15)\qquad \Delta\tau_i(t) = \sum_{n=0}^{N-1}(-1)^{d(N-n,i)}\tau_n(t)$$

and:

$$(16)\qquad \mathbf{U}_i = \prod_{n=0}^{N-1}\mathbf{U}_{N-n,d(N-n,i)}\ .$$

In (14)-(15) above: τ_n is the fixed DGD of the n-th component in the cascade, $d(n,i)$ is the value of the n-th digit of the N-bit binary representation of the index i, and $\mathbf{U}_{n,0}$ and $\mathbf{U}_{n,1}$ are the $\mathbf{U}_0$ and $\mathbf{U}_1$ of the n-th component, respectively.

The result in (16) can be easily generalized to a time-variant case when all DGDs in the cascade are allowed to vary with time. From (14) one gets the following expression for the CTVIR of a cascade of time-variant DGD components:

$$(17)\quad \hat{\mathbf{h}}_e(t,t_2) = \sum_{i=0}^{2^N-1}\delta\left(t_2-\frac{\Delta\tau_i(t)}{2}\right)\mathbf{U}_i\exp\left(-j\omega_0\frac{\Delta\tau_i(t)}{2}\right)$$

The first term in (17) represents time delays which the complex envelope undergoes due propagation through the cascade while the second one describes the associated phase shifts of the optical carrier.

B. Complex time-varying impulse response of a polarization scrambler

There may be a variety of realizations of polarization scramblers. In order to rotate polarization of light one of following two devices can be utilised: a space rotatable waveplate or, a tunable phase shifter, typically stacked in a cascade. Polarization scramblers build with the two types of components are equivalent. In the following only the polarization scrambler based on tunable phase shifters will be considered without loss of generality.

A three axis scrambler can be build as a cascade of three variable DGD components playing the role of phase shifters (at a given optical frequency DGD changes manifest as phase shifts to a harmonic wave). Two of them shall have the same orientation of their birefringence axes, say 0 rad with respect to *XOY* axes, and the third shall be sandwiched between the two and have birefringence axes rotated by $\frac{1}{4}\pi$. This rule is kept observed while adding extra stages to a scrambler. One conclusion from the above considerations is that a polarization scrambler, being a cascade of variable DGD components, has its CTVIR in general described by (17).

In the following the (17) will be specialized to express CTVIR of an N-stage scrambler (a stage is regarded a single phase shifter). For a phase shifter with birefringence axes aligned with *X0Y* axes the matrices $\mathbf{U}_0$ and $\mathbf{U}_1$ are as follows:

(18) $$\mathbf{U}_0^s = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \mathbf{U}_1^s = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

while for a phase shifter with birefringence axes rotated by $\frac{1}{4}\pi$ the matrices are given by:

(19) $$\mathbf{U}_0^s = 0.5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \mathbf{U}_1^s = 0.5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

In (18) and (19) above the upper script *'s'* lables matrices which correspond to the scrambler.

Moreover, it shall be noted that for any practical scrambler the $|\Delta\tau(t)|$ remains in the range of 10^{-2} picoseconds which is two orders of magnitude lower than DGD of a typical fibre segment. Consequently, in practical applications to an excellent approximation the polarization scrambling can be modelled with neglected effect of time shifting the signal's complex envelope:

(20) $$\hat{\mathbf{h}}_s(t,t_2) = \delta(t_2)\mathbf{S}(t).$$

where $\mathbf{S}(t)$ represents phase shifts due to polarization scrambling:

(21) $$\mathbf{S}(t) = \sum_{i=0}^{2^N-1} \mathbf{U}_i \exp(-j0.5\omega_0 \Delta\tau_i(t)).$$

In typical applications scramblers are driven by sinusoidal signals (particularly when fast scrambling, in the range of Megahertz, is applied). For an N-stage scrambler controlled by sinusoidal signals we have:

(22)
$$\mathbf{S}(t) = \sum_{l=0}^{2^N-1} \left\{ \mathbf{U}_i \exp\left[-j \sum_{n=0}^{N-1} (-1)^{d(n,l)} j\Delta\varphi_n \sin(\omega_n t + \vartheta_n) \right] \right\}$$

In the above formula ω_n, ϑ_n are the angular frequency and the initial phase of the sinusoidal signal controlling the n-the stage of the scrambler and the $\Delta\varphi_n$ is the amplitude of this control signal; $\Delta\varphi_n=\pi$ for odd n and $\Delta\varphi_n=\frac{1}{2}\pi$ for even n.

C. Complex impulse response of a fibre segment

Here, PMD in any fibre segment constituting a fibre line with polarization scramblers is considered time-invariant, at least in the time scale of interest. Dynamic behaviour of such a fibre segment is described by its time-invariant complex impulse response. Limiting to the case in which modelling PMD in a fibre by the first order model (frequency invariant DGD) is sufficient, the frequency domain transfer function of a fibre segment as well as its complex impulse response function can be described by (10) and (14), respectively. Any fibre segment is fully characterized by its DGD (denoted as τ) and orientation of the principal states of polarization (PSPs) described by angle of rotation α with respect to *XOY* axes and phase retardation θ between a PSP projections onto *XOY* axes. Under these assumptions the matrices $\mathbf{U}_0$ and $\mathbf{U}_1$ of a fibre are given by:

(23) $$\begin{gathered}\mathbf{U}_0 = \frac{1}{2}\begin{bmatrix} 1+\cos 2\alpha & e^{-j\theta}\sin 2\alpha \\ e^{j\theta}\sin 2\alpha & 1-\cos 2\alpha \end{bmatrix} \\ \mathbf{U}_1 = \frac{1}{2}\begin{bmatrix} 1-\cos 2\alpha & -e^{-j\theta}\sin 2\alpha \\ -e^{j\theta}\sin 2\alpha & 1+\cos 2\alpha \end{bmatrix}\end{gathered}.$$

D. Complex time-varying impulse response of a fibre optic line with distributed polarization scramblers

With the use of (9), (13) and (20) the CTVIR of a concatenation of a polarization scrambler followed by a fibre segment can be derived as:

(24)
$$\hat{\mathbf{h}}_{fs}(t,t_2) = \delta\left(t_2 - \frac{\tau}{2}\right)\mathbf{H}_0(t)e^{-j\omega_0\frac{\tau}{2}} + \delta\left(t_2 + \frac{\tau}{2}\right)\mathbf{H}_1(t)e^{j\omega_0\frac{\tau}{2}}$$

where:

(25) $$\mathbf{H}_0(t) = \mathbf{U}_0^f \mathbf{S}(t) \qquad \mathbf{H}_1(t) = \mathbf{U}_1^f \mathbf{S}(t).$$

The upper script *'f'* is introduced to differentiate the matrices $\mathbf{U}_0$ and $\mathbf{U}_1$ of a fibre segment from the ones of a polarization scrambler.

Because (24) has the same form as (13) one can use the same reasoning that was used in the sub-section A above to draw the conclusion that the formula for the CTVIR for a cascade of M fibre segments, each preceded with a polarization scrambler, has the general form of (14) with time varying $\mathbf{U}_i$ factors. The corresponding CTVIR can be written as:

(26) $$\hat{\mathbf{h}}_e(t,t_2) = \sum_{i=0}^{2^M-1} \delta\left(t_2 - \frac{\Delta\tau_i}{2}\right)\mathbf{H}_i(t)\exp\left(-j\omega_0\frac{\Delta\tau_i}{2}\right),$$

where:

(27) $$\mathbf{H}_i(t) = \prod_{m=0}^{M-1} \mathbf{U}^f_{M-m,d(M-m,i)} \mathbf{S}^s_{M-m}(t),$$

and $\Delta\tau_i$ are linear combinations of DGDs of all fibre segments calculated according to (15).

E. First order model of complex time-variant impulse response of a fibre optic line with polarization scramblers

The (26) represents effects of all-order PMD of a fibre opic line with polarization scramblers. For OOK signalling and up to 10Gbps bitrates limitation to first-order effects can prove sufficient. According to the first order model of PMD the distortion to the propagating wave at any time instance can be fully described by differential group delay between two particular polarizations called principal states of polarization (PSPs) [6]. In order to calculate an arbitrarily polarized narrowband light pulse at the output of a transmission medium described by the first-order PMD model one should translate the polarization of the input pulse to the two input PSP components and then apply to them the $\tau_{g1}=\frac{1}{2}\tau$ and $\tau_{g2}=-\frac{1}{2}\tau$ group delays. The output signal is the composition of the two output PSPs. If the output polarization is expressed in the same coordinate system which is used to describe the input polarization the "first-order" complex impulse response of a time invariant medium can be given by (11) and (23).

In case of fibre optic line with polarization scramblers the differential group delay τ as well as elements of the matrices $\mathbf{U}_0$ and $\mathbf{U}_1$ (related to input PSPs) are time varying in tact with scrambling signals. So, the "first-order" CTVIR of the entire fibre optic line with scramblers, equivalent to (26), can be expressed as:

(28)
$$\hat{\mathbf{h}}_e(t,t_2)=\delta\left[t_2-\frac{\tau(t)}{2}\right]\mathbf{U}_0(t)e^{-j\omega_0\frac{\tau(t)}{2}}+\delta\left[t_2+\frac{\tau(t)}{2}\right]\mathbf{U}_1(t)e^{j\omega_0\frac{\tau(t)}{2}}.$$

To find the formula for the time varying DGD and the time varying input PSPs of the system represented by (26) one can consider time variant Jones matrix which can describe temporal variation of spectral transmission properties of the system provided the variation is slow compared to the optical bandwidth. This is the typical case in scrambling as frequencies of scrambling signals are in Megahertz or tens of Megahertz range while the optical bandwidths of interest are at least 10 GHz.

The time variant Jones matrix is the Fourier transform in the domain of t_2 of the time variant impulse response $\boldsymbol{h}_e(t,t_2)$ related to the CTVIR by:

(29)
$$\hat{\mathbf{h}}_e(t,t_2)=\mathbf{h}_e(t,t_2)e^{j\omega_0 t}.$$

From (26) and (29) we have:

(30)
$$\mathbf{U}_e(t,\omega)=\sum_{i=0}^{2^M-1}\exp\left[-j\omega\frac{\Delta\tau_i}{2}\right]\mathbf{H}_i(t).$$

Note, that $\boldsymbol{U}_e(t,\omega)$ can be expressed in the Caley/Klein form which means that only two elements of this matrix: $u_{11}(t,\omega)$ and $u_{12}(t,\omega)$ are independent [6]. From (30) we can derive both elements:

(31)
$$u_{11}(t,\omega)=\sum_{i=0}^{2^M-1}\exp\left[-j\omega\frac{\Delta\tau_i}{2}\right]h_{11i}(t)$$
$$u_{12}(t,\omega)=\sum_{i=0}^{2^M-1}\exp\left[-j\omega\frac{\Delta\tau_i}{2}\right]h_{12i}(t)$$

as well as their frequency derivatives:

(32)
$$u'_{11}(t,\omega)=-j\sum_{i=0}^{2^M-1}\frac{\Delta\tau_i}{2}\exp\left[-j\omega\frac{\Delta\tau_i}{2}\right]h_{11i}(t)$$
$$u'_{12}(t,\omega)=-j\sum_{i=0}^{2^M-1}\frac{\Delta\tau_i}{2}\exp\left[-j\omega\frac{\Delta\tau_i}{2}\right]h_{12i}(t)$$

which all will be used in calculation of $\tau(t)$ and parameters of the PSPs.

The time variant DGD is given by [6]:

(33)
$$\tau(t)=2\sqrt{u'_{11}(t,\omega_0)u'^{*}_{11}(t,\omega_0)+u'_{12}(t,\omega_0)u'^{*}_{12}(t,\omega_0)},$$

where the asterisk means complex conjugation.

The time varying parameters of the input PSPs necessary for (28) can be derived from the components of the output PMD vector (representing PMD in the Stokes space). Let $\mathbf{s}(t)=[s_1(t),s_2(t),s_3(t)]^T$ be the time variant PMD vector. The vector components are given by [6]:

(34)
$$\begin{aligned}s_1(t)&=2j\left[u'_{11}(t,\omega_0)u^{*}_{11}(t,\omega_0)+u'_{12}(t,\omega_0)u^{*}_{12}(t,\omega_0)\right]\\ s_2(t)&=2\,\mathrm{Im}\left\{u'_{11}(t,\omega_0)u_{12}(t,\omega_0)-u'_{12}(t,\omega_0)u_{11}(t,\omega_0)\right\}\\ s_3(t)&=2\,\mathrm{Re}\left\{u'_{11}(t,\omega_0)u_{12}(t,\omega_0)-u'_{12}(t,\omega_0)u_{11}(t,\omega_0)\right\}\end{aligned}.$$

The output PSP parameters: α_o - the alignment angle of PSPs to the coordinate system, θ_o - the phase shift between the PSPs, can be calculated from (33) and (34) as shown below:

(35)
$$\alpha_o(t)=\frac{1}{2}\arccos\left[\frac{s_1(t)}{\tau(t)}\right],\quad \theta_o(t)=\arg\left[s_2(t)+js_3(t)\right]$$

where $arg(z)$ returns the argument of a complex number z. The input PSPs with parameters: α_i - the alignment angle of PSPs to the coordinate system, θ_i - the phase shift between the PSPs, are related to the output PSPs by the following formulas:

(36)
$$\begin{aligned}\begin{bmatrix}a(t)\\ b(t)\end{bmatrix}&=U_e^{-1}(t,\omega_0)\begin{bmatrix}\cos(\alpha_O(t))\\ \sin(\alpha_O(t))e^{j\theta_O(t)}\end{bmatrix}\\ \alpha_i(t)&=\arccos[a(t)]\\ \theta_i(t)&=\arg[b(t)]\end{aligned}.$$

In some applications utilising the first order model of PMD the input PMD vector $\boldsymbol{\tau}_{in}(t)$ (in Stokes space) is used. Combining the results in (33) and (36) can provide the required quantity:

(37)
$$\boldsymbol{\tau}_{in}(t)=\tau(t)\begin{bmatrix}\cos(2\alpha_i(t))\\ \sin(2\alpha_i(t))\cos(\theta_i(t))\\ \sin(2\alpha_i(t))\sin(\theta_i(t))\end{bmatrix}.$$

The formula (28) can be further simplified if applications of CTVIR are limited to direct detection systems. In such systems the received signal $r(t)$ is proportional to the envelope of the optical signal (momentary optical power). This in turn is related to the complex envelope $\hat{\mathbf{w}}(t)$ by:

(38)
$$r(t)=\left|\hat{w}_1(t)\right|^2+\left|\hat{w}_2(t)\right|^2,$$

where: 1 and 2 denote two orthogonal arbitrary polarizations. It shall be noticed, that the output state of polarization does not affect the output envelope of a received wave unless polarization sensitive detector is used, which is uncommon in direct detection systems. Consequently in these applications, the CTVIR of the system can be modified in such a way that splitting output PSPs into the polarizations aligned with the axes of the arbitrary coordinate system is omitted. This shall be reflected by modification of the matrices $\mathbf{U}_0$ and $\mathbf{U}_1$ in (28). Now, they shall be given by:

(39)
$$\mathbf{U}_0^{(PSP)}=\begin{bmatrix}\cos\alpha_i & e^{-j\theta_i}\sin\alpha_i\\ 0 & 0\end{bmatrix},\quad \mathbf{U}_1^{(PSP)}=\begin{bmatrix}0 & 0\\ -\sin\alpha_i & e^{-j\theta_i}\cos\alpha_i\end{bmatrix}$$

where: α_i and θ_i have the meaning as explained in (36). The (PSP) upperscript denotes alignment of output polarizations to PSPs. When (28) with $\mathbf{U}_0$ and $\mathbf{U}_1$ defined by (39) are used in (5) the resultant output complex envelope can be expressed as:

(40) $$\begin{bmatrix}\hat{w}_1(t)\\ 0\end{bmatrix}=\mathbf{U}_0^{(PSP)}(t)e^{-j.5\omega_0\tau(t)}\hat{\mathbf{v}}[t-0.5\tau(t)]$$
$$\begin{bmatrix}0\\ \hat{w}_2(t)\end{bmatrix}=\mathbf{U}_1^{(PSP)}(t)e^{j.5\omega_0\tau(t)}\hat{\mathbf{v}}[t+0.5\tau(t)] .$$

Having looked onto (38) one can deduce that in (28) the *exp[jω₀τ(t)]* and *exp[-jω₀τ(t)]* components can be dropped if (28) shall be used only to calculate output envelope. Hence, for the purpose of applications in direct detection systems the first order CTVIR can be simplified to:

(41) $$\hat{\mathbf{h}}_e^{(PSP)}(t,t_2)=$$
$$\delta\left(t_2-\frac{\tau(t)}{2}\right)\mathbf{U}_0^{(PSP)}(t)+\delta\left(t_2+\frac{\tau(t)}{2}\right)\mathbf{U}_1^{(PSP)}(t).$$

The above form of CTVIR can be used in calculations of received signal in a direct detection system using a fibre optic line with polarization scramblers.

Applications

One important application of CTVIR is calculation of received signal in direct detection OOK systems in order to find temporal variation of BER due to temporal changes in the transmission medium. Consequently, CTVIR can prove useful in analysis of a system of this type which uses fibre optic line with polarization scramblers.

The probability of erroneous detection of a bit is given by:

(42) $$p(1/0)=\sigma_L\int_{\sqrt{SNR_L}}^{+\infty} g_n(x\sigma_L,\sigma_L)dx$$
$$p(0/1)=\sigma_H\int_{\sqrt{SNR_H}}^{+\infty} g_n(x\sigma_H,\sigma_H)dx ,$$

for conditional probalility of detecting 1 instead of 0 and vice versa, respectively. In the above formulas: $g(x,\sigma)$ is the distribution of noise in the receiver front-end, σ_L is the RMS noise when receiving 0, σ_H is is the RMS noise when receiving 1. The values of the signal-to-noise ratio can be calculated according to:

(43) $$SNR=\left(\frac{r_{th}-r(t_d)}{\sigma}\right)^2 ,$$

where: σ takes σ_L or σ_H, as appropriate, $r(t_d)$ is the received bit pulse at detection time instance (typically at the peak of the pulse), r_{th} is the detection threshold.

In a medium with time variant PMD the transmitted pulses arrive at the medium end delayed by a time variant amount $\Delta t(t)$. The detection time instances cannot then occur simply at kT for k-th bit, where T is the bit slot duration (pulses are assumed centred in the bit slots). The receivers monitor variations of the delay and sample received signal at the time instances when it really reaches maximum. So, for practical receivers the t_d shall be expressed:

(44) $$t_{d\,k}=kT+\Delta t(t_{d\,k})\approx kT+\Delta t(kT) .$$

In a fibre optic line with scramblers the $\Delta t(t)$ changes enough slowly to justify approximation made on the right in the above formula. The $\Delta t(t)$ can be easily calculated if PMD can be approximated by the first order model. It is given by [6]:

(45) $$\Delta t(kT)=0.5\boldsymbol{\tau}_{in}(kT)\cdot\mathbf{p}_{in} .$$

where: $\tau_{in}(kT)$ is the input PMD vector at the time instance kT, $\mathbf{p}_{in}$ is the polarization vector of the input light (both vectors are in Stokes space), the dot denotes vector scalar product. The above quantities can be used to calculate the RMS pulse width spread σ_w [8]:

(46) $$\sigma_w=0.5\tau(t)\left|1-\left(\boldsymbol{\tau}_{in}\cdot\mathbf{p}_{in}\right)^2\right| .$$

The Fig. 1 illustrates effects of scrambling on temporal variations of bit error probalility (Fig 1a). The received signal sampled at detection time instances is shown in the Fig. 1c.

The PMD effects were calculated from the presented model. For the purpose of this illustration $\sigma_L=\sigma_H$ and $SNR_L=SNR_H$ (thermal noise limited receiver) were assumed.

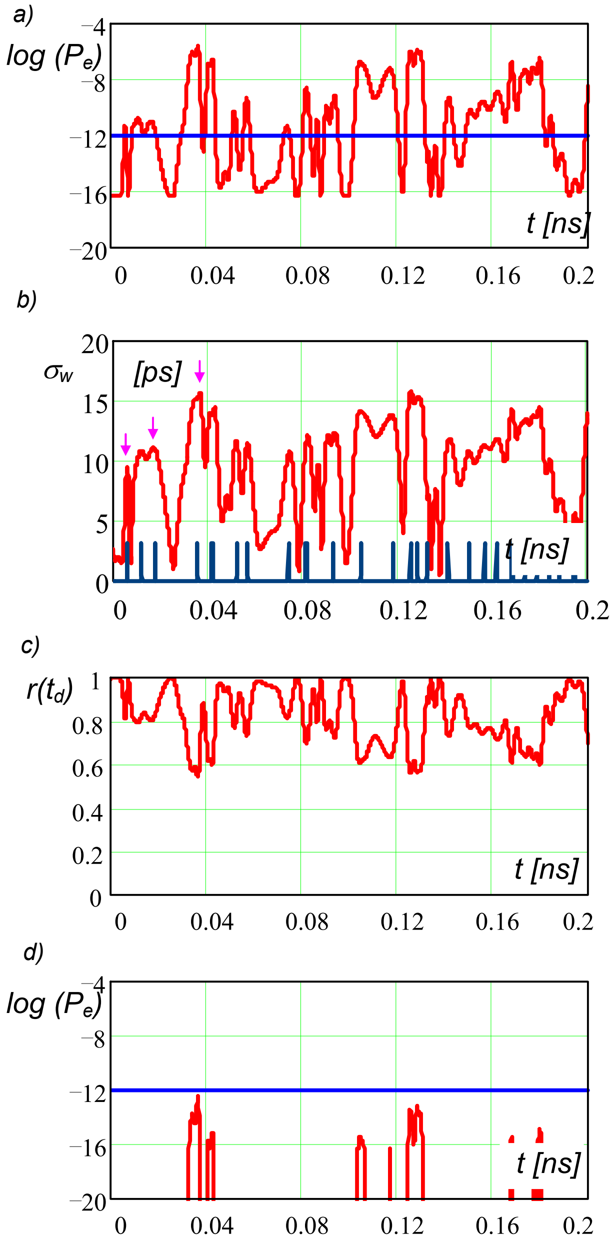

Fig.1. a) log of bit error probability vs time; the blue line marks targetted BER; b) the rms pulse width spread vs time; c) received signal samples vs time; d) log of bit error probability vs time after margin increased to model 4dB code gain

The targeted BER was 10^{-12}. 1.5 dB margin for power penalty was added which system designers used to allocate in order to provide acceptable probability of outage due to

PMD fluctuations. A 5 segment fibre line was simulated, each segment with equal DGD value of 7.5ps. This corresponded to the maximum compound DGD 37.5ps which was 37.5% of the bit signalling interval (100 ps). No PDL was assumed. Each fibre segment was preceded by a 3-axis polarization scrambler driven by sinusoidal signals. The frequencies of the signals driving polarization rotators were set to 50, 51 and 52 MHz – in the first scrambler, 53, 54 and 55 in the second one, and so on with the last scrambler driven by 62, 63 and 64 MHz. The modulation format was RZ with 33% duty cycle at 10Gbps data transmission speed.

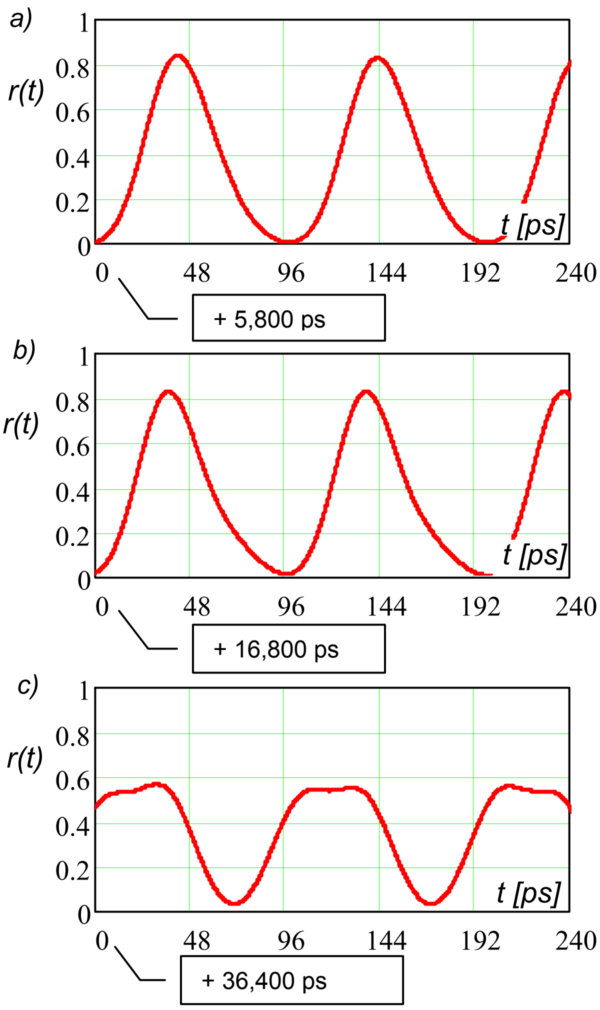

Fig.2. Received signal at three different time instances when pulse spread reaches maxima marked by arrows in the Fig 1b. Time shift to the origin indicated below each plot.

In the Fig. 1a shots of unacceptably high bit error probability are apparently visible which reveals insufficient margin. The shots maxima correspond to the maxima in the RMS pulse width spread curve (Fig. 1b). The Fig. 2 exemplifies pulse distortions which are effected due to scrambling. The three plots relate to the three local maxima of the pulse RMS pulse width pointed by arrows in the Fig. 1b. The plots evidence that the distortions do not only depend on instantaneous DGD but the variation of power split between the two orthogonal polarizations also matters.

The shots can be cleared by increasing the power margin. Alternatively, the equivalent effect can be obtained if a forward error correction code (FEC) is applied. A rough estimation of the effect of using FEC can be based on the code's gain figure. Increasing the original signal-to-noise ratio by code gain can give approximate bit error probability which could result if FEC is used. For example, according to this approximation the targeted BER of 10^{-12} could be achieved in the timescale as in the Fig. 1 if the code gain were 4 dB, which is illustrated in the Fig. 1d. More thorough estimation may account for grouping of errors within the shots and FEC's capability to correct burts errors. Hence, what FEC should be applied to reduce the probability to the targeted level can be revealed by extensive inspection of the bit error probability versus time. Calculation of the received signal at each bit slot would require large workload. The amount of calculations could be reduced if the received signal is computed only at those time instances when the PMD resultant pulse spread reaches maxima. The pulse spread curve exhibits relatively slow (directly related to the speed of scrambling) temporal dependence what suggests that it can be possibly quickly searched for maxima even over long time scopes. A gradient based variable step with quadratic approximation search method shown approximately 12-fold workload reduction. The maxima locations found by the algorithm are marked by spikes in the Fig. 1b.

Conclusions

A complex time-variant impulse response - CTVIR can be considered a tool for analysis of temporal behaviour of a fibre optic transmission line with polarization scramblers driven by real world signals. The CTVIR variant that accounts only for the first-order PMD effects is advantageous in such analysis as it involves only three real valued parameters that influence the output signal. The model can facilitate a search for FEC required to clear the bit error probability ripple resulted from scrambling.

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