

# Performance comparison of wavelet thresholding techniques on weak ECG signal denoising

**Abstract.** The Electrocardiogram (ECG) signal is a biological non-stationary signal which contains important information about rhythms of heart. ECG signals can be buried by various types of noise. These types can be electrode movement, strong electromagnetic effect and muscle noise. Noisy ECG signal has been denoised using signal processing. This paper presents a weak ECG signal denoising method based on interval-dependent thresholds of wavelet analysis. Several experiments were conducted to show the effectiveness of the interval-dependent thresholding method and compared the results with the soft and hard wavelet thresholding methods for denoising. The results are evaluated by calculating the root mean square error and the correlation coefficient.

**Słowa kluczowe.** W artykule przedstawiono metodę odzuszumiania sygnałów elektrokardiografu w oparciu o analizę falkową. W rozwiązaniu zastosowano progowanie przedziałowo-zależne. Na podstawie poczynionych eksperymentów oraz wyznaczonych wartości RMS błędu i współczynnika korelacji wykazano jego skuteczność. Dodatkowo dokonano porównania otrzymanych wyników z działaniem metod miękkiego i twardego progowania falkowego. (Porównanie działania metod progowania falkowego w odzuszumianiu sygnałów EKG).

**Keywords:** ECG signal; wavelet denoising; interval-dependent thresholding.

**Słowa kluczowe:** sygnały EKG, odzuszumianie falkowe, progowanie przedziałowo-zależne.

## 1. Introduction

The ECG signal identifying rhythms of the heart is one of the most important biomedical signals [1]. This signal is corrupted by various types of noise [2]. These corruptions can be removed by using various denoising techniques [3]. If the signal-to-noise ratio (SNR) is under -10dB, the signal may refer to as weak signal [3]. Thus, the idea of denoising has been emerging and this may be utilized at various practical application fields such as radar, physic, communication, earthquake and biomedical signals [3]. In literature can be found in various denoising methods. Some methods such as Wiener filtering and Kalman filtering methods have been proposed to remove the additive noises [4-5]. However, frequency domain filters could cause distortion in a transient interval of the signal and important clinical information may be lost because nature of the ECG signal is non-stationary. [6]. To protect the signal features, wavelet transform is the most popular [7, 8, 9, 6]. Donoho and Johnstone have proposed several wavelet shrinkage approaches [10, 11]. Karel suggested two performance criteria to measure the quality of a wavelet based on the principle of maximization of variance [12]. Mahmoodabadi et al. proposed an ECG feature extraction system based on the multi-resolution wavelet transform [13]. Shantha et al. suggested designing optimal discrete wavelet a method which uses based orthogonal filter bank for cardiac signal [14]. Nikolaev et al. suggested a two-step algorithm for remove of Electromyogram (EMG) artifacts from the ECG using Wavelet Domain Wiener Filtering has been investigated [15].

In this paper, an interval-dependent thresholding [16] method was used to remove the noise from the weak ECG signals. Correlation coefficients and Root Mean Square Error (RMSE) calculated to evaluate the performance of the wavelet based interval-dependent thresholding method for denoising the weak ECG signals. It also was realized a comparative study to show the effectiveness of the interval-dependent thresholding method with hard and soft thresholding techniques for different SNR values.

This paper is organized as follows: In Sect. 2 the basic concepts of wavelet transformation and denoising by using wavelet transform are described. In Sect. 3 the interval-dependent thresholding method is explained. Experimental results are presented and discussed in Sect.4. Finally, conclusion is given in Sect. 5.

## 2. Wavelet Transform

Wavelet transform is a multiresolution analysis as developed by Mallat [17]. Classical methods like Fourier Transform are analyzed in frequency, only. The wavelet transform analyzes signals in both time and frequency domains. It is an important tool for analysis of ECG signals because the ECG signal is time-varying non-stationary [18]. Short-Time Fourier Transform (STFT) uses fixed time-frequency to find spectrogram, whereas wavelet transform presents best time resolution for low and high frequency components [2].

The wavelet transform is change width of the window, which is the most important characteristic of the wavelet transform. There are two functions scaling function and mother wavelet. A mother wavelet  $\psi(t)$  is a function of zero average and as follows [2, 18, 19].

$$(1) \quad \int_{-\infty}^{\infty} \psi(t) dt = 0$$

The Continuous Wavelet Transform (CWT) given by [19]:

$$(2) \quad CWT(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt$$

Where  $a, b$  are the scale and position parameters.

### 2.1. The Denoising Algorithm

This algorithm is applied to denoising a noisy signal. Assume that the received signal  $y(n)$  is given as follows.

$$(3) \quad y[n] = x[n] + d[n]$$

where  $x(n)$  will denote an unknown signal to be detected, and  $d(n)$  is a white Gaussian noise. The original signal may also have high frequency features. High frequency characteristics of original signal with wavelet transform is preserved. Therefore, the wavelet transform is an effective method for denoising the noisy signal [2, 20]. Standard wavelet thresholding techniques, consists of hard thresholding and soft thresholding functions [21-22]. Wavelet Transformation, threshold selection is very important. The signal if the threshold value is too large or

too small cannot be estimated accurately [23]. Soft thresholding given as following [16];

$$(4) \quad \hat{X} = \begin{cases} y - \text{sgn}(y)T & \text{if } |y| \geq T \\ 0 & \text{if } |y| < T \end{cases}$$

Hard threshold [16];

$$(5) \quad \hat{X} = \begin{cases} y & \text{if } |y| \geq T \\ 0 & \text{if } |y| < T \end{cases}$$

The steps of the algorithm using soft and hard thresholding are as listed below [2, 16].

1- Firstly, the received signal levels are separated by wavelet transform. Then, the received signal wavelet coefficients are calculated up to the desired level.

2- The variance ( $\sigma^2$ ) of the noise is calculated using the wavelet coefficients.

$$(6) \quad \hat{\sigma} = \frac{\text{med}(|W_{j,k}|)}{0.6745}$$

where  $\text{med}(\cdot)$  denotes the median.

3- The threshold value is calculated using the variance.

$$(7) \quad T = \sigma \sqrt{2 \cdot \log(n)}$$

where T is the threshold value and n is the length of signal.

4- Thresholding is performed using Eq. 4 or Eq. 5 after the calculation of the threshold value.

5- The original signal is reconstructed using the inverse wavelet transform and retained coefficients.

### 3. Interval-dependent thresholding method

The noisy signal decomposed with the detail coefficients and the approximation coefficients. Low-frequency components are shown with large coefficients and high-frequency components are shown with small coefficients. Wavelet coefficients that is smaller than the threshold value is removed. As a result, the original signal is obtained from the noisy signal. Method in this article, the threshold values are obtained separately for each level of wavelet transformation [16]. Because, high-frequency and low-frequency parts of the signals have different features such as mean value and standard deviation. Therefore interval-dependent threshold value is calculated separately for each level and each interval is denoised. In this study, the noisy ECG signals were tested effectiveness of this method. The block diagram of the interval-dependent thresholding method can be seen in Fig. 1.

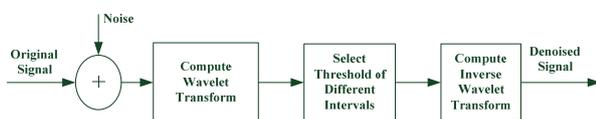


Fig.1. The block diagram of the interval-dependent thresholding method

The algorithm of the wavelet based interval-dependent denoising is described as following [16];

**Step1:** Decomposing of the noisy signal using the wavelet transform.

**Step 2:** Noise variance at each wavelet scale is calculated using Eq. 6.

**Step 3:** The threshold is calculated at each level using Eq. 7.

**Step 4:** Hard or soft threshold values are calculated using interval-dependent thresholding method of in the different intervals by using Eq. 4 or 5.

**Step 5:** The original signal is reconstructed from the retained coefficients using the inverse wavelet transform.

The noisy signal using wavelet transform is decomposed into 8 levels. Then, instead of a threshold for all levels, the threshold value is determined separately for each level. The wavelet coefficients of the noise are eliminated. The original signal is obtained from the retained coefficients. The most important feature of this method is to determine the threshold for each level separately. This feature improves the performance of the algorithm.

### 4. Experimental studies

In this paper, the ECG signals are assumed to be corrupted by white Gaussian noise with low SNRs. In the experiments, ECG signals were taken from Harvard-MIT Division of Health Sciences and Technology [24]. Signals 8-channel instrumentation tape-recorded and later transferred to digital media [24]. The ECG signals are resampled at 44100 samples. An ECG signal can be seen in Fig. 2.

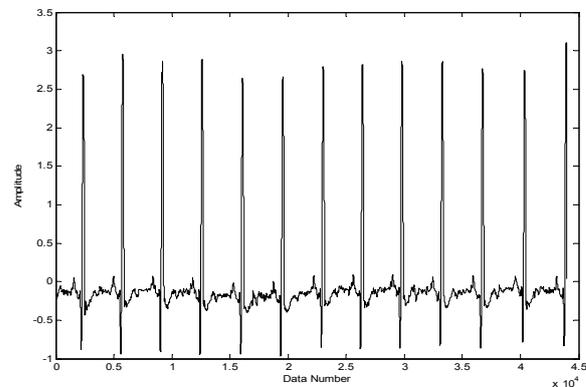


Fig.2. An ECG signal

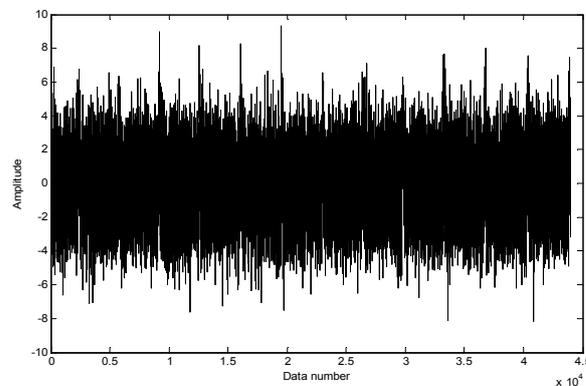


Fig.3. The noisy ECG signal (SNR=-10dB)

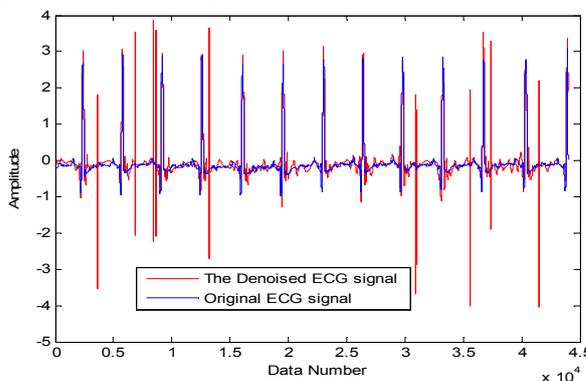


Fig.4. The denoised ECG signal using hard thresholding (SNR=-7dB)

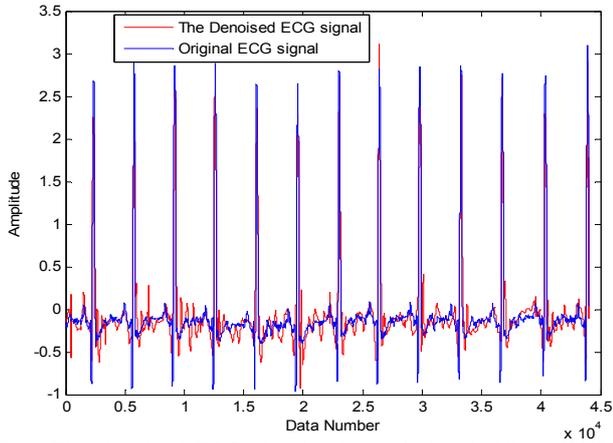


Fig.5. The denoised ECG signal using soft thresholding (SNR=-7dB)

A white Gaussian noise is generated and added to the ECG signals. A noisy ECG signal is shown in Fig. 3. for SNR value -10 dB.

Firstly, noisy ECG signal is tested with the popular soft and hard thresholding methods for denoising. Soft and hard threshold methods are described briefly in section 2. For a fair comparison, the wavelet transform parameters are fixed for all methods. db3 wavelet decomposition family and 8 level are selected. Figure 4 and 5 shows denoised signal by the popular hard and soft thresholding methods respectively. Figure 6 shows the denoised ECG signal using the proposed method for SNR value -7 dB.

We compared our proposal with the popular hard and soft thresholding methods [25]. Table 1, 2 and 3 presents a complete comparison with hard and soft thresholding methods [25] for the weak ECG signals that are corrupted by white Gaussian noise. To evaluate the performance of the proposed method and the compared methods [25] several evaluation tests such as correlation coefficients and RMSE methods were employed.

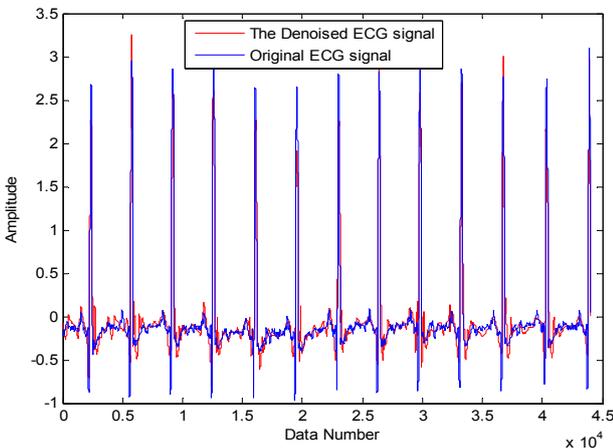


Fig.6. The denoised ECG signal using interval-dependent thresholding (SNR=-7dB)

The Correlation coefficient is a statistical concept [26]. It can measure of how well the predicted signal follows the original signal [2, 26]. The Correlation coefficient is used to evaluate the methods, as is defined as following:

$$(8) \quad r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}}$$

Root Mean Square Error will be used as a performance measure. The RMSE is defined as [27]:

$$(9) \quad RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (G_{(i)} - T_{(i)})^2}$$

Where,  $G_{(i)}$  is the original ECG signal,  $T_{(i)}$  is the denoised signal and  $n$  is the number of the samples.

From the plots shown in figures 4, 5 and 6, it can be noted that the interval-dependent thresholds of wavelet analysis give better results than soft and hard thresholding methods. The results of this analysis are shown in the related tables. Table 3 gives the performance measure values for the interval-dependent thresholds of wavelet analysis with different SNR rates ranging from -3 dB to -14.5dB. When the SNR rate is -3 dB, the correlation coefficient and the RMSE values are 0.9821 and 0.1073 respectively. Similarly, when the SNR rate is -14.5 dB, the correlation coefficient and the RMSE values are 0.8280 and 0.3314.

Table 1 and 2 gives the performance measure values for the hard and soft thresholding of wavelet analysis with different SNR rates ranging from -3 dB to -14.5dB. These two thresholding methods also achieved denoising when the SNR rate is between -3 to -10 dB. From the tables shown in Tables 1 and 2, it can be noted that the soft thresholding method a better result than hard thresholding when we consider the related correlation coefficients and RMSE values.

Table 1. RMSE and correlation coefficient values for various SNR using hard thresholding [25]

SNR (dB)	RMSE	Correlation coef.
-3	0.1186	0.9779
-4	0.1243	0.9761
-5	0.1293	0.9739
-6.46	0.1483	0.9659
-7.13	0.1781	0.9525
-8.15	0.2489	0.9158
-9.01	0.3617	0.8386
-9.51	0.4461	0.7775
-9.96	0.5334	0.7150
-14.41	2.1042	0.2627

Table 2. RMSE and correlation coefficient values for various SNR using soft thresholding [25]

SNR (dB)	RMSE	Correlation coef.
-3	0.1898	0.9449
-4	0.1887	0.9466
-5	0.1970	0.9412
-6.52	0.1973	0.9393
-7.1886	0.1964	0.9416
-8.12	0.2005	0.9372
-9.05	0.2078	0.9308
-9.54	0.2334	0.9118
-10.03	0.2338	0.9114
-14.50	0.6698	0.5741

Table 3. RMSE and correlation coefficient values for various SNR using interval-dependent thresholding

SNR (dB)	RMSE	Correlation coef.
-3	0.1073	0.9821
-4	0.1191	0.9778
-5	0.1204	0.9776
-6.49	0.1577	0.9607
-7.21	0.1637	0.9583
-8.15	0.1683	0.9555
-9.05	0.1936	0.9415
-9.58	0.2078	0.9310
-10.03	0.2159	0.9297
-14.50	0.3314	0.8280

It is evident that the interval-dependent thresholding of wavelet analysis is performing notably well in comparison with soft and hard thresholding methods.

## 5. Conclusions

The interval based wavelet denoising method for denoising the ECG signals are used. The interval-dependent threshold value is calculated separately for each level. The interval based wavelet denoising method has been tested for SNR down to -14.5 dB. When the SNR rate is about -14 dB, method was shown that could achieve notable denoising. The results of the simulation show that it has advantages comparing with the wavelet analysis based on hard and soft threshold. All computer simulations were performed in MATLAB environment. The results are evaluated by calculating the RMSE and the correlation coefficient. It can be noted that the interval-dependent thresholds of wavelet analysis a better result than soft and hard thresholding methods. In all SNR rates, the interval based wavelet denoising method shows superior performance of the ECG signals.

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