

The maintenance costs estimation of electrical lines with the use of interpretability-oriented genetic fuzzy rule-based systems

Abstract. The paper demonstrates an evolutionary technique to design an interpretability-oriented fuzzy rule-based system and its application to estimate the maintenance costs of medium voltage electrical lines. The main goal of the proposed technique is to design the system with not only a relatively high accuracy for estimating the costs, but also with a clear and transparent structure which is easy to interpret by humans. The structure includes easily readable and understandable fuzzy logic rules that represent the knowledge about the considered problem.

Streszczenie. Praca demonstruje ewolucyjną technikę projektowania przejrzystych systemów regułowo-rozmytych i jej zastosowanie do szacowania kosztów utrzymania linii energetycznej średniego napięcia. Zaprojektowany system posiada względnie wysoką dokładność i przejrzystą strukturę w formie czytelnych, zrozumiałych i łatwych do interpretacji przez człowieka reguł logicznych, będących kwintesencją wiedzy o rozważanym problemie. (Szacowanie kosztów utrzymania linii energetycznych z wykorzystaniem systemów regułowo-rozmytych zorientowanych na przejrzystość)

Keywords: the maintenance costs of electrical line, genetic fuzzy rule-based systems, accuracy vs. interpretability.

Słowa kluczowe: koszty utrzymania linii energetycznej, genetyczne systemy regułowo-rozmyte, dokładność a przejrzystość.

Introduction

The estimation of the maintenance costs of electrical lines at the initial stage of their design process is one of the important problem in the electrical engineering area. On the one hand, the solution is inherently a difficult task, because it depends on many factors. On the other hand, it has a significant impact on a final decision about the profitability of future investment. For this reason, there is a necessity to find effective theoretical tools, especially from the field of computer science, to solve the problem. These tools may be used to develop decision support systems [1], which are very useful in supporting engineers, designers, managers, and other employees of institutions operating in the energy market in making proper decisions.

As early as in 1998, the above outlined problem was raised by O. Cordon, F. Herrera and L. Sanchez from the University of Granada, Spain. In [2, 3] they presented a set of numerical data representing the relationship between the maintenance cost of medium voltage electrical lines in the cities of Spain and selected parameters of the environment in which the lines operated. The data were applied to test several methods of solving the considered problem. However, many papers published since 1998 (e.g. [4 - 12]) prove that the problem is still open. It seems that the computational intelligence techniques based on various combinations of genetic algorithms, fuzzy sets and neural networks (see e.g. [13 - 20]) are particularly suited to effectively address this problem.

Our paper demonstrates an original technique based on a genetic algorithm and the Pittsburgh approach [21 - 23] to design a linguistic, interpretability-oriented fuzzy rule-based system and its application to estimate the maintenance costs of medium voltage electrical line. The main goal of the proposed technique is to design the system with not only a relatively high accuracy for estimating the costs, but also with a clear and transparent structure which is easy to interpret by humans. The structure is presented in the form of easily readable and understandable fuzzy logic rules enclosing the essence of the knowledge about the considered problem.

First, some aspects of the system design from data are discussed. Then, a test of the proposed technique with the use of above mentioned benchmark data set, available at the server of the KEEL project (Knowledge Extraction based on Evolutionary Learning) at <http://sci2s.ugr.es/keel>, is presented. Finally, a comparative analysis with several

alternative methods of designing fuzzy rule-based systems is carried out.

Fuzzy rule-based system - on outline

In the paper, a system with n inputs (attributes) x_i ($x_i \in X_i$, $i = 1, 2, \dots, n$) and an output y ($y \in Y$) is considered (X_i and Y are the data domains of i -th input and output of the system, respectively). A collection of fuzzy sets $A_i = \{A_{i1}, A_{i2}, \dots, A_{iq_i}\} = \{A_{ik}\}_{k=1}^{q_i}$ is assigned to i -th input (q_i is the number of fuzzy sets in the collection). The fuzzy set A_{ik} is defined as follows:

$$(1) \quad A_{ik} = \{(x_i, \mu_{A_{ik}}(x_i)) \mid x_i \in X_i\}; \quad A_{ik} \in F(X_i),$$

where $F(X_i)$ denotes a family of all fuzzy sets defined on the universe X_i , and $\mu_{A_{ik}} : X_i \rightarrow [0, 1]$ is the membership function of A_{ik} . In turn, an analogous collection of fuzzy sets $B = \{B_1, B_2, \dots, B_q\} = \{B_r\}_{r=1}^q$ is assigned to the output, where

$$(2) \quad B_r = \{(y, \mu_{B_r}(y)) \mid y \in Y\}; \quad B_r \in F(Y).$$

In general case, the numbers q_i , $i = 1, 2, \dots, n$ and q of fuzzy sets for particular inputs and the output are adjusted independently - usually, they do not exceed 7 (± 2) sets [24]. In our experiments, each input and the output are described by three fuzzy sets corresponding to verbal terms: *Small* (S -type fuzzy set), *Medium* (M -type fuzzy set) and *Large* (L -type fuzzy set), respectively. Their membership functions are expressed by the following formulas:

$$\mu_{S_i}(x_i) = \exp[-0.5(x_i - c_{S_i})^2 / \sigma_{S_i}^2] \quad \text{for } x_i \geq c_{S_i} \text{ and } 1 \\ \text{elsewhere,} \quad \mu_{M_i}(x_i) = \exp[-0.5(x_i - c_{M_i})^2 / \sigma_{M_i}^2] \quad \text{and} \\ \mu_{L_i}(x_i) = \exp[-0.5(x_i - c_{L_i})^2 / \sigma_{L_i}^2] \quad \text{for } x_i \leq c_{L_i} \text{ and } 1 \\ \text{elsewhere; } \sigma_{S_i} > 0, \quad \sigma_{M_i} > 0, \quad \sigma_{L_i} > 0, \quad i = 1, 2, \dots, n.$$

$\mu_S(y)$, $\mu_M(y)$, $\mu_L(y)$ are defined in an analogous way. The parameters of all the membership functions constitute a *data base* of the system.

The system has a *rule base* containing q fuzzy rules in the conjunctive normal form (q is also the number of fuzzy sets in the collection \mathbf{B}):

$$(3) \quad \text{IF } [(x_1 \text{ is } A_{11})_{act} \text{ AND } \dots \text{ AND } (x_n \text{ is } A_{n1})_{act}] \text{ OR } \dots \text{ OR} \\ [(x_1 \text{ is } A_{1q_1})_{act} \text{ AND } \dots \text{ AND } (x_n \text{ is } A_{nq_n})_{act}] \\ \text{THEN } (y \text{ is } B_r), \quad r = 1, 2, \dots, q,$$

where index $act=0$ represents a non-active (that is, removed from the rule base) part of the rule antecedent whereas $act=1$ denotes an active (that is, occurring in the rule) part. At least one part $(x_i \text{ is } A_{iq_i})$ is active in a given rule. In different rules, different parts of the rule antecedents (3) are active. It is worth emphasizing that during the learning process the rule-antecedent parts corresponding to particular inputs may be disabled and re-enabled in order to select the essential attributes in a given rule. Therefore, the final rules contain only the selected set of input attributes upon which the consequents B_r of the rules depend. Moreover, in (3) A_{ik} denotes one of the linguistic terms *Small*, *Medium*, or *Large*, that is represented by k -th fuzzy set in the collection A_i assigned to i -th input (analogously for B_r for the system output).

For the purpose of a more clear presentation of the experimental results shown later in the paper, a single rule in the conjunctive normal form (3) will be represented by an equivalent set of rules in the pure conjunctive form (without connectors "OR" in the rule antecedent):

$$\text{IF } [(x_1 \text{ is } A_{11})_{act} \text{ AND } \dots \text{ AND } (x_n \text{ is } A_{n1})_{act}] \\ \text{THEN } (y \text{ is } B_r), \\ (4) \quad \text{IF } [(x_1 \text{ is } A_{12})_{act} \text{ AND } \dots \text{ AND } (x_n \text{ is } A_{n2})_{act}] \\ \text{THEN } (y \text{ is } B_r), \dots, \\ \text{IF } [(x_1 \text{ is } A_{1q_1})_{act} \text{ AND } \dots \text{ AND } (x_n \text{ is } A_{nq_n})_{act}] \\ \text{THEN } (y \text{ is } B_r).$$

An alternative and more compact form of the rule (3) - for computational purposes - can be presented as follows:

$$(5) \quad \text{IF } \mathbf{x} \text{ is } A'_r \text{ THEN } y \text{ is } B_r,$$

where $\mathbf{x} = [x_1, x_2, \dots, x_n]$ and $A'_r \in F(X_1 \times X_2 \times \dots \times X_n)$ is a fuzzy relation with the membership function (\times stands for Cartesian product of ordinary sets):

$$(6) \quad \mu_{A'_r}(\mathbf{x}) = [\mu_{A_{11}}(x_1) \star_T \dots \star_T \mu_{A_{n1}}(x_n)] \star_S \\ [\mu_{A_{12}}(x_1) \star_T \dots \star_T \mu_{A_{n2}}(x_n)] \star_S \dots \\ [\mu_{A_{1q_1}}(x_1) \star_T \dots \star_T \mu_{A_{nq_n}}(x_n)].$$

The symbols \star_T and \star_S - corresponding to AND- and OR-connectives in the rule (3) - denote t -norm and s -norm operators, respectively (e.g. [16]). Obviously, for non-active parts of the rule antecedents in (3) (with $act=0$), the corresponding membership functions disappear from (6). In this paper, the product t -norm: $a \star_T b = ab$ and the probabilistic sum s -norm: $a \star_S b = a + b - ab$ are used (e.g. [16]).

The system's non-fuzzy response y^0 when the vector $\mathbf{x}^0 = [x_1^0, x_2^0, \dots, x_n^0]$ is presented to its inputs is calculated on the basis of a modified *Center of Area* (COA) method:

$$(7) \quad y^0 \text{ such that } \int_{y=y_{\min}-0.5\sigma_S}^{y^0} \mu_{B^0}(y) dy = \int_{y=y^0}^{y_{\max}+0.5\sigma_L} \mu_{B^0}(y) dy,$$

where

$$(8) \quad \mu_{B^0}(y) = \bigoplus_{r=1}^q I[\mu_{A'_r}(\mathbf{x}^0), \mu_{B_r}(y)],$$

is the membership function of the fuzzy set $B^0 \in F(Y)$ being a fuzzy response of the system for \mathbf{x}^0 (see e.g. [25] for similarity based fuzzy reasoning scheme), $I[a, b]$ is the fuzzy implication operator, \oplus is the aggregation operator, y_{\min} and y_{\max} denote the minimum and the maximum values of the output, and σ_S and σ_L are the parameters of the membership functions representing S -type and L -type fuzzy sets, assigned to the output of the system. In this paper, the product operation $I[a, b] = ab$ plays the role of the fuzzy implication and the arithmetic mean $\bigoplus_{r=1}^q b_r = \frac{1}{q} \sum_{r=1}^q b_r$ is used as the aggregation operator (e.g. [16]).

Some theoretical aspects of the learning process

The considered system is designed from the learning data in the form of L input-output data samples:

$$(9) \quad D = \{\mathbf{x}_l, d_l\}_{l=1}^L,$$

where $\mathbf{x}_l = [x_{1l}, x_{2l}, \dots, x_{nl}] \in \mathbf{X} = X_1 \times X_2 \times \dots \times X_n$ is the set of input attributes, and $d_l \in Y$ is the desired response at the system's output, when \mathbf{x}_l is presented to the inputs.

During the learning process, fuzzy rules and fuzzy sets evolve simultaneously in the framework of a genetic algorithm and the modified Pittsburgh approach (see also [22, 23]). The fitness function evaluates the systems encapsulated by the chromosomes in the population, in terms of both the accuracy and the transparency of their structures. The fitness function is defined as follows:

$$(10) \quad ff = (1 - \alpha) \cdot ff_{ACU} + \alpha \cdot ff_{INT},$$

where $\alpha \in [0, 1]$ is the accuracy-interpretability coefficient (it regulates the level of the compromise between both aspects of the system design).

The accuracy component $ff_{ACU} \in [0, 1]$ has the form:

$$(11) \quad ff_{ACU} = 1 - \frac{Q_{RMSE}}{y_{\max} - y_{\min}},$$

$$(12) \quad Q_{RMSE} = \sqrt{\frac{1}{L} \sum_{l=1}^L (y_l - d_l)^2},$$

where y_l and d_l denote the actual and desired responses of the system, respectively, when input vector \mathbf{x}_l is presented to the inputs, whilst Q_{RMSE} is the root-mean-squared-error cost function.

The interpretability component $ff_{INT} \in [0, 1]$ is the following:

$$(13) \quad ff_{INT} = 1 - Q_{CPLX} = 1 - \frac{ACAR + NNR}{2},$$

where $ACAR$ and NNR refer to the mean complexity of fuzzy rules in the rule base and the normalized number of rules in the rule base, respectively. They are defined as follows:

$$(14) \quad ACAR = \frac{1}{R} \sum_{r=1}^R \frac{NA_r - 1}{n - 1}, \quad n > 1; \quad NNR = \frac{R - R_{\min}}{R_{\max} - R_{\min}},$$

where R is the number of rules in the form (4), NA_r is the actual number of antecedents in r -th rule ($r = 1, 2, \dots, R$), n is the number of inputs in the system (it is also the number of all possible antecedents allocated in the rule), whilst R_{\min} and R_{\max} are the minimum and maximum number of rules, respectively. For special case $n=1$, the coefficient $ACAR$ is not calculated ($Q_{CPLX} = NNR$).

A single chromosome consists of two parts, representing the rule base and the data base of the system, respectively. During the evolution process, randomly selected pairs of chromosomes recombine with each other in order to form their offspring. The operation is executed separately both for the pair of data bases and for the pair of rule bases belonging to the corresponding chromosomes. In the first case, a recombination of the rule bases is performed using specialized crossover operators, which operate directly on the rules (there is no special coding of the rule base into a chromosome). In the second case, a recombination of the membership function parameters is achieved by means of traditional binary crossover operators (parameters are encoded in the form of a bit string). Similarly, the specialized mutation operators for fuzzy rules and the binary mutation operators for the parameters are used to randomly change the chromosomes during the evolution. The operation of these genetic operators are described in detail in [26, 27].

The experimental results

The application of the proposed technique to design fuzzy rule-based system for estimating the maintenance cost of the medium voltage electrical line will now be presented. For this purpose, a well-known Maintenance Electrical benchmark data set (ELE-2), that is available at the server of the *Knowledge Extraction based on Evolutionary Learning* project (<http://sci2s.ugr.es/keel>) will be applied. The data set contains a representative sample of 1059 records regarding the maintenance costs of the medium voltage electrical line in the cities in Spain. The paper [3] indicates that the costs estimation on the basis of four, easily measurable attributes (see Table 1) of the considered data set is sufficiently effective. For the calculations performed in this paper, the original data set has been divided into two parts: 66.6% of randomly selected records and 33.4% of remaining records have been allocated to the learning and test data sets, respectively.

Table 1. The attributes of the maintenance cost estimation system

Name	Range	Meaning
x_1	[0.5, 11.0]	Total length of all the streets in the town [km]
x_2	[0.15, 8.55]	Total area of the town [km ²]
x_3	[1.64, 142.5]	Total area occupied by buildings [km ²]
x_4	[1.0, 165.0]	The energy supply to the town [MWh]
y	[64.47, 8546.03]	The maintenance costs of the medium voltage line [millions of pesetas]

In our experiment, at the beginning of the learning process, a population of 100 chromosomes representing 100 competing with each other fuzzy rule-based systems

has been prepared. Each input and the output of a given system are characterized by three fuzzy sets (one S -type, one M -type and one L -type). The parameters of their membership functions have been randomly initialized at the start of the genetic algorithm. The chromosomes evolve simultaneously with the crossover and mutation probabilities equal to 0.8 and 0.5, respectively. A tournament method (with the tournament size equal to 2) supported by the elitist strategy has been applied to the selection of parent chromosomes in every single generation of the evolution.

A special strategy to control the learning process has been used in order to avoid local maxima of the fitness function (10). The learning consists of sixteen stages; each of them lasts 1000 generations (see Fig. 1). At the beginning of each stage, α coefficient occurring in (10) (which is responsible for the level of the compromise between the interpretability and the accuracy of the system) is adjusted in advance and then remains constant during the whole stage. In the first stage (for $\alpha = 0$), the learning process is focused on the design of the system with the highest accuracy. In the next stages, α is gradually increased (see Fig. 1 for details) in order to progressively direct the learning process to eliminate unnecessary rules and also to generate the systems with higher transparency (at the expense of a small degradation in the accuracy). Fig. 1 demonstrates the changes of accuracy ff_{ACU} (11) and interpretability ff_{INT} (13) components of the fitness function (10) during the learning process.

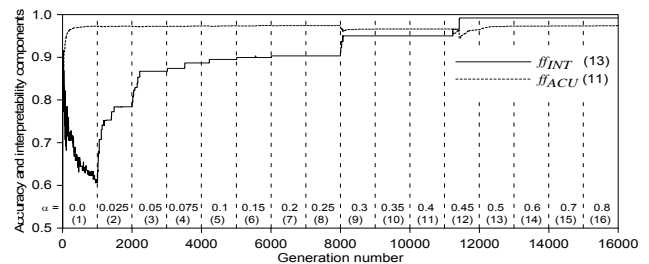


Fig. 1. Accuracy ff_{ACU} (11) and interpretability ff_{INT} (13) components of the fitness function (10) vs. generation number, as well as the numbers of stages (in the brackets) and the coefficient α in a given stage.

Figs. 2, 3 and 4 show the changes of the system's accuracy (represented by the cost function Q_{RMSE} (12)), the number of fuzzy rules in the system and the number of fuzzy sets used in the fuzzy rules during the learning process, respectively. Finally, in the sixteen stage (for $\alpha = 0.8$), the learning process ends when the system achieves a sufficient transparency (the rule base contains 5 rules operating on 2 input attributes), while maintaining a relatively high accuracy (Q_{RMSE} (12) equal to 221.46 for the learning data set and 217.37 for the test data set).

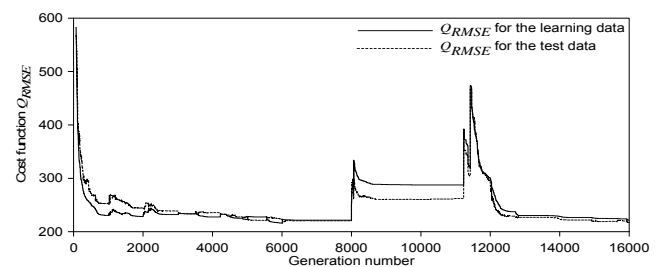


Fig. 2. Cost function Q_{RMSE} (12) for the learning and test data sets vs. generation number.

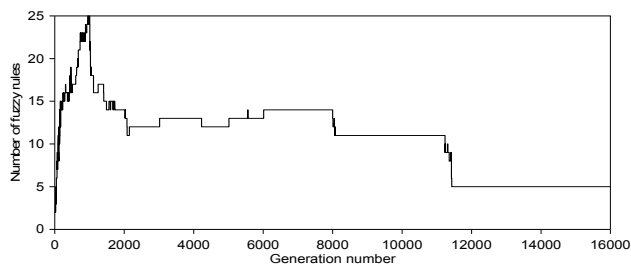


Fig. 3. The number of fuzzy rules R in the rule base vs. generation number.

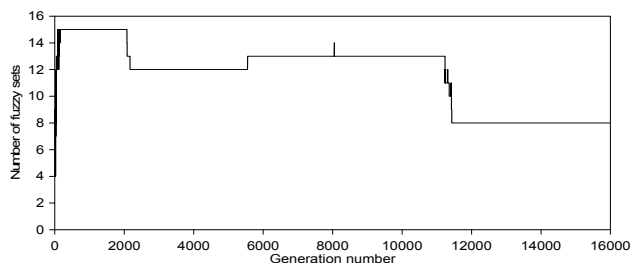


Fig. 4. The number of fuzzy sets used in all fuzzy rules occurring in the system vs. generation number.

The obtained system represents – in the form of the transparent and easy-to-understand rules – a synthesis of the most important aspects regarding the considered cost estimation problem. Fig. 5 shows the fuzzy sets representing the antecedents and the consequents of the system, while its rule base, in the form (4), is as follows:

$$\begin{aligned}
 &\text{IF } (x_3 \text{ is } \textit{Small}) \quad \text{THEN } (y \text{ is } \textit{Small}), \\
 &\text{IF } (x_4 \text{ is } \textit{Small}) \quad \text{THEN } (y \text{ is } \textit{Small}), \\
 (15) \quad &\text{IF } (x_3 \text{ is } \textit{Medium}) \quad \text{THEN } (y \text{ is } \textit{Medium}), \\
 &\text{IF } (x_3 \text{ is } \textit{Large}) \quad \text{THEN } (y \text{ is } \textit{Large}), \\
 &\text{IF } (x_4 \text{ is } \textit{Large}) \quad \text{THEN } (y \text{ is } \textit{Large}).
 \end{aligned}$$

As far as the expressions (15) and (4) are concerned, in (15), obviously, only the active parts of the rule antecedents are exposed, that is, these for which, in the general expression (4) the act -parameters are equal to 1. All the remaining non-active parts (with $act=0$) of the rule antecedents in the general description (4), in the expression (15) simply do not occur in the rule base.

For the purpose of comparative analysis, several alternative methods presented in [2, 3, 10] have been chosen (see Table 2). Additionally, due to some aspects of the comparison, a multilayer perceptron (MLP) with 50 neurons in the hidden layer has been applied to solve the considered problem. It achieved the highest accuracy in comparison with all the remaining approaches (Q_{RMSE} (12) equal to 87.86 for the learning data and 88.38 for the test data). A similar accuracy has the system based on Takagi-Sugeno-Kang (TSK) model [15] (Q_{RMSE} (12) equal to 148.82 for the learning data and 153.85 for the test data). However, both approaches are useless as tools supporting the decision making for estimating the maintenance costs of the electrical lines. Their structures are too complex and incomprehensible to humans (301 weights in neural network or 268 rules in TSK system). A similar conclusion can be drawn for other alternative methods (see Table 2). They have the worse accuracies than the above ones and also relatively complex structures. For example, the Mamdani fuzzy model [3] requires as many as 63 rules providing the accuracy measures Q_{RMSE} (12) equal to 201.58 for the learning data and 235.01 for the test data.

Traditional techniques of linear and polynomial regression and MLP with 5 neurons in the hidden layer are not accurate enough.

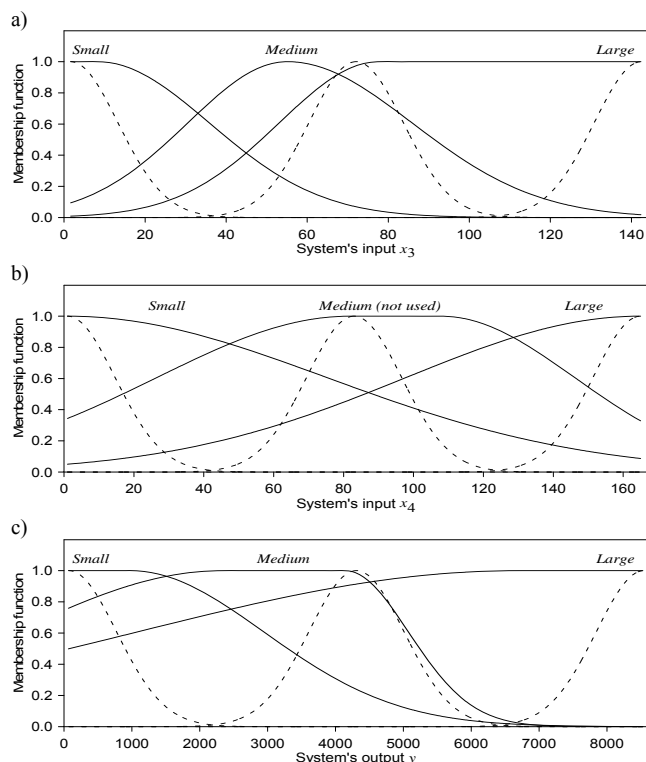


Fig. 5. The initial (the broken curve plots) and final (the continuous curve plots) membership functions of fuzzy sets occurring in fuzzy rules of the system (with $\alpha = 0.8$) for input attributes x_3 (a) and x_4 (b) as well as for output attribute y (c).

Table 2. The results of comparative analysis

Method	Complexity of the system	Q_{RMSE} for the learning data set	Q_{RMSE} for the test data set
Linear regression [2]	17 nodes, 5 parameters	573.86	271.36
2th order polynomial regression [2]	77 nodes, 15 parameters	453.94	301.10
MLP with 5 neurons in the hidden layer [2] ¹⁾	35 parameters	415.85	257.31
MLP with 50 neurons in the hidden layer ¹⁾	301 parameters	87.86	88.38
GA-P [2] ²⁾	50 nodes, 5 parameters	190.62	209.21
Interval GA-P [2] ²⁾	15 nodes, 4 parameters	180.34	191.44
Mamdani fuzzy model [2]	63 rules, 4 inputs	201.58	235.01
Wang and Mendel fuzzy model [3]	66 rules, 4 inputs	198.38	212.56
Takagi-Sugeno-Kang fuzzy model [3]	268 rules, 4 inputs	148.82	153.85
MOEA [10] ³⁾	29 rules, 4 inputs	185.01	199.17
Our technique	5 rules, 2 inputs	221.46	217.37

¹⁾ MLP = Multilayer Perceptron, ²⁾ GA-P = Genetic Algorithm-Programming, ³⁾ MOEA=Multi-Objective Evolutionary Algorithm

It should be clearly pointed out, that the proposed method generates not only the system with a relatively high accuracy for estimating the costs, but also with a clear and transparent structure which is easy to interpret by humans. The structure is presented in the form of a few easily readable and understandable fuzzy rules. The rules encapsulate the essence of the knowledge about the

relationship between the maintenance cost of electrical lines and the properties of the environment in which the lines operate. As already mentioned above, this knowledge has a significant impact on the decision making process regarding the profitability of the investment.

Conclusions

In this paper, the evolutionary method of designing fuzzy rule-based systems for the estimation of the maintenance costs of electrical lines has been proposed. The experiment shows a high usefulness of the method to design interpretability-oriented systems, that is, with a relatively high accuracy as well as a clear and transparent structure in the form of easily interpretable and understandable linguistic rules. In the future, the proposed technique will be applied to construct the systems from more complex data (containing more input attributes than the data used in this paper) representing similar problems from the electrical engineering area. These systems – synthesizing the knowledge about the nature of the problem from data – can be very useful in developing important components of the decision support platforms for managers, engineers and other employees of the energy sector.

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