Dariusz Janiszewski

Poznan University of Technology

Particle Filter Approach for Permanent Magnet Synchronous Motor State Estimation

Abstract. *The paper describes an observer for a permanent magnet synchronous motor based on Particle Filter techniques. Based on introduced theory an observer of shaft position, speed and load torque is proposed. Preliminary research show good properties of proposed estimation structure.*

Streszczenie. *W artykule przedstawiono obserwację zmiennych stanu silnika synchronicznego o magnesach trwałych przy wykorzystaniu techniki filtracji cząsteczek. W oparciu o przedstawioną teorię zaproponowano obserwator położenia wału, prędkości i momentu obciążenia części mechanicznej napędu. Początkowe badania symulacyjne pokazują dobre właściwości zaproponowanej struktury estymacji.*
(Wykorzystanie techniki filtru cząsteczkowego do estymacji stanu silnika synchronicznego o magnesach trwałych)

Keywords: permanent magnet synchronous motor, sensorless control, observers, Particle Filter
Słowa kluczowe: silnik synchroniczny o magnesach trwałych, sterowanie bezczujnikowe, obserwatory , filtr cząsteczkowy

doi:10.12915/pe.2014.06.11

Introduction

Filtering is the problem of estimating the states (also parameters or hidden variables) of a system as a set of observations becomes online. To solve this problem necessary is modeling the system and the noises in system and measurements. The resulting models show complex nonlinearities and noise distribution, often real non-Gaussian, sometimes excluding analytical approaches. The right algorithm to solve the problem of estimaton state with nonlinearities is nonlinear family of Kalman Filters, include Extended and Unscented [1, 2, 3, 4]. These solutions bases on knowing model of system, with linearizion – EKF or with aproximation of Gaussian noise distribution – UKF. Theres is some extension to use EKF and UKF for parameter and disturbances estimation[5, 6, 7]. Another popular solutions strategy for solve filtering problem is to use Sequential Monte Carlo methods, known as Particle Filter[8, 9]. These methods allow for complete representation of the posteriori distribution of state variables, so that any statistical tools are easy computed. They can therefore handle with any nonlinearities and complex distributions.

Permanent Magnet Synchronous Motor are widely used in industrial high precision and high dynamic mechatronics drives. Their inherent high torque to inertia ratio, small sizes and enhanced dynamic performances are predominant features[10, 11, 12]. Systems called *sensorless* have the potential to get rid of any mechanical sensors placed on the machine shaft, usually of position and velocity measurements[11, 13]. Some method have been developed in order to obtain mechanical quantities of PMSM and induction motors[11, 14, 15, 16, 13, 17, 18, 19]. Interesting issue is drive parameter estimation estimation[20]. In simple model of electromechanicals one axis drive system, the natural state variables are shaft position and speed with disturbance which is load torque[6, 7]. So it is possible to obtain information about position, speed and torque.

The paper presents widely approach of Particle Filter apply for Permanent Magnet Synchronous Motor state estimation.

Mathematical model of PMSM

The general mathematical model of system in statistical approach can be broken into state transition and state measurements models:

$$p\left(\underline{x}_t|\underline{x}_{t-1},\underline{u}_t\right) \tag{1}$$

$$p\left(\underline{y}_t|\underline{x}_t\right) \tag{2}$$

where $\underline{x}_t \in \mathbb{R}^{n_x}$ denotes the states (hidden variables or/and parameters) of the system in time t, and $\underline{y}_t \in \mathbb{R}^{n_y}$ observations. The states follow a first order Markov chain process and the observations are assumed to be independent given the states. Based on this approach the model can be expressed as follows:

$$\underline{x}_t = f\left(\underline{x}_{t-1},\underline{u}_t,\underline{v}_{t-1}\right)\ , \tag{3}$$

$$\underline{z}_t = h\left(\underline{x}_t,\underline{n}_t\right)\ , \tag{4}$$

where $\underline{u}_t \in \mathbb{R}^{n_u}$ denotes the input observations, $\underline{v}_t \in \mathbb{R}^{n_x}$ and $\underline{n}_t \in \mathbb{R}^{n_y}$ are noises for process and measurements. In state space approach above equations can be presented as:

$$\underline{x}_t = \mathbf{F}_t(\underline{x}_{t-1})\underline{x}_{t-1} + \mathbf{B}_t(\underline{x}_{t-1})\underline{u}_t + \underline{v}_{t-1}\ , \tag{5}$$

$$\underline{z}_t = \mathbf{H}_t(\underline{x}_t)\underline{x}_t + \underline{n}_t\ , \tag{6}$$

Mathematical model of PMSM concerns in three main parts: electrical network, electromechanical torque production and third is mechanical subsystem[12]. The stator of PMSM and Induction Motor are similar. The rotor consists permanent magnets, there are a modern rare-earth magnets with high strength.

During investigations some simplified assumptions are made: saturation is neglected, inducted electromagnetic force is sinusoidally, eddy currents and hysteresis losses are neglected, no dynamical dependencies in air-gap, no rotor cage. With these assumptions, the rotor oriented dq electrical network equations of PMSM can be described as:

$$u_d = R_s i_d + L_d \frac{di_d}{dt} - p\omega_r L_q i_q\ , \tag{7}$$

$$u_q = R_s i_q + L_q \frac{di_q}{dt} + p\omega_r L_d i_d + p\omega_r \Psi_m\ . \tag{8}$$

where: u_d, u_q are dq axis voltages, i_d, i_q are dq axis currents, L_d, L_q are dq axis inductances, R_s is stator resistance, and

Ψ_m is magnetic flux produced by permanent magnets placed on rotor.

The value of produced electromagnetic torque is given by equation:

$$T_e = \frac{3}{2} \cdot p \left[\Psi_m - (L_q - L_d)\, i_d \right] \cdot i_q \,, \tag{9}$$

where p is number pole pairs, and fraction $\frac{3}{2}$ stems from frame conversion: perpendicular stator $\alpha\beta$ into rotor dq.
Drive dynamics can be describes as:

$$T_e - T_l = J \frac{d\omega_r}{dt}, \tag{10}$$

where T_l is load torque and J is summary moment of inertia of kinematic chain.

Based on (9) and (10) movement equation is:

$$\frac{d\omega_r}{dt} = \frac{p}{J} \left[\frac{3}{2} \left(\Psi_m - (L_q - L_d)\, i_d \right) \cdot i_q \right] - \frac{T_e}{J} \,. \tag{11}$$

Rotor position – γ can be described by derivative equation of rotational speed:

$$\frac{d\gamma}{dt} = p \cdot \omega_r \,. \tag{12}$$

For this work there are true assumption that load torque T_l is invariable in a narrow interval:

$$\frac{d}{dt} T_l \approx 0 \,. \tag{13}$$

Model can be described as classical discrete function model, with sample time T_s:

or as state space model:

System matrix $\mathbf{F}_t$ is:

$$\mathbf{F}_t(\hat{\underline{x}}_t) = \begin{bmatrix} 1 - T_s \cdot \frac{R_s}{L_d} & T_s \cdot \omega_r \frac{L_q}{L_d} & 0 & 0 & 0 \\ -T_s \cdot \omega_r \frac{L_d}{L_q} & 1 - T_s \cdot \frac{R_s}{L_q} & -T_s \cdot \frac{\Psi_m}{L_q} & 0 & 0 \\ 0 & T_1 & 0 & 0 & -T_s \cdot \frac{1}{J} \\ 0 & 0 & T_s & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{14}$$

where:

$$T_1 = T_s \cdot \frac{3}{2} \frac{p}{J} \left[\Psi_f - (L_q - L_d)\, i_d \right]$$

Output matrix $\mathbf{H}_t$ is a rotating matrix – Clark/Parck transformation:

$$\mathbf{H}_t(\hat{\underline{x}}_t) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \end{bmatrix}, \tag{15}$$

and matrix $\mathbf{B}_t$:

$$\mathbf{B}_t(\hat{\underline{x}}_t) = \begin{bmatrix} T_s \cdot \frac{1}{L_d} \cos\gamma & T_s \cdot \frac{1}{L_d} \sin\gamma \\ -T_s \cdot \frac{1}{L_q} \sin\gamma & T_s \cdot \frac{1}{L_q} \cos\gamma \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{16}$$

Particle filtering

The main assumption of Particle Filtering find sources in Monte Carlo simulation and importance sampling[22, 23, 24, 8, 9]. *In Monte Carlo simulation*, set of weighted particles, obtain from posteriori distribution, can be used to map integrals to discrete sum, the posteriori can be approximated as:

$$\hat{p}\left(\underline{x}_{0:t} | \underline{y}_{1:t}\right) = \frac{1}{N} \sum_{i=1}^{N} \delta\left(\underline{x}_{0:t}^{(i)} - \underline{x}_{0:t}\right) \tag{17}$$

where the random samples $\{\underline{x}_{0:t}; i = 1 ... N\}$, are computed from posteriori distribution and $\delta(d\cdot)$ denotes the Dirac delta function. So expectations of the form:

$$\mathbb{E}\left(g_t(\underline{x}_{0:t})\right) = \int g_t\left(\underline{x}_{0:t}\right) p\left(\underline{x}_{0:t} | \underline{y}_{1:t}\right) d\underline{x}_{0:t} \tag{18}$$

may be approximated by mean:

$$\overline{\mathbb{E}\left(g_t(\underline{x}_{0:t})\right)} = \frac{1}{N} \sum_{i=1}^{N} g_t\left(\underline{x}_{0:t}\right), \tag{19}$$

In order to compute a sequential estimate of posteriori distribution at time t without modifying the previously simulated states $\underline{x}_{0:t-1}$, proposal distributions of the following form can be used:

$$q\left(\underline{x}_{0:t} | \underline{y}_{1:t}\right) = q\left(\underline{x}_{0:t-1} | \underline{y}_{1:t-1}\right) q\left(\underline{x}_t | \underline{x}_{0:t-1}, \underline{y}_{1:t}\right), \tag{20}$$

the ate the making the assumption that the current state is not depend on the future observations (filtering only without smoothing). Under the assumptons that the states corespond to Markov Chain and that the observations are condittionally independent given the states:

$$p\left(\underline{x}_{0:t}\right) = \underline{x}_0 \prod_{j=1}^{t} \left(\underline{x}_j | \underline{x}_{j-1}\right), \tag{21}$$

and outputs:

$$p\left(\underline{y}_{1:t} | \underline{x}_{0:t}\right) = \prod_{j=1}^{t} p\left(\underline{y}_j | \underline{x}_j\right). \tag{22}$$

Substituting equations 21 and 22, and Bayesian importance weights[9, 24]:

$$\underline{w}_t = \frac{p\left(\underline{y}_{1:t}\right) p\left(\underline{x}_{0:t}\right)}{q\left(\underline{x}_{0:t} | \underline{y}_{1:t}\right)} \tag{23}$$

can obtain:

$$\underline{w}_t = \underline{w}_{t-1} \frac{p\left(\underline{y}_t | \underline{x}_t\right) p\left(\underline{x}_t | \underline{x}_{t-1}\right)}{q\left(\underline{x}_t | \underline{x}_{0:t-1}, \underline{y}_{1:t}\right)}. \tag{24}$$

Proposed in equation 24 mechanism to sequentially update the importance weights give an appropriate choice of proposal distributions $q\left(\underline{x}_t | \underline{x}_{0:t-1}, \underline{y}_{1:t}\right)$. This form of distribution is critical design issue. This procedure, known as *sequential importance sampling* (SIS), allows to obtain estimates importance weights $w_{t,i}$ simpler for every particle as:

$$w_{t,i} = p\left(\underline{y}_t | \underline{x}_{t-1,i}\right) w_{t-1,i} \tag{25}$$

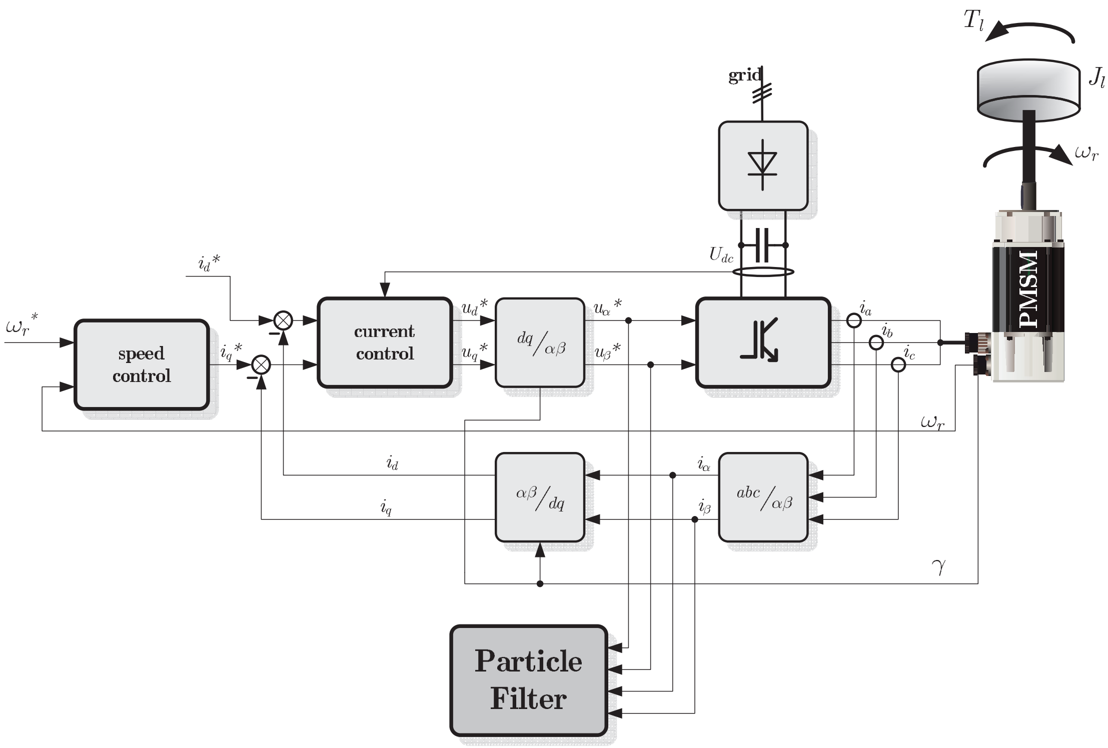

Fig. 1. Drive control scheme with Particle Filter

with necessity of normalize to $\sum_i w_{t,i} = 1$ by:

$$w_{t,i} = \frac{w_{t,i}}{\sum_{i=1}^{N} w_{t,i}} . \tag{26}$$

In case that weight $w_{t,i}$ is bigger then assumed there is replaced by neighboring $w_{t,i-1}$ – resampled.

Other important way to obtain importance weights is *Residual Resampling* [8, 9, 24] as an efficient means to decrease the variance due to resampling. In this approach for every particle i in every time t is decomposed:

$$N_{t,i} = \lfloor n w_{t,i} \rfloor + \overline{N}_{t,i} , \tag{27}$$

where $\lfloor\ \rfloor$ denotes integer part and set of $\overline{N}_{t,1:N}$ according to cumulative distribution function (CDF) up into N components. Important weights can be determined:

$$w_{t,i} = \frac{n w_{t,i} - \lfloor n w_{t,i} \rfloor}{n - \sum_{i=1}^{N} \lfloor n w_{t,i} \rfloor} \tag{28}$$

During resampling every weights $w_{t,i}$ are maintained.

The purpose of the Metropolis–Hastings algorithm[22, 21], also used in proposed solution, is to generate a collection of states according to a desired distribution $p(\underline{x}_t)$. To accomplish this, the algorithm uses a Markov process which, under certain conditions, asymptotically reaches a unique stationary distribution $\pi(\underline{x}_t)$.

A Markov process is uniquely defined by its transition probabilities, the probability $P(x \to x')$ of transitioning between any two of its states $\underline{x}_t$ to new $\underline{x}'_t$. It is possible to say that:

$$\pi(\underline{x}_t) p(\underline{x}_t \to x') = \pi(x') p(\underline{x}'_t \to \underline{x}_t) . \tag{29}$$

Introducing the concept of acceptance distribution $A(\underline{x}_t \to \underline{x}'_t)$, and proposal distributions $g(\underline{x}_t \to \underline{x}'_t)$ the conditional probability to accept the proposed state $\underline{x}'_t$ given $\underline{x}_t$:

$$P(\underline{x}_t \to \underline{x}'_t) = g(\underline{x}_t \to \underline{x}'_t) A(\underline{x}_t \to \underline{x}'_t) . \tag{30}$$

Recombining complementary pair 29 into:

$$\frac{A(\underline{x}_t \to \underline{x}'_t)}{A(\underline{x}'_t \to \underline{x}_t)} = \frac{p(\underline{x}'_t)}{p(\underline{x}_t)} \frac{g(\underline{x}'_t \to \underline{x}_t)}{g(\underline{x}_t \to \underline{x}'_t)} , \tag{31}$$

it is possible to define *Metropolis choice* by:

$$A(\underline{x}_t \to \underline{x}'_t) = \min\left(1, \frac{p(\underline{x}'_t)}{p(\underline{x}_t)} \frac{g(\underline{x}'_t \to x)}{g(\underline{x}_t \to \underline{x}'_t)}\right) . \tag{32}$$

If above *mininal* is fulfilled new acceptance probability is valid, otherwise remains.

Validation of proposed estimation scheme

Open-loop analysis of Particle Filter for PMSM drive was performed on simulated drive system of rated power equal 1.35 kW working with vector control sensor strategy. An overview scheme of the proposed PMSM sensorless speed control is shown in figure 1, with the classic subordinate speed and torque control[11]. The complete drive control system, which is used in this approach, includes: PWM IGBT inverter, current sensors and converters, and elements of processor code. There are PI controllers of current and speed, frame reference transformations modules (dq/ab, ab/dq, abc/ab) and Unscented Kalman Filter are fully implemented in DSP.

The PI speed controller feeds current i_q^* in q axis in order to keep Field Oriented Control. The demanded current is computed by using the difference between requested speed ω_r^* and speed $\hat{\omega}_r$ estimated in Kalman filter. Motor working does not required the field weakening, as assumed. Therefore desired current in d axis is maintained to zero $i_d^* = 0$. These signals are inputs of PI current controller, which provides desired voltages in dq references frame. Basing on the shaft position $\hat{\gamma}$, voltages are converted into the stationary two axis frame $\alpha\beta$, and control PWM inverter. The main component is *Particle Filter* that provides estimated signals of angular position $\hat{\gamma}$, speed $\hat{\omega}_r$ and load torque $\hat{T}_l$.

The research proof was performed on simulated setup with PMSM on parameters:

- nominal speed 3000 rpm,
- nominal still load torque $4,3$ Nm,
- max load torque $11,7$ Nm,
- moment of inertia $2,5$ kg $\cdot$ cm^2
- still current $2,45$ A,
- electromechanical constant $1,6\ \frac{\mathrm{Nm}}{\mathrm{A}}$,
- EMF constant $98\ \frac{\mathrm{V}}{1000\mathrm{rpm}}$.

As a result of the combination of two motor the moment of inertia reduced to the motor shaft has increased the value of $J = 35,36$ [kg $\cdot$ cm^2].

During particles generation assume noise covariances $\mathbf{Q}$ i $\mathbf{R}$ in accordance with the rules for the expected normal Gaussian distribution for $v_{t,i}$ and $n_{t,i}$:

$$\mathbf{Q} = \mathbf{diag}\{\ q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5\ \}, \tag{33}$$

$$\mathbf{R} = \mathbf{diag}\{\ r_1 \quad r_2\ \}, \tag{34}$$

where q_1 and q_2 are covariances of motor currents (d and q axis),and q_3 – speed ω_r, q_4 – angular position γ, and q_5 means in as maximum possible change of of T_l during one computation period T_s. Coefficients r_1 and r_2 are variances of measurements motor currents i_d and i_q.
Presented results bases on 100 particles.

The investigations presented below are in some sense introduction confirming the use of Particle Filter to observe the electrical and mechanical variables of the drive. There can provide the possibility of using these variables to the full sensorless control.

The entire experiment presented on figure 2 during which the observer was tested for 100 particles, sequential importance sampling, in motor vector sensor control strategy. Main stages of experiment consist reference speed ω_r^* changing:

- freely start up to desired speed $\omega_r = 1000\ \frac{\mathrm{obr}}{\mathrm{min}}$, what is $\frac{1}{3}$ nominal speed,
- reverse of established speed $\omega_r = 1000\ \frac{\mathrm{obr}}{\mathrm{min}}$ into $\omega_r = -1000\ \frac{\mathrm{obr}}{\mathrm{min}}$,
- breaking to steady of shaft.

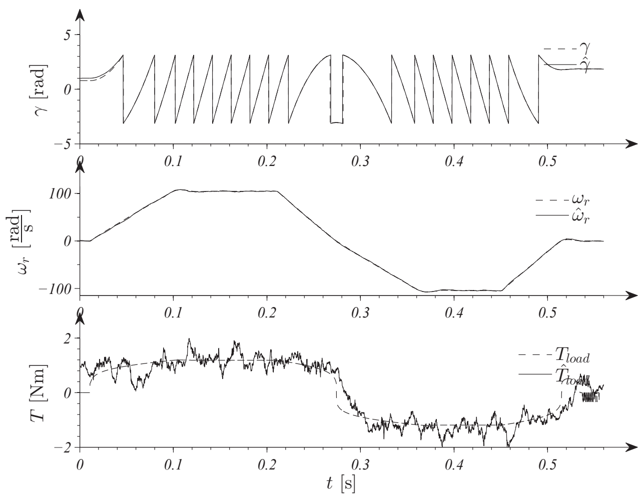

Fig. 2. Speed response $\omega_r = 104.72\ \frac{\mathrm{rad}}{\mathrm{s}}$ during start, reverse and stop

In addition, the figure 3 shows the errors of observation each important variables: $\hat{\omega_r}$, $\hat{\gamma}$, $\hat{T}_{load}$.

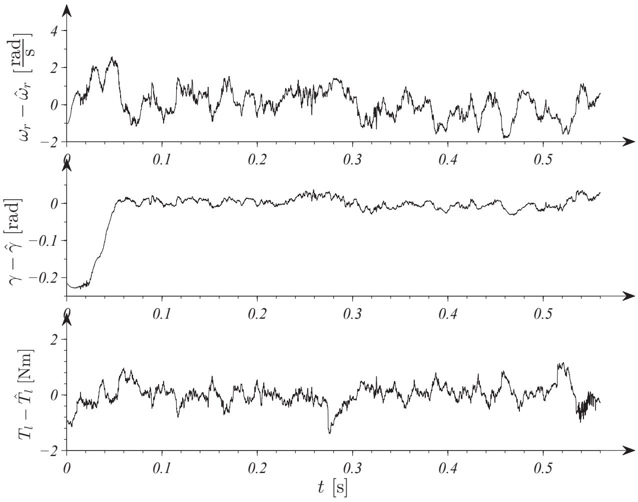

Fig. 3. Errors during speed response test for $\omega_r = 104.72\ \frac{\mathrm{rad}}{\mathrm{s}}$

It can be seen that most of the errors of observation is disclosed in a dynamic cases of the drive. It is worth mentioning that the design and the way the selection of noise particles are determined only preliminary. Practical experience shows that it is possible to better tuning of the filter by changing the value of the matrix $\mathbf{Q}$ and $\mathbf{R}$ which is the subject of the current work of the author.

To further the nature of the work putting shown waveforms on figure 4 speed value for a group of 100 particles. It is easy to be compared particles to the actual and estimated speed.

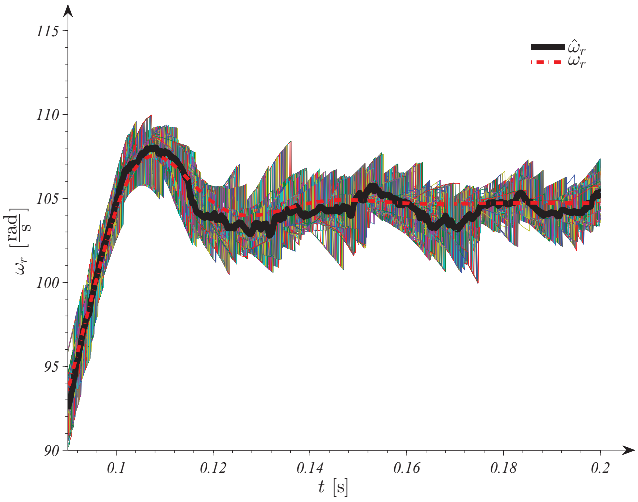

Fig. 4. Speed values from Particle Filter family – part of experiment

During preliminary investigation examined a few types of resamplig methods, there are *sequential importance sampling* and *residual resampling*. Details are presented in the theoretical part above. Other examined algorithm was with additional *Metropolis-Hasting* step.

Table 1 presented RMS errors for state variable estimation with this same conditions for presented simulated system and proposed model of particles. For examined cases the better solution is *Particle Filter* with *sequential importance sampling* and supported by *Metropolis-Hasting* step.

Summary

This paper presents the design and simulation verification observer based on Particle Filter for state estimation of non-linear object, which is a Permanent Magnet Synchronous Motor. The estimated variables can be used in a subordi-

Table 1. Average RMS error for state variable estimation

x	RMS			
	PF		PF-MH	
	res	SIS	res	SIS
i_d	0.376	0.375	0.376	0.341
i_q	0.1097	0.105	0.115	0.085
ω_r	0.847	0.718	0.8616	0.636
γ	0.057	0.057	0.058	0.054
T_l	0.357	0.3263	0.370	0.285

nate cascade control system of currents and speed in vector control strategy. Particle Filter approach has many advantages compared to Extended Kalman Filter and Unscented Kalman, which are trouble free tuning, to find the initial value, as shown during investigation.

There is shown that the resulting filter is capable to estimate the rotor position in the full speed range, including the standstill.

Worth noting on the basis of preliminary simulation tests that errors play strictly depend on the tuning filter. Such a system is an interesting object of research, and also can find its use in industrial applications.

REFERENCES

[1] Kalman R.E. *New Approach to Linear Filtering and Prediction Problems* Transactions of the ASME-Journal of Basic Engineering, 1960, pp. 374-382.

[2] Julier S. J., Uhlmann J. K. *Unscented filtering and nonlinear estimation* Proceedings of the IEEE, 2004, 92, pp. 401-422,

[3] Welch G., Bishop G. *An Introduction to the Kalman Filter* SIGGRAPH 2001; Proceedings on, Computer Graphics, Annual Conference on Computer Graphics and Interactive Techniques, ACM Press, Addison-Wesley, 2001, 8, 3-42,

[4] Zawirski K., Janiszewski D. Muszynski R. *Unscented and extended Kalman filters study for sensorless control of PM synchronous motors with load torque estimation* Bulletin of the Polish Academy of Sciences. Technical Sciences, 2013, 61(4), pp. 793-801.

[5] Van der Merwe R., Wan, E. *The square-root unscented Kalman filter for state and parameter-estimation* Acoustics, Speech, and Signal Processing, 2001. Proceedings. (ICASSP '01). 2001 IEEE International Conference on, 2001, 6, pp.3461-3464,

[6] Janiszewski, D. *Load torque estimation in sensorless PMSM drive using Unscented Kalmana Filter* Proc. Industrial Electronics (ISIE), 2011 IEEE International Symposium on, 2011, pp.643 -648,

[7] Janiszewski D. *Extended Kalman Filter Based Speed Sensorless PMSM Control with Load Reconstruction* Proc. IECON 2006 - 32nd Annual Conf. IEEE Industrial Electronics, 2006, 1465-1468,

[8] Doucet A., De Freitas N., Gordon N. (Eds.) *Sequential Monte Carlo methods in practice* Springer, 2001,

[9] van der Merwe R., de Freitas N., Doucet A., Wan, E. *The Unscented Particle Filter* Advances in Neural Information Processing Systems 13, 2001,

[10] Brock S., Zawirski K. *New approaches to selected problems of precise speed and position control of drives* In Proc. IECON 2012 - 38th Annual Conf. IEEE Industrial Electronics Society, 2012, pp. 6291-6296.

[11] Vas P. *Sensorless Vector and Direct Torque Control* Oxford University Press, Oxford – New York – Tokyo 1998,

[12] Pillay P., Krishnan R. *Modeling, simulation, and analysis of permanent-magnet motor drives. I. The permanent-magnet synchronous motor drive* Industry Applications, IEEE Transactions on, 1989, 25, pp. 265-273,

[13] Schroedl, M. *Operation of the permanent magnet synchronous machine without a mechanical sensor* Power Electronics and Variable-Speed Drives, 1991., Fourth International Conference on, 1990, pp. 51-56,

[14] Ko J.-S., Han B.-M. *Precision Position Control of PMSM Using Neural Network Disturbance Observer on Forced Nominal Plant* Mechatronics, 2006 IEEE International Conference on, 2006, pp. 316-320,

[15] Schrodl M., Simetzberger C. *Sensorless control of PM synchronous motors using a predictive current controller with integrated INFORM and EMF evaluation* Proc. on Power Electronics and Motion Control Conference, 2008. EPE-PEMC 2008. 13th, 2008, pp. 2275-2282,

[16] Peroutka, Z. *Development of Sensorless PMSM Drives: Application of Extended Kalman Filter* Industrial Electronics, 2005, Proceedings of the IEEE International Symposium on, June 20-23, 2005, 4, pp. 1647-1652,

[17] Guzinski J., Abu-Rub H., Diguet M., Krzeminski Z., Lewicki A. *Speed and Load Torque Observer Application in High-Speed Train Electric Drive* Industrial Electronics, IEEE Transactions on, 2010, 57, pp.565 -574,

[18] Comanescu, M *An Induction-Motor Speed Estimator Based on Integral Sliding-Mode Current Control* Industrial Electronics, IEEE Transactions on, 2009, 56, pp. 3414-3423,

[19] Janiszewski D. *Extended Kalman Filter Based Speed Sensorless PMSM Control with Load Reconstruction* The 32nd Annual Conference of the IEEE Industrial Electronics Society (IECON06), November 7-10 2006, Paris, France, pp. 1465-1468.

[20] Szabat K., Orlowska-Kowalska T. *Application of the Kalman filters to the high performance drive system with elastic coupling* Industrial Electronics, IEEE Transactions on, 59(11) 2012, pp. 4226-4235,

[21] Chib S., Greenberg E. *Understanding the Metropolis-Hastings Algorithm* The American Statistician, 1995, 49, pp. 327-335,

[22] Arulampalam M., Maskell S., Gordon N., Clapp T. *A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking* Signal Processing, IEEE Transactions on, 2002, 50, pp. 174-188,

[23] Gordon N., Salmond D., Smith, A. *Novel approach to nonlinear/non-Gaussian Bayesian state estimation* Radar and Signal Processing, IEE Proceedings, 1993, 140, pp. 107-113,

[24] Douc, R., Cappe O. *Comparison of resampling schemes for particle filtering* Image and Signal Processing and Analysis, 2005. ISPA 2005. Proceedings of the 4th International Symposium on, 2005, pp. 64-69,

***Autor**: Dariusz Janiszewski, PhD,*
Poznan University of Technology,
Institut of Control and Information Engineering,
3a Piotrowo St., PL60965 Poznan,
email: dariusz.janiszewski@put.poznan.pl