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Numerical method of computing impedances of a three-phase busbar system of rectangular cross section

Abstract. *In this paper, a new numerical method of calculating rectangular busbar system impedances is proposed. This method is based on the partial inductance theory. In particular, the impedances of a three-phase system of rectangular busbars with the neutral busbar, and the use of the method are described. Results for resistance and reactance for this systems of multiple rectangular conductor have been obtained, and the skin and proximity effects have also been taken into consideration. Finally, two applications of a three-phase system are described.*

Streszczenie. *W artykule przedstawiono nową numeryczna metodę obliczania impedancji układów szyn prostokątnych. Metoda ta oparta jest na teorii indukcyjności cząstkowych. W szczególności opisano impedancje szynoprzewodów prostokątnych w układzie trójfazowym z przewodem neutralnym. Wyznaczono rezystancje i reaktancje takiego wieloprzewodowego układu szynoprzewodów prostokątnych z uwzględnieniem zjawiska naskórkowości i zbliżenia. Wyznaczono impedancje dla dwóch przykładów układów trójfazowych z szynoprzewodami prostokątnymi.* (***Numeryczna metoda obliczania impedancji trójfazowego układu szynoprzewodów prostokątnych.***)

Key words: Rectangular busbar, high-current bus duct, impedance, numerical method
Słowa kluczowe: Prostokątny przewód szynowy, tor wielkoprądowy, impedancja, metoda numeryczna

Introduction

High-current air-insulated bus duct systems with rectangular busbars are often used in power generation and substation, due to their easy installation and utilisation. The increasing power level of these plants requires an increase in the current-carrying capacity of the distribution lines (usually 1-10 kA). The medium voltage level of the generator terminals is 10-30 kV. The construction of busbar is usually carried out by putting together several flat rectangular bars in parallel for each phase in order to reduce thermal stresses. The spacing between the bars is made equal to their thickness for practical reasons, and this leads to skin and proximity effects [1-6].

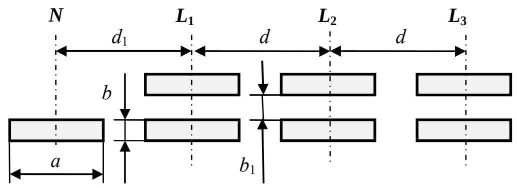


Fig. 1. Three phase high-current bus duct of rectangular cross-section with two busbars per phase and one neutral busbar

The busbar resistance and reactance are not normally sufficiently large to affect the total impedance of a power system and hence is not included in the calculations when establishing the short-circuit currents and reactive volt drops within a power system. The exception to this is when considering certain heavy current industrial applications such as furnaces, welding sets, or roll heating equipment for steel mills. In these cases the reactance may be required to be known for control purposes, or to obtain busbar arrangements to give minimum or balanced reactance. This may be important because of its effect on both volt drop and power factor, and hence on the generating plant kVA requirement per kW of load, or on the tariffs payable where the power is purchased from outside [4].

The inductances and the effective resistances, in other words the impedances, of a system of busbars at a certain frequency are closely related to the current distribution over the cross-section of each busbar generally known as “skin effect” and “proximity effect” of nearby busbars. Both the skin effect and proximity effect will generally cause the resistance of the busbars to increase and the inductance to decrease. If the current distribution is not uniform over the cross section of busbar, the determination of skin and proximity effects becomes complex. Hence the computation of the resistance and inductance of busbars is also complex [7,8]. The analytical formulae are possible for round wires and tubes [9-12], very long and thin (tapes or strips) rectangular busbars [7, 13-20] or for d.c. cases (current densities are assumed to be uniform) [13-15, 21-27]. In the other cases of rectangular busbars analytically-numerical and numerical methods must be applied [5, 11, 14, 15, 28-36]. These impedances can also be determined by experimental way [37-39].

Integral equation

The integral formulation is well known [9, 10, 21-25, 40] and is derived by assuming sinusoidal steady-state, and then applying the magnetoquasistatic assumption that the displacement current $\varepsilon\omega\underline{\boldsymbol{E}}$, is negligible. Given this, the complex vector potential $\underline{\boldsymbol{A}}$ can be related to the complex current density $\underline{\boldsymbol{J}}$, by

$$(1)\qquad \underline{\boldsymbol{A}}(X)=\frac{\mu_0}{4\pi}\int_v\frac{\underline{\boldsymbol{J}}(Y)}{\rho_{XY}}\,\mathrm{d}v$$

where $X=X(x_1,y_1,z_1)$ is the point of observation, $Y=Y(x_2,y_2,z_2)$ is the source point, v is the volume of all conductors, $\rho_{XY}=\sqrt{r_{XY}^2+(z_2-z_1)^2}$ is the distance between the point of observation X and the source point Y (Fig.2), where $r_{XY}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

Also, directly from Faraday’s law and the definition of $\underline{\boldsymbol{A}}(X)$, it follows that

$$(2)\qquad \underline{\boldsymbol{E}}(X)=-\mathrm{j}\,\omega\underline{\boldsymbol{A}}(X)-\mathbf{grad}\underline{V}(X)$$

where $\underline{V}(X)$ is referred to as the complex scalar electric potential and ω is the angular frequency.

Assuming the ideal conductor constitutive relation, $\underline{\boldsymbol{J}}(X)=\sigma\underline{\boldsymbol{E}}(X)$, and combining this relation with (1) and (2) results in

(3) $$\frac{\underline{\boldsymbol{J}}(X)}{\sigma}+\frac{\mathrm{j}\omega\,\mu_0}{4\pi}\int_v\frac{\underline{\boldsymbol{J}}(Y)}{\rho_{XY}}\,\mathrm{d}v=-\mathbf{grad}\underline{V}(X)$$

In the case of N straight parallel conductors with length l, conductivity σ_i (i=1, 2,…, N), cross section S_i with sinusoidal current input function with angular frequency ω and complex value $\underline{I}_i$ respectively flowing in direction of *Oz*, the complex current density has one component along the *Oz* axis, that is $\underline{\boldsymbol{J}}_i(X)=\mathbf{a}_z\,\underline{J}_i(X)$. The component $\underline{J}_i(X)$ is independent of variable *z* and in a general case, depends on the self current and on the currents in the neighboring conductors – there are skin and proximity effects. Then also the vector magnetic potential $\underline{\boldsymbol{A}}(X)=\mathbf{a}_z\,\underline{A}(X)$ and the electric field $\underline{\boldsymbol{E}}(X)=\mathbf{a}_z\,\underline{E}(X)$.

The potential $\underline{V}(X)$ is a function of variable *z* only and must satisfy Laplace's equation

(4) $$\frac{\mathrm{d}^2\underline{V}(z)}{\mathrm{d}z^2}=0$$

the solution of which is

(5) $$\underline{V}(z)=\frac{\underline{V}(0)-\underline{V}(l)}{l}\,z+\underline{V}(0)$$

By introducing a unit voltage drop (in $\mathrm{V\cdot m^{-1}}$) in a conductor

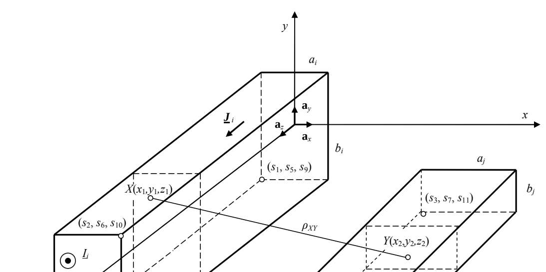
(6) $$\underline{u}=\frac{\underline{V}(l)-\underline{V}(0)}{l}$$

we obtain from the formula (5) that

(7) $$\underline{V}(z)=-\underline{u}\,z+\underline{V}(0)$$

and finally

(8) $$-\mathbf{grad}\underline{V}(X)=\underline{u}$$



Fid. 2. The i^{th} and p^{th} conductors of a system of N parallel busbars of rectangular cross section

Thus, in the case of N parallel conductors (Fig.1), the integral equation for i^{th} conductor is given as following one

(9) $$\frac{\underline{J}_i(X)}{\sigma_i}+\frac{\mathrm{j}\omega\,\mu_0}{4\pi}\sum_{j=1}^{N}\int_{v_j}\frac{\underline{J}_j(Y)}{\rho_{XY}}\,\mathrm{d}v_j=\underline{u}_i$$

or

(10) $$\frac{\underline{J}_i(X)}{\sigma_i}+\frac{\mathrm{j}\omega\mu_0}{4\pi}\int_{v_i}\frac{\underline{J}_i(Y)}{\rho_{XY}}\,\mathrm{d}v_i+\frac{\mathrm{j}\omega\mu_0}{4\pi}\sum_{\substack{j=1\\ j\neq i}}^{N}\int_{v_p}\frac{\underline{J}_j(Y)}{\rho_{XY}}\,\mathrm{d}v_j=\underline{u}_i$$

where i,j =1, 2,…, N.

Then, by simultaneously solving (9) or (10) with the current conservation, $\nabla\cdot\underline{\boldsymbol{J}}(X)=0$, the conductor current densities and the unit voltage drops can be computed.

In the case shown in Fig. 1 for each busbar the integral equation can be written as

(11) $$\frac{\underline{J}_{i,k}(X)}{\sigma_i}+\frac{\mathrm{j}\omega\,\mu_0}{4\pi}\sum_{j=1}^{N_c}\sum_{l=1}^{N_j}\int_{v_{i,l}}\frac{\underline{J}_{j,l}(Y)}{\rho_{XY}}\,\mathrm{d}v_{j,l}=\underline{u}_i$$

where:

- N_c is the number of phases plus the neutral and i, j =1, 2,…, N_c (N_c=4),
- N_j is the number of busbars belonging to one phase or the neutral and k,l =1, 2,…, N_j .

Multiconductor model of the busbars

In this model, each phase and neutral busbars is divided in several thin subbars or filaments [5, 20, 32, 35], as shown in Fig. 3.

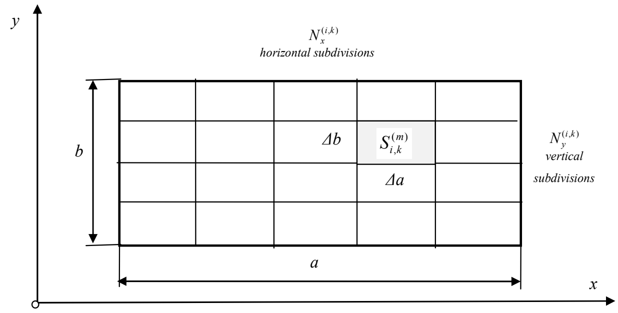


Fig. 3. The k^{th} bar of the i^{th} phase divided into $N_{i,k}=N_x^{(i,k)}N_y^{(i,k)}$ subbars

This division of the k^{th} bar of the i^{th} phase or the neutral into subbars is carried out separately for the horizontal (*Ox* axis) and vertical (*Oy* axis) direction of its cross-sectional area. Hence, subbars are generally rectangular in the cross-section, with the length and width, respectively, defined by the following relations:

(12) $$\Delta a=\frac{a}{N_x^{(i,k)}}\quad\text{and}\quad\Delta b=\frac{b}{N_y^{(i,k)}}$$

where *a* and *b* are the width and the thickness of the busbar respectively, $N_x^{(i,k)}$ and $N_y^{(i,k)}$ are the number of divisions along the busbar width and thickness respectively. So the total number of subbars of the k^{th} bar of the i^{th} phase is $N_{i,k}=N_x^{(i,k)}N_y^{(i,k)}$, and they are numbered by $m=1,2,...,N_{i,k}$. For the l^{th} bar of the j^{th} phase or the neutral we have the total number of subbars $N_{j,l}=N_x^{(j,l)}N_y^{(j,l)}$ numbered by $n=1,2,...,N_{j,l}$. All subbars have the length *l*. If the area $S_{i,k}^{(m)}=\Delta a\cdot\Delta b$ of the m^{th} subbar is very small and the diagonal $\sqrt{(\Delta a)^2+(\Delta b)^2}$ of it is not greater than skin depth, we can neglect the skin effect and assume that the complex current density can be considered uniform, i.e.

(13) $$\underline{J}_{i,k}^{(m)}=\frac{\underline{I}_{i,k}^{(m)}}{S_{i,k}^{(m)}}$$

where $\underline{I}_{i,k}^{(m)}$ is the complex current flowing through the m^{th} subbar.

Busbar impedances

For the m^{th} subbar the integral equation (10) can be written as

$$\frac{\underline{J}_{i,k}^{(m)}(X)}{\sigma_i} + \frac{\mathrm{j}\omega\mu_0}{4\pi}\sum_{j=1}^{N_c}\sum_{l=1}^{N_j}\sum_{n=1}^{N_{j,l}}\int_{v_{t,l}}\frac{\underline{J}_{j,l}^{(n)}(Y)}{\rho_{XY}}\,\mathrm{d}v_{j,l}^{(n)} = \underline{u}_i \tag{14}$$

where $v_{j,l}^{(n)}$ is the volume of the n^{th} subbar of the l^{th} bar of the j^{th} phase or the neutral.

Now, we can divide Eq. (14) by the area $S_{i,k}^{(m)}$ and integrate over the volume $v_{i,k}^{(m)}$ of the m^{th} subbar. Then we obtain the following equation

$$R_{i,k}^{(m)}\underline{I}_{i,k}^{(m)} + \mathrm{j}\omega\sum_{j=1}^{N_c}\sum_{l=1}^{N_j}\sum_{n=1}^{N_{j,l}}M_{(i,k)(j,l)}^{(m,n)}\underline{I}_{j,l}^{(n)} = \underline{U}_i \tag{15}$$

where $\underline{U}_i$ is the voltage drop across of all subbars of the i^{th} phase or the neutral (they are connected in parallel), and the resistance of the m^{th} subbar is defined by

$$R_{i,k}^{(m)} = \frac{l}{\sigma_i S_{i,k}^{(m)}} \tag{16}$$

and the self or the mutual inductance is expressed as

$$M_{(i,k)(j,l)}^{(m,n)} = \frac{\mu_0}{4\pi\, S_{i,k}^{(m)} S_{j,l}^{(n)}}\int_{v_{i,k}^{(m)}}\int_{v_{j,l}^{(n)}}\frac{\mathrm{d}v_{i,k}^{(m)}\,\mathrm{d}v_{j,l}^{(n)}}{\rho_{XY}} \tag{17}$$

The exact closed formulae for the self and the mutual inductance of rectangular conductor of any dimensions, including any length, are given in [21] and [22] respectively. We do not use here the geometric mean distance and the formula for mutual inductance between two filament wires as well.

The set of equations like as (15), written for all subbars, form the following general system of complex linear algebraic equations

$$\underline{\boldsymbol{U}} = \underline{\boldsymbol{Z}}\cdot\underline{\boldsymbol{I}} \tag{18}$$

where $\underline{\boldsymbol{U}}$ and $\underline{\boldsymbol{I}}$ are column vectors of the voltages and currents respectively of all subbars, and $\underline{\boldsymbol{Z}}$ is the symmetric matrix of self and mutual impedances (the impedance matrix) of all subbars and it can be expressed as

$$\underline{\boldsymbol{Z}} = \left[\underline{Z}_{(i,k)(j,l)}^{(m.n)}\right] \tag{19}$$

where the element of $\underline{\boldsymbol{Z}}$ is

$$\underline{Z}_{(i,k)(j,l)}^{(m.n)} = \begin{cases} R_{i,k}^{(m)} + \mathrm{j}\omega M_{(i,k)(j,l)}^{(m,n)} & \text{for } m=n, i=j, k=l \\ \mathrm{j}\omega M_{(i,k)(j,l)}^{(m,n)} & \text{for } m\neq n \end{cases} \tag{20}$$

The matrix $\underline{\boldsymbol{Z}}$ can be rearranged and rewritten as

$$\underline{\boldsymbol{Z}} = \left[\underline{Z}_{(i,k)(j,l)}^{(m.n)}\right] = \left[\hat{\underline{Z}}_{u,v}\right] \tag{21}$$

where the number of the u^{th} row is

$$u = \sum_{r=1}^{i}\sum_{p=1}^{k-1}N_{r,p} + (s-1)N_y^{(i,k)} + \alpha \tag{22}$$

and the number of the v^{th} column is

$$v = \sum_{r=1}^{j}\sum_{p=1}^{l-1}N_{r,p} + (t-1)N_y^{(j,l)} + \beta \tag{23}$$

where $N_{r,p}$ is the total number of subbars of the p^{th} bar of the r^{th} phase and $s,t = 1,2,\ldots,N_x^{(i,k)}$ as well $\alpha,\beta = 1,2,\ldots,N_y^{(i,k)}$.

Then, we can find the admittance matrix $\underline{\boldsymbol{Y}}$ which is the inverse matrix of the impedance matrix $\underline{\boldsymbol{Z}}$ and it is expressed as

$$\underline{\boldsymbol{Y}} = \left[\underline{Y}_{(i,k)(j,l)}^{(m.n)}\right] = \left[\hat{\underline{Y}}_{u,v}\right] = \underline{\boldsymbol{Z}}^{-1} = \left[\underline{Z}_{(i,k)(j,l)}^{(m.n)}\right]^{-1} = \left[\hat{\underline{Z}}_{u,v}\right]^{-1} \tag{24}$$

After calculating the admittance matrix it is possible to determine the current of the m^{th} subbar of the k^{th} bar of the i^{th} phase or the neutral as

$$\underline{I}_{i,k}^{(m)} = \sum_{j=1}^{N_c}\sum_{l=1}^{N_j}\sum_{n=1}^{N_{j,l}}\underline{Y}_{(i,k)(j,l)}^{(m,n)}\underline{U}_j \tag{25}$$

The total current of the i^{th} phase or the neutral is

$$\underline{I}_i = \sum_{k=1}^{N_i}\sum_{m=1}^{N_{i,k}}\underline{I}_{i,k}^{(m)} \tag{26}$$

By substituting Eq. (25) into Eq. (26), we obtain

$$\underline{I}_i = \sum_{j=1}^{N_c}\underline{Y}_{i,j}\underline{U}_j \tag{27}$$

where

$$\underline{Y}_{i,j} = \sum_{k=1}^{N_i}\sum_{m=1}^{N_{i,k}}\sum_{l=1}^{N_j}\sum_{n=1}^{N_{j,l}}\underline{Y}_{(i,k)(j,l)}^{(m,n)} \tag{28}$$

From the admittance matrix with elements given by Eq. (28), we can determine the impedance matrix of a three-phase system busbars with the neutral busbar as follows

$$\underline{\boldsymbol{Z}} = \left[\underline{Z}_{i,j}\right] = \underline{\boldsymbol{Y}}^{-1} = \left[\underline{Y}_{i,j}\right]^{-1} \tag{29}$$

Since each $\underline{Z}_{i,j}$ is obtained from a matrix whose elements are comprised of information related only to construction and material, its value is not affected by the busbar current. In spite of that the skin and proximity effects are taken into consideration.

Numerical examples

The first numerical example selected for this paper features a three-phase system of rectangular busbars with one neutral busbar, whose cross-section is depicted in Fig.1. According to the notations applied in this figure, the following geometry of the busbars has been selected: the dimensions of the phase rectangular busbars and the neutral busbars are $a = 60\,\mathrm{mm}$, $b = b_1 = 5\,\mathrm{mm}$ $d = d_1 = 90\,\mathrm{mm}$. The phase busbars and the neutral are made of copper, which has the electric conductivity of $\sigma = 56\,\mathrm{MS\cdot m^{-1}}$. The frequency is 50 Hz. All phases have two busbars per phase - $N_1 = N_2 = N_3 = 2$ and the neutral has one busbar - $N_4 = 1$. The length of the busbar system is assumed to be $l = 1\,\mathrm{m}$ and $l = 10\,\mathrm{m}$.

In the numerical procedure, each phase busbar is divided into $N_x^{(i,k)} = 30$ and $N_y^{(i,k)} = 5$ subbars, which gives 150 for each busbar. Hence, all three phase and the neutral busbars have 1050 total subbars. With the chosen division, each rectangular subbar has dimensions of $2 \times 1\,\mathrm{mm}$.This allows for the fact that the current density is uniform on the cross-section of the subbars. The results of computations are shown in Table 1.

Table 2. Self and mutual impedances in $\mathrm{m}\Omega$ of a three phase high-current bus duct of rectangular cross-section with a neutral busbar depicted in Fig.1.

l [m]	N_i \ N_j	1	2	3	4
1	1	0.038+j 0.233	0.002+j 0.126	-0.002+j 0.079	0.001+j 0.126
	2	0.002+j 0.126	0.038+j 0.232	0.001+j 0.127	-0.001+j 0.079
	3	-0.002+j 0.079	0.001+j 0.127	0.036+j 0.234	-0.003+j 0.048
	4	0.001+j 0.126	-0.001+j 0.079	-0.003+j 0.048	0.065+j 0.240
10	1	0.377+j 3.801	0.014+j 2.771	-0.019+j 2.348	0.010+j 2.771
	2	0.014+j 2.771	0.378+j 3.791	0.007+j 2.775	-0.016+j 2.343
	3	-0.019+j 2.348	0.007+j 2.775	0.361+j 3.813	-0.025+j 2.087
	4	0.010+j 2.771	-0.016+j 2.343	-0.025+j 2.087	0.647+j 3.868

The second configuration of a three phase busbar system, the impedances of which are investigated, is shown in Fig. 4. It has only one busbar per phase and neutral - $N_1 = N_2 = N_3 = 1$ and also $N_4 = 1$. The length of the busbar system is assumed to be $l = 1\,\mathrm{m}$ and $l = 10\,\mathrm{m}$. In the numerical procedure, each phase busbar is divided into $N_x^{(i,k)} = 30$ and $N_y^{(i,k)} = 5$ subbars, which gives 150 for each busbar. Hence, all three phase and the neutral busbars have 600 total subbars. With the chosen division, each rectangular subbar has still dimensions of $2 \times 1\,\mathrm{mm}$.

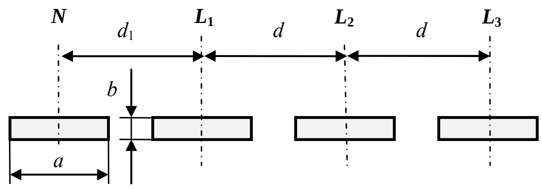


Fig. 4. Three phase high-current bus duct of rectangular cross-section with one busbar per phase and one neutral busbar

The results of computations are shown in Table 2.

Table 2. Self and mutual impedances in $\mathrm{m}\Omega$ of a three phase high-current bus duct of rectangular cross-section with a neutral busbar depicted in Fig.4.

l [m]	N_i \ N_j	1	2	3	4
1	1	0.066+j 0.241	0.002+j 0.127	-0.001+j 0.078	0.001+j 0.127
	2	0.002+j 0.127	0.066+j 0.241	0.001+j 0.127	-0.001+j 0.078
	3	-0.001+j 0.078	0.001+j 0.127	0.064+j 0.242	-0.002+j 0.046
	4	0.001+j 0.127	-0.001+j 0.078	-0.002+j 0.046	0.064+j 0.242
10	1	0.658+j 3.875	0.014+j 2.775	-0.014+j 2.334	0.008+j 2.780
	2	0.014+j 2.775	0.658+j 3.875	0.008+j 2.780	-0.014+j 2.334
	3	-0.014+j 2.334	0.008+j 2.780	0.641+j 3.887	-0.020+j2 .074
	4	0.008+j 2.780	-0.014+j 2.334	-0.020+j 2.074	0.641+j 3.887

Conclusions

A novel approach to the solution of impedances of the high-current bus ducts of rectangular cross-section has been presented in this paper. The proposed approached combines filament methods with the exact closed formulae for a.c. self and mutual inductances of rectangular conductors of any dimensions, which allows the precise accounting for the skin and proximity effects. Complete electromagnetic coupling between the phase busbars and the neutral busbar is taken into account as well.

As tables 1 and 2 show, both the skin effect and proximity effect will generally cause the resistance of the busbars to increase and the inductance to decrease. In addition, impedances of a three-phase busbar system are not proportional to its length, but the proposed method allows us to calculate the phase impedances of a set of parallel rectangular busbars of any dimensions including any length. For the industrial frequency, 100% increase of the number of total subbars changes impedances less than 0.02 % .

The validity of our numerical method has been successfully compared with a classical finite element method (FEM) such a FLUX2D software in the case of 2D busbar systems, particularly for the long busbars.

The model is strikingly simple, from a logical stand-point and can be applied in general to conductors of any section while conventional methods, being much more complicated, always require a greater or lesser degree of symmetry. From the practical stand-point of the calculations involved, the model requires the solution of a rather large set of linear simultaneous equations. However, this solution is well within the range of the possibility of existing computers, even those of overage capacity.

Acknowledgment

This work was supported by the National Science Centre, Poland, under Grant No. N N511 312540.

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