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An optimisation of the reflector shape subject to the linear inequalities

***Abstract**. The paper presents the results of an investigation into the calculation of a specular reflector which ensures a predetermined uniformity of illuminance distribution on a given surface. Optimisation calculations were carried out with a genetic algorithm with constraints of the searched area.*

***Streszczenie.** W artykule przedstawiono wyniki badań dotyczących obliczania kształtu zwierciadlanego odbłyśnika, który zapewnia uzyskanie rozkładu natężenia oświetlenia o zakładanej równomierności na zadanej powierzchni. Obliczenia wykonano z wykorzystaniem optymalizacji opierającej się na algorytmie genetycznym, w którym zastosowano ograniczenia liniowe (**Optymalizacja kształtu odbłyśnika z zastosowaniem liniowych ograniczeń**) .*

Keywords: calculation of luminaires, optimization, ray tracing method.
Słowa kluczowe: obliczanie opraw oświetleniowych, optymalizacja, metoda śledzenia promienia.

Introduction

The search for the best shape of luminaire optical components in the proposed method consists in simulations for a luminaire model which changes in successive iterations [1, 2]. The modification of the luminaire model consists in changing the shape of the reflector. The reflector profile is described by an interpolating polynomial which changes the profile shape when changes occur in the coordinates of the points which constitute interpolating nodes. The coordinates of interpolating nodes give the decisive variables Ci on which the optimisation algorithm operates. The search for the reflector shape is complete when the adopted assumptions are met. Since the optimisation algorithm searches for the function minimum, the adoption of assumption is to formulate the objective as a mathematical function. This work is intended to obtain the highest possible value of illuminance on a given surface at the assumed level of uniformity ratio of illuminance. The paper describes the results of an investigation which uses a genetic algorithm. Photometric parameters of the luminaire for the model of optical elements adopted in a given iteration are calculated with the use of a genuine calculation technique which employs ray tracing method. The optical elements of the calculated luminaire are formed as a reflector the active surface of which has the properties of ideal mirror reflection (i.e. it is a specular reflector).

The reflector model

The reflector profile is described by the Hermite interpolating polynomial [3]. The points $P1$, $P2$, $P3$, $P4$ and $P5$ (Fig. 1) are the interpolation nodes through which a curve passes that describes the reflector profile (Fig. 1).

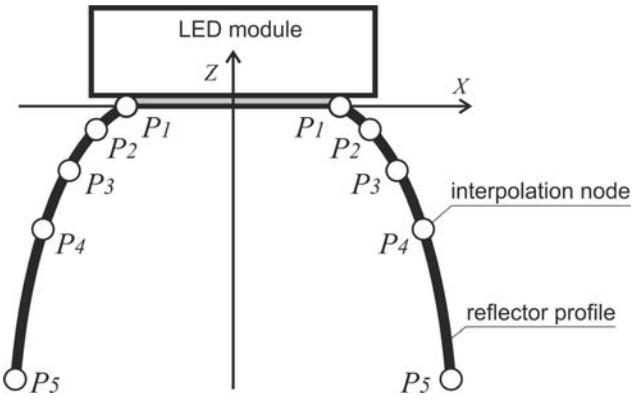

Fig. 1 The reflector model: LED module (light source), $P1$, $P2$, $P3$, $P4$, $P5$ – interpolation nodes

Two cases of the reflector model were studied with five interpolation nodes per each case. In the first case (Fig. 2), the two extreme nodes ($P1$, $P5$) do not change their position and hence they explicitly define the reflector dimensions (i.e. height and width). Between the extreme points are three nodes ($P2$, $P3$, $P4$) which have fixed and invariable axis X coordinates, whereas they can change axis Z coordinates. The Z coordinate values of the points are three variables on which the optimisation algorithm operates (z_2, z_3, z_4). The permissible change range of the points P shown in Figure 2 results from the assumption that the reflector profile curve should be convex upward.

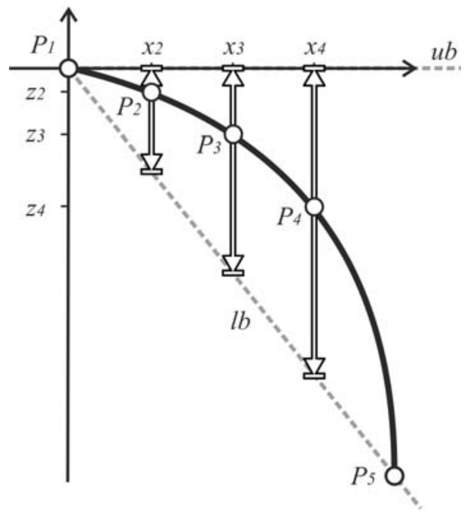

Fig. 2 The reflector profile (version I); the illustration of limiting conditions of the permissible positions of interpolation nodes (points $P2$, $P3$, $P4$)

In the successive steps which should result in finding the objective function minimum, the optimisation algorithm changes the values of the decisive variable Ci (i.e. the corresponding coordinates of the points $P2 \div P5$). The reflector profile shape is interpolated between the interpolation nodes by third order Hermite polynomials. A set of polynomials for all intervals forms a spline. The interpolation employed the Hermite polynomial with the shape preservation based on the method by Fritsch and Carlson [4].

The reflector design had the top opening and the bottom opening in the form of a square (Fig. 4). The shape of the four side walls is formed by a profile curve described with the Hermite interpolating polynomial. The top opening features a model of a LED module whose parameters meet the technical specifications of a Fortimo LED DLM 2000 module. The lamp of the module is a circular surface with a diameter of 6 cm and coated with a phosphor. The luminous flux distribution of the surface is nearly Lambertian [5].

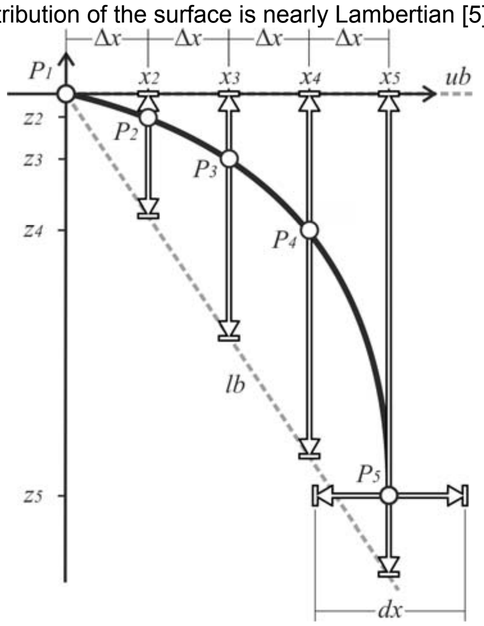

Fig. 3 The reflector profile (version II); the illustration of the limiting conditions of the permissible positions of interpolation nodes (points *P1, P2, P3, P4, P5*)

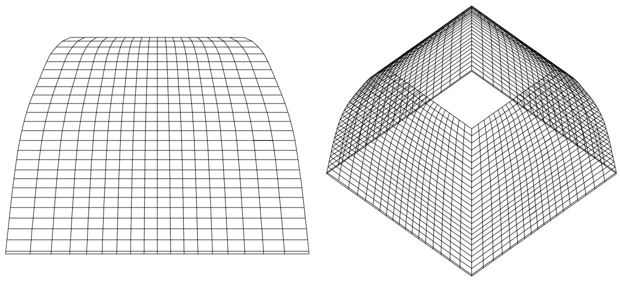

Fig. 4 The figure of the reflector model made of flat elementary surfaces. Side view and isometric view

The genetic algorithm

Genetic algorithms are searching methods which use mechanisms of natural selection and inheritance patterned on the theory of evolution. A genetic algorithm differs in the following ways from classic optimisation methods based on differentiation of the studied function:
- it generates a population of points, not a single point per iteration,
- it generates successive generations with random-number generators.

The genetic algorithm repeatedly changes the population of individual solutions. At each stage the algorithm randomly selects individuals from the current population which become parents and generate children for the next generation. The individual evolves during successive generations towards an optimal solution. The genetic algorithm can be applied to solve various optimisation problems which are not well adapted to standard optimisation algorithms, including the problems in which the objective function is discrete, indifferentiable, stochastic or highly non-linear. The genetic algorithm uses three main operations at each stage of creation of a new generation from the current population [6]:
- selection, i.e. the choice of parents among the individuals in the given population,
- crossover, i.e. the mating of parents who give individuals for the next generation,
- mutation, i.e. introduction of random variations for some of the selected parents.

Classic genetic algorithms operate on a binary representation of individuals. However, this method used a floating-point representation. The main purpose of this solution is to approximate the algorithm to the problem space. The variables in the problem are the coordinates of the interpolation nodes which form the reflector profile. In the floating-point representation, two points positioned close to each other in the representation space will also be close to each other in the problem space (and vice versa). This is usually not possible in binary representation [6].

Selection is a process of selecting parents from among individuals of a given population. The individuals which are more adapted have a higher chance of selection and they can introduce more children to the next generation. Several superior individuals which occur in the beginning of the calculation characteristic may result in a premature convergence and termination of the process without searching an adequately large area. On the other hand, at the end of the calculation characteristic and albeit the population may show a high variety, a small difference between the average and maximum fitness function leads to allocation of the same number of children to both average and best individuals. Then the evolutionary rule of survival of the fittest is replaced by a random walk among average individuals. In order to prevent these unfavourable processes, objective function scaling must be introduced. The scaling transforms the results obtained for the objective function into values from a range appropriate for the selection operation. The higher probability of selection belongs with the individuals with higher scaled values.

A ranking method was used to scale the objective function. Scaling involves arranging the population by values of the objective function. The number of copies produced for each individual depends on the individual's place in the series (i.e. its rank). The most adapted individual is at the beginning of the series (rank 1). The ranking method follows the two following rules:
- the scaled value of the rank n individual is directly proportional to $n^{-0,5}$,
- the sum of scaled values in the entire population is equal to the number of parents needed to create the next generation.

Scaling with ranking levels the scores of less adapted individuals while preserving a high variety in the population that is required to search an adequately large area.

In the selection process a given individual can be chosen as a parent more than once. Then it passes its genes to a larger number of children. In a uniform stochastic selection, each parent occupies a certain length of the line which is proportional to its scaled value. The algorithm moves along the line in steps of identical lengths, finds a specific parent in each step and assigns it to the next population.

By creating the next generation, the algorithm transfers two most adapted individuals to the next generation. This maintains the best solutions. The remaining free places in the population are for the individuals created by the parents by crossover and mutation. Eighty percent of the places are populated by individuals produced by crossing of the parents, while the remaining share is for the individuals produced by mutation.

Heuristic crossover creates a child which lies on the line with two parents at a small distance from a more adapted parent in the direction away from a less adapted parent [6, 7]. The parameter R defines how far the child is from the more adapted parent (1). This type of crossover is significantly related to the problem and allows searching in a promising direction. The completed simulations show that when compared to e.g. single-point or multi-point crossover, heuristic crossover accelerates searching the given area.

(1) $$c_3 = c_2 + R(c_1 - c_2)$$

where: c_3 – the child created by two parents c_1 and c_2.

Mutation involves introduction of small random changes in an individual. A random number selected from the Gaussian distribution is added to that individual. The size of mutation proportional to the standard deviation is reduced in each new generation. The size of mutation can be controlled with two variables. The variable s defines the standard deviation of the mutation in the first generation. If the permissible range of changes in the first generation of individuals is defined for the vector v with two rows and with the number of columns i equal to the number of variables, the initial standard deviation σ is equal to (2):

(2) $$\sigma = s(v(i,2) - v(i,1))$$

The variable h controls the rate of change of the average mutation size in successive generations. The standard deviation is decreased in a linear manner so that the end mutation size is equal to $(1 - h)$ times its initial value. If the default value of h is 1, the mutation size in the last generation is equal to 0. The standard deviation $\sigma_{i,k}$ for the generation k at total number of generations n_g is equal to (3) [7]:

(3) $$\sigma_{i,k} = \sigma_{i,k-1}(1 - h(k / n_g))$$

The optimisation of the reflector shape

The calculations of the reflector profile were carried out with the optimisation method based on the genetic algorithms [7]. The photometric values (i.e. luminous intensity and illuminance) were calculated with the use of a proprietary method which employs the ray tracing algorithm implemented in the Radiance system [8]. The method has been described in several publications [9] which present calculation results of luminaires with optical elements of various photometric properties. The accuracy of the method was confirmed by a series of calculations for the cases in which accuracy verification can employ analytical calculations and measurements of the luminaire model.

The optimisation was carried out with the genetic algorithm function in Matlab. The optimisation problem involved finding the reflector shape which ensures the maximum value of average illuminance with the adequate uniformity ratio of illuminance on the illuminated surface. The adopted assumption is that the reflector illuminates a square surface with the side length of 3 metres. The luminaire model is located 3 m above the centre of the surface. In order to reduce the number of calculation points, the illuminance distribution is calculated only for the lines which cross along the surface centre and its diagonal (Fig. 5), not for the entire illuminated surface area. This positioning of the calculation points is sufficient due to the symmetrical nature of the luminous intensity distribution of the adopted luminaire model.

The objective function was built to account for the required highest value of average illuminance at the assumed level of uniformity ratio of illuminance R (4), where the uniformity ratio of illuminance is the ratio of the minimum illuminance E_{min} to the average illuminance E_{av}.

(4) $$F(C) = -E_{av} + f_k$$

where: f_k – the penalty function in the following form (5):

(5) $$f_k = \begin{cases} \left(50\left(R - \frac{E_{min}}{E_{av}}\right)\right)^2 & if \ \left(R - \frac{E_{min}}{E_{av}}\right) > 0 \\ 0 & \end{cases}$$

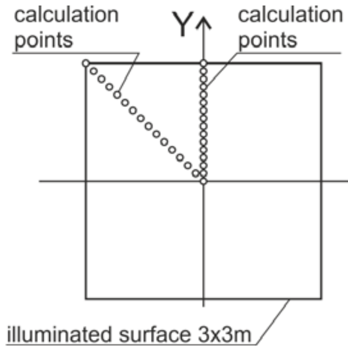

Fig. 5 The positions of 31 calculation points on the illuminated surface

Since the optimisation algorithm searches for the function minimum and the problem involves obtaining the highest possible illumination value, the negative sign is introduced in the objective function equation. It was assumed that the minimum uniformity ratio of illuminance R is 0,7. This means that the penalty function value (5) will be above zero if the calculated uniformity ratio of illuminance is less than 0,7. All results for which the uniformity ratio of illuminance is equal to or more than 0,7 are accepted without penalty.

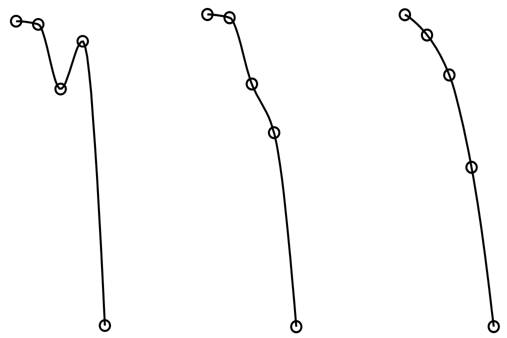

Fig. 6 The examples of the reflector profile curves generated in successive iterations of the unlimited optimisation process (from the left): the 3rd, 10th and 50th iteration; the circles mark the positions of interpolation nodes

The only constraints introduced in the search for the objective function minimum in the studies to date [1, 2, 3] involved defining the permissible range of changes in the coordinates Z of the points which are the interpolation nodes. The lower limit of the range is the straight line lb, and the upper limit is the straight line ub (Fig. 2 and 3). The limits can be formulated as (6, 7) for the first reflector model and as (8, 9) for the second reflector model.

(6) $$[-0.0175 \ -0.0375 \ -0.056] \leq C \leq [0 \ 0 \ 0]$$

(7) $$C = [z_2 \ z_3 \ z_4]$$

(8) $$[-0,025 \ -0,050 \ -0,075 \ -0,100 \ -0,1] \leq C \leq [0 \ 0 \ 0 \ 0 \ 0,1]$$

(9) $$C = [z_2 \ z_3 \ z_4 \ z_5 \ dx]$$

This notation of limits is intended to make the reflector profile curve convex upward.

However, the lack of relation in the positions between points Pi may result in a condition where the reflector profile curve does not remain monotonic in the entire interval. This occurs when e.g. the point $P4$ is above $P3$. Figure 6 presents several examples of the reflector profile curve generated in successive iterations of the optimisation process.

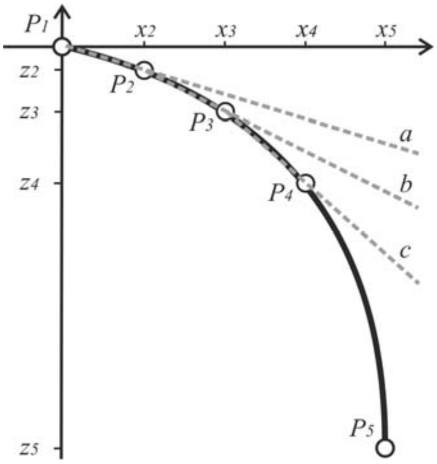

Fig. 7 The illustration of the conditions which allow preserving the monotony of the reflector profile curve

Creation of a reflector profile the curve of which is not monotonic may result in increased calculation times due to expansion of the search area by an area in which no optimum solution is found and the production of such reflector will not be technically possible. In order to improve the effectiveness of the optimisation algorithm and to preserve monotony of the reflector profile curve, additional constraints are introduced (Fig. 7):

- the point P3 should be below the straight line a which crosses the points P1 and P2,
- the point P4 should be below the straight line b which crosses the points P2 and P3,
- the point P5 should be below the straight line c which crosses the points P3 and P4 (only for the second reflector model).

The constraints can have the following notation for the second reflector model (10):

$$\begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_2 \\ z_3 \\ z_4 \\ z_5 \\ dx \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Table 1 lists the calculation results for the first reflector model. The calculations were carried out with two algorithm types. The first algorithm does not include the constraints which ensure monotony of the reflector profile curve (Table 1, no constraints). The second algorithm includes the constraints (Table 1, with constraints). The adopted calculation methods employ random-generated numbers which gives slightly differing results from each calculation. Hence, in order to obtain the average values which enable a statistical analysis of results, ten calculation series were carried out for each algorithm.

Table 1 The list of calculation results for the first reflector model. The objective function $F(C)$ value, the average illuminance E_{av}, the uniformity ratio of illuminance and the calculation time for the algorithm with and without constraints

No	Name	No constraints		With constraints	
		Average	Best	Average	Best
1	$F(C)$	-91,9	-98,6	-91,9	-99,7
2	Standard deviation of $F(C)$	4,2	-	6,7	-
3	E_{av} [lx]	92,5	99,1	92,2	100,7
4	E_{min} / E_{av}	0,68	0,67	0,70	0,68
5	No of iteration	440	-	441	-
6	CPU time [s]	539	-	595	-
7	Iterations / time elapsed [s]	0,82	-	0,74	-

The introduction of constraints did not improve the effectiveness of the algorithm. The best result was obtained for the algorithm with constraints, yet the difference between the algorithm and the algorithm without constraints is only 1%. Moreover, the results from the algorithm with constraints have a larger dispersion around the average value. The cause of failure to improve the effectiveness may result from improper selection of the position of the point $P5$ which defines the reflector dimensions. In the second reflector model (Fig. 3), releasing the point $P5$ may generate a reflector which will ensure a better result. Table 2 lists the calculation results for the second reflector model (with ten series for each algorithm).

The results obtained for the second reflector model show that algorithm effectiveness is improved. Releasing the point $P5$ and the freedom to change the reflector height and width allowed better results in comparison to the calculations carried out for the first reflector model. However, the introduction of constraints (Table 2) deteriorated the algorithm effectiveness. The average values and the best values of the ten series were obtained for the algorithm without constraints. The introduced constraints produce a reflector profile described by a monotonic curve. The choice of this reflector shape is sometimes forced by compliance with technological requirements.

Table 2 The list of calculation results for the second reflector model. The objective function $F(C)$ value, the average illuminance E_{av}, the uniformity ratio of illuminance and the calculation time for the algorithm with and without constraints

No	Name	No constraints		With constraints	
		Average	Best	Average	Best
1	$F(C)$	-102,3	-113,6	-95,6	-106,9
2	Standard deviation of $F(C)$	7,4	-	7,2	-
3	E_{av} [lx]	102,8	116,4	98,7	108,4
4	E_{min} / E_{av}	0,72	0,67	0,71	0,60
5	No of iteration	454	-	446	-
6	CPU time [s]	679	-	734	-
7	Iterations / time elapsed [s]	0,67	-	0,61	-

When the manufacturing process allows building the reflectors with a non-monotonic profile curve, the reflector design should have constraint options removed to verify if the produced solution is better. Figure 8 shows the reflector profile calculated with the algorithm without constraints in the series which produces the best result. This is an example of a tri-curve reflector in which three parts can be distinguished and each is described by a different curve. Multi-curve reflectors sometimes allow better results in comparison to single-curve reflectors [10].

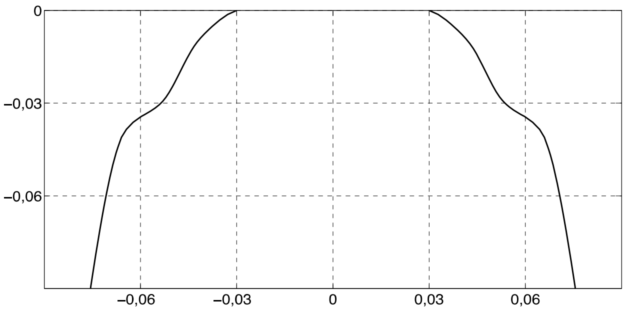

Fig. 8 The figure of a tri-curve reflector profile calculated with the algorithm without constraints

Conclusions

The concluded investigation reveals that the proposed method of reflector shape optimisation [1, 2, 3] can also be used in problems which involve obtaining a uniform illuminance distribution on a given surface.

Introduction of constraints can have unfavourable effects consisting in the rejection of solutions which might ensure a better result. If the processing capacities of reflector production allow designing multi-curve reflectors, the reflector calculations should have constraint options removed and the solution produced for the reflectors with non-monotonic profile curves should be verified. It seems that the profile representation accuracy in multi-curve reflectors is more significant than in single-curve reflectors [11]. Hence, further investigations will attempt to introduce a larger number of interpolation nodes through which the reflector profile curve passes.

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