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# Calculation of Forces in AC Dielectrophoretic Separation

***Abstract*** *In this paper two-dimensional dielectrophoresis is described. First electric field distribution in particle and surrounding fluid is calculated and next dipole moment and surface charge density is derived. These values are fundamental for force calculation in two-dimensional dielectrophoresis and in simulation velocity distribution in interdigitated electrodes.*

***Streszczenie.*** *W tej publikacji omówiono zjawisko dielektroforezy w dwóch wymiarach. Najpierw odpowiednie równania pole zostaną rozwiązane analityczni, a następnie zostaną wyprowadzone wzory na wartość momentu dipolowego cząsteczki zanurzonej w płynie dielektrycznym. Wielkości te mają podstawowe znaczenie w obliczaniu sił i momentów działających na cząsteczkę oraz na wyznaczanie rozkładu prędkości w urządzeniach do separacji cząstek. (**Obliczanie sił w urządzeniach do separacji cząstek**)*

**Keywords:** dielectrophoresis, dipole force calculations
**Słowa kluczowe:** dielektroforeza, obliczanie wartości sił działających na dipol.

## Introduction

Despite this growing importance of dielectrophoresis is, little attention has been paid to the theoretical and analysis. Although dielectrophoresis is only possible in strong divergent electric fields, theoretical analyses are usually based on equations derived from uniform field behavior. The calculation of DEP force acting on particle has been reported as a difficult task unless in many cases simplifying assumptions and very simple geometries are considered and is usually based on the dipole approximation first introduced by Pohl. Pohl derived an expression for the dielectrophoretic force acting on cells by modeling the cell as a solid spherical dielectric particle placed in a fluid medium. A more realistic geometries for biological particles has been used by a number of scientists, which includes a spherical dielectric shell employed usually for the dielectric properties of the plasma-membrane.

It is well-known that an electrically neutral but polarizable particle, suspended in a dielectric or conducting fluid, under the influence of a non-uniform electric field tends to move towards the region of highest electric field intensity. This migration caused by dielectric polarization forces is discovered by [1] and named as dielectrophoresis. During the past years dielectrophoresis has proved to be of very important in many applications such as, for example, industrial filtration of liquids, dielectric solid - solid separations and biological analyses.

There are many reasons for studying a behavior of particles and fluid globules immersed fluid suspension and placed in electric fields. Among different the chemical engineering applications [2] are the determination of forces acting on droplets exiting electrospray nozzles, the enhancement of heat and mass transfer in emulsions by the imposition of electric fields [3], electrically driven separation of particles techniques, dielectrophoretic and electro-rotational manipulation of living and death cells, and the control of electrorheological fluids.

Dielectrophoretic (DEP) traps use the force acting on an induced multipole with a nonuniform steady or alternating electric field to create electric forces that will change position of particles. DEP forces can trap different kind of particles on or between special electrodes – among others including micron and submicron polymer beads, cells, viruses, and bacteria. With the appropriate electrode geometry design and careful control of the potentials conditions, single particle trapping can be attained.

The Finite Element Method (FEM) is useful method for analyzing electromagnetic fields in devices, because these can model complicated geometries and non-linear electric properties with relatively short computing time. In spite of these advantages, in many papers have been proved that obtaining an accurate force or torque from FEM computation can be inaccurate, particularly when geometry is enough complex, such as in the case of dielectrophoretic traps with multiple particles. Unfortunately, force and torque calculations are influenced by the approximate nature of the discretisation used in FEM meshes.

## Equations describing the electromagnetic field

Field equations for potential distribution in harmonic case. It is assumed that frequency and conductivities are sufficiently small and therefore magnetic part of electromagnetic field can be neglect. Let us start with first Maxwell equation:

(1) $$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Taking divergence of both sides of the above equation and utilizing relations $\mathbf{J} = \sigma\mathbf{E}$, $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{E} = -\nabla V$ we have

(2) $$\nabla \cdot \left( -\sigma \nabla V - \varepsilon \frac{\partial}{\partial t} \nabla V \right) = 0$$

where $\sigma$ is conductivity and $\varepsilon$ permittivity. Assuming that we have sinusoidal excitation and steady state potential has the value

(3) $$V(t) = \hat{V} e^{j\omega t}$$

where $\hat{V}$ is the complex amplitude of the potential. Introducing this relation to (2) we get

(4) $$\nabla \cdot \left( -\sigma \nabla \hat{V} - j\omega\varepsilon \nabla \hat{V} \right) = 0$$

We can assume that $\hat{V}$ hes real and imaginary parts

(5) $$\hat{V} = V_r + jV_i$$

This gives as two equations for real and imaginary parts of the potential:

(6) $$\nabla \cdot \left( -\sigma \nabla V_r + \omega\varepsilon \nabla V_i \right) = 0$$

(7) $$\nabla \cdot \left( -\sigma \nabla V_i - \omega\varepsilon \nabla V_r \right) = 0$$

Electric field strength also has real and imaginary parts

(8) $$\hat{\mathbf{E}} = \left( E_{xr} + jE_{xi} \right) \mathbf{a}_x + \left( E_{yr} + jE_{yi} \right) \mathbf{a}_y$$

On the other hand

(9) $$\hat{\mathbf{E}} = -\nabla \hat{V} = -\nabla\left(V_r + jV_i\right)$$

what gives as relations

(10) $$E_{xr} = -\frac{\partial V_r}{\partial x} \quad E_{xi} = -\frac{\partial V_i}{\partial x} \quad E_{yr} = \frac{\partial V_r}{\partial y} \quad E_{yi} = -\frac{\partial V_i}{\partial y}$$

The second square of the modulus of the vector $\hat{\mathbf{E}}$ can be derived as

(11) $$\left|\hat{\mathbf{E}}\right|^2 = \hat{\mathbf{E}} \cdot \hat{\mathbf{E}}^* = E_{xr}^2 + E_{yr}^2 + E_{xi}^2 + E_{yi}^2$$

Boundary conditions in time domain are given by

(12) $$\left(\mathbf{J}_1 - \mathbf{J}_2\right) \cdot \mathbf{n}_{12} = -\frac{\partial \rho_s}{\partial t}$$

(13) $$\left(\mathbf{D}_1 - \mathbf{D}_2\right) \cdot \mathbf{n}_{12} = \rho_s$$

where $\mathbf{n}_{12}$ is a unit vector perpendicular to boundary and directed from domain 1 to domain 2. Taking into account relations between current densities, flux densities and potential and taking symbolic transformation we get

(14) $$\hat{\varepsilon}_1 \frac{\partial \hat{V}_1}{\partial r} = \hat{\varepsilon}_2 \frac{\partial \hat{V}_2}{\partial r}$$

where symbolic dielectric permeabilities have values:

(15) $$\hat{\varepsilon}_1 = \varepsilon_1 - j\frac{\sigma_1}{\omega} \qquad \hat{\varepsilon}_2 = \varepsilon_2 - j\frac{\sigma_2}{\omega}$$

**Force acting on single particle**

Let us calculate the mean value of force acting on particle when all fields are sinusoidal in steady state. In time domain the force is given by

(16) $$\mathbf{f}(t) = \left(\mathbf{p}(t) \cdot \nabla\right)\mathbf{E}(t)$$

Polarization vector in complex domain is given by formula

(17) $$\mathbf{p}(t) = \mathbf{P}_{\mathrm{m}} \cos(\omega t + \varphi) = \frac{1}{2}\left(\hat{\mathbf{P}}_{\mathrm{m}} e^{j\omega t} + \hat{\mathbf{P}}_{\mathrm{m}}^* e^{-j\omega t}\right)$$

Likewise for electric field we can write

(18) $$\mathbf{E}(t) = \frac{1}{2}\left(\hat{\mathbf{E}}_{\mathrm{m}} e^{j\omega t} + \hat{\mathbf{E}}_{\mathrm{m}}^* e^{-j\omega t}\right)$$

Thus, force acting on particle in time domain has the value

(19) $$\mathbf{f}(t) = \frac{1}{4}\Big[\left(\hat{\mathbf{P}}_{\mathrm{m}} \cdot \nabla\right)\hat{\mathbf{E}}_{\mathrm{m}} e^{2j\omega t} + \left(\hat{\mathbf{P}}_{\mathrm{m}}^* \cdot \nabla\right)\hat{\mathbf{E}}_{\mathrm{m}} + \left(\hat{\mathbf{P}}_{\mathrm{m}} \cdot \nabla\right)\hat{\mathbf{E}}_{\mathrm{m}}^* + \left(\hat{\mathbf{P}}_{\mathrm{m}}^* \cdot \nabla\right)\hat{\mathbf{E}}_{\mathrm{m}}^* e^{-2j\omega t}\Big]$$

Integral the first and last term over one time period $T$ is equal zero, so

(20) $$\left\langle \mathbf{f}(t) \right\rangle = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2}\mathrm{Re}\left[\left(\hat{\mathbf{P}}_{\mathrm{m}}^* \cdot \nabla\right)\hat{\mathbf{E}}_{\mathrm{m}}\right]$$

Because equivalent dipole moment of single particle in two dimensions is given by

(21) $$\hat{\mathbf{P}} = 2\pi\varepsilon_1 r_0^2 \left(\hat{K}\hat{\mathbf{E}} \cdot \nabla\right)\hat{\mathbf{E}}^*$$

and dividing Clausius-Mosotti coefficient $\hat{K}$ into real and imaginary part, we get the total force acting on particle in two dimensions. Electric field is considered as root mean square value.

(22) $$\mathbf{F} = \pi\varepsilon_1 r_0^2 \left\{K_{\mathrm{R}} \nabla\left|\hat{\mathbf{E}}\right|^2 + 2K_{\mathrm{I}}\left[\nabla \times \left(\mathbf{E}_{\mathrm{I}} \times \mathbf{E}_{\mathrm{R}}\right)\right]\right\}$$

Real and imaginary parts of the Clausius-Mosotti coefficient are given by:

(23) $$\hat{K}\left(\hat{\varepsilon}_1, \hat{\varepsilon}_2\right) = K_{\mathrm{R}} + jK_{\mathrm{I}}$$

(24) $$K_{\mathrm{R}} = \frac{\omega^2\left(\varepsilon_2^2 - \varepsilon_1^2\right) + \left(\sigma_2^2 - \sigma_1^2\right)}{\omega^2\left(\varepsilon_2 + \varepsilon_1\right)^2 + \left(\sigma_2 + \sigma_1\right)^2}$$

(25) $$K_{\mathrm{I}} = \frac{2\omega\left(\varepsilon_2\sigma_1 - \varepsilon_1\sigma_2\right)}{\omega^2\left(\varepsilon_2 + \varepsilon_1\right)^2 + \left(\sigma_2 + \sigma_1\right)^2}$$

where all permittivities are equal $\varepsilon_{\mathrm{w}}\varepsilon_0$.

**Results**

The simulated chamber is modeled as a two-dimensional model, where we need, because of longitudinal symmetry, to consider only a single pair of electrodes, one with positive $V_z$ = 10 V and one with zero voltage. The extension of the interdigitated electrode array beyond the considered region can be simulated by applying periodic boundary conditions to the left and right of the problem boundary model. Figure 1 shows a cross-sectional geometry of the canal, which includes the substrate and channel covers, the interdigitated electrodes and a fluid. The finite element calculations with triangle elements was used for following geometrical dimensions: A-B = 60 μm, A-C = 160 μm, the electrode dimensions are $a$ = 40 μm, $b$ = 40 μm, $h$ = 4 μm. High of the chamber with fluid is 45 μm. Cylindrical particle has radius $r_1$ = 3 μm and relative permittivity of the particle amounts $\varepsilon_2$ = 160. All forces are calculated for particle 1 meter long in $y$ direction.

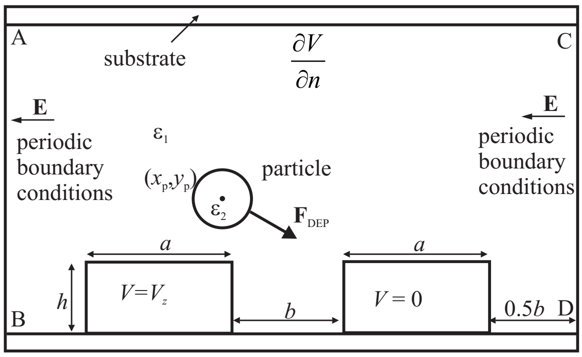



Fig.1. Single electrode arrangement in dielectrophoretic trap.

The conducting fluid, where particle moves, has relative permittivity $\varepsilon_1$ = 80 and angular frequency of the source has value $\omega = 30 \cdot 10^3$ rd/s.

After solution of the Laplace equations in complex domain (6) and (7) for potential, electric field was calculated. Next modulus of the electric field according to equation (11) and its gradient were obtained. This allows us to compute the force acting on dielectric particle immersed in dielectric and conducting fluid. The particle itself in two-dimensional computations has cylindrical form and is placed parallel to *z* axes, that is perpendicular to figure 1 plane.

Dependence of the real part of the Clausius-Mosotti coefficient from frequency, is depicted in Fig.2. It was calculated according equation (24). The sign of this constant depends from relative permittivity, as it is shown in figure below. For enough great value of this coefficient, Clausius-Mosotti coefficient changes his sign, what means that force acting on particle also changes its direction.

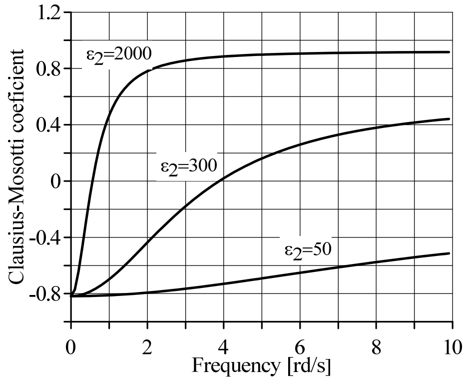



Fig.2. Dependence of the real part of the Clausius-Mosotti coefficient from frequency. Relative permittivity of particle is as parameter.

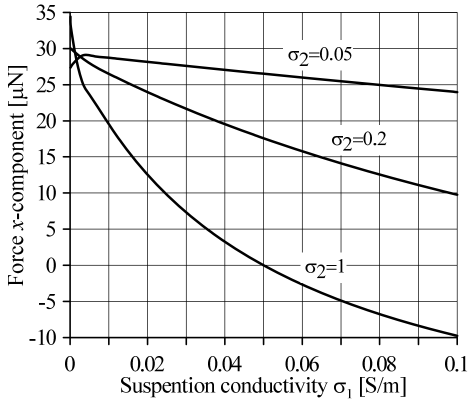
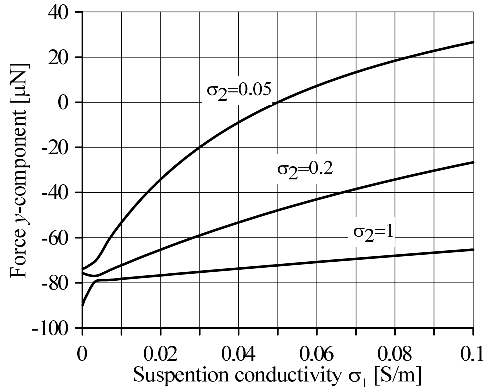



Fig.3. Dependence of the $x$-component of the force $\mathbf{F}$ acting on particle from fluid conductivity. Conductivity of the particle in $\mathrm{S/m}$ is a parameter.

Next dependence of force acting on particle according with equation (22) was for different fluid and particle parameters calculated. The second part of this equation can be neglected, because it is in this case equal zero. In Fig. 3 dependence of the $x$-component of the force $\mathbf{F}$ acting on particle from fluid conductivity is shown.

For values of the conductivity $\sigma_1$ higher then $1\mathrm{S/m}$,. the $x$-component of the force becomes negative. Generally this force diminishes with particle conductivity. In Fig. 4 the $y$-component of the total force is shown. For the values of the particle conductivity greater than $0.05\ \mathrm{S/m}$, this component is always negative, that is it is directed to the trap.

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## Conclusions

In this article, cylindrical particle in uniform electric field perpendicular to the particle was considered. The Clasius-Mosotti coefficient allows us to deduce the direction of the total force component acting on particle and to investigate the influence of different fluids on particle parameters on the magnitude and the direction of the resulting force.

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