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Calculation in 3D of magnetic fields generated by underground cable of complex geometry

***Abstract**. Magnetic fields generated by underground cables can be high enough that some utility customers are concerned about their health effects and electromagnetic interference. The lay-out of the underground cables is much more similar to a broken line than to straight line. In the study the magnetic flux densities above the earth surface produced by underground cables of complex geometry are estimated. It is assumed, that the currents induced in the earth can be neglected, so the magnetic field can be obtained using the Biot-Savart law. The analytical formulas for calculating the 3D magnetic field with respect to a convenient and unique reference system are derived.*

Streszczenie.** Praca przedstawia metodę obliczania pola magnetycznego w otoczeniu kabli podziemnych o złożonej geometrii. Trasę kabla aproksymuje się odcinkami linii łamanej, pomija się prądy indukowane w ziemi oraz wyznacza indukcję magnetyczną stosując prawo Biota-Savarta i zasadę superpozycji. Uzyskane zależności analityczne umożliwiają analizę trójwymiarowego pola magnetycznego w dowolnie przyjętym układzie odniesienia.* (Obliczanie rozkładu pola magnetycznego w otoczeniu kabli podziemnych o złożonej geometrii***).

Keywords: magnetic field, underground cable, complex geometry, Biot-Savart law, 3D calculation
Słowa kluczowe: pole magnetyczne, kabel podziemny, złożona geometria, prawo Biota-Savarta

Introduction

Over the few decades many studies have been undertaken in an attempt to analyze the potential health hazards that may arise from human exposure to electric and magnetic fields. The International Commission on Non-Ionizing Radiation, Protection (ICNIRP) in cooperation with the Environmental Division of the World Health Organization have assessed the available knowledge and published in 1998 the guidelines for safe public nad occupational exposure [1]. In 1999 the recommendation of the European Union Council on the limitation of exposure of the general public to electromagnetic fields [2] was published. In this recommendation, the Council adopted the limit values of ICNIRP's guidelines after their ratification from the Scientific Steering Committee of the European Committee. In 2004, the Directive of the European Parliament and Council [3] was published, in which the limit values of ICNIRP for the occupational exposure were adopted, whereas the World Health Organization in 2007 has concluded a review of the health implications of extremely low frequency (ELF) fields [4].

In numerous papers, magnetic fields in the vicinity of power facilities have been extensively surveyed in recent years. Underground cables are one of the major sources of magnetic field. Managing magnetic field levels in the space surrounding the cables [5 – 17] is connected with studies which involve the evaluation of the magnetic field intensity. In order to predict the worst case field exposure theoretical models need to be developed which will be based on analytical or numerical methods [18 – 26].

If the magnitude and phase angles of the currents in the cable system are known, a simple application of Ampere's circuital law, will give the value of the 2D magnetic field in the vicinity of a transmission cable. The usual procedure is to assume that some positive sequence currents are flowing in the cable circuit under analysis and then calculate any current that may be induced in the cable/sheaths for a multipoint grounded cable system. In order to determine the magnetic flux density B, due to currents flowing in these cables, the following assumptions were usually made: the earth has no effect on the magnetic field produced by the cable (i.e. μ_r = 1), the total magnetic field at any point is determined by linear superposition of the magnetic field produced by the currents flowing in each individual conductor, the effect of induced shield/sheath currents on the magnetic field is negligible, each cable is considered to be infinitely long and straight, and the currents induced in the earth can be neglected.

However, many cable structures (e.g. underground cables used in residential distribution systems) have complex geometries for which the magnetic field in the volume around these structures cannot be assessed using the 2D approach. For this type of structures it is necessary to define a 3D model of the geometry and calculate the magnetic field distribution in the cable surrounding by use of the freely available software [21].

The objective of the paper is to present a method (as an alternative to the numerical approach) for calculation of the 3D magnetic fields generated by underground cable of complex geometry. It is assumed that the lay-out of the underground cable is much more similar to a broken line than to a straight line. Furthermore it is supposed, that the currents induced in the earth can be neglected, so the magnetic field can be obtained using the Biot-Savart law and the superposition principle. In the paper analytical formulas for calculating the 3D magnetic field with respect to a convenient and unique reference system are derived. The formulas can be used by a software tool to model the magnetic fields generated by the cables.

Calculation of magnetic field generated by underground cables

Consider the arbitrary configuration of the underground cable, as shown in Figure1.

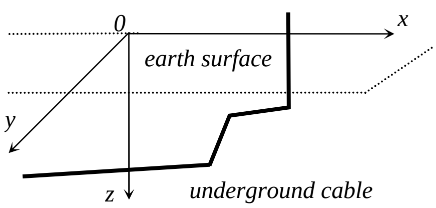

Fig.1. Underground cable with complex geometry

The magnetic field in the observation point $P(x,y,z)$ produced by a current path c as in Figure 2 can be computed using the Biot-Savart law:

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi} \int_c \frac{I(\vec{dl} \times 1_r)}{r^2} \tag{1}$$

where I is a phasor current, the vector element $\vec{dl}$ coincides with the direction of the current I, 1_r is a unit vector in the direction of the vector $\vec{r}$, r is the distance between the source point $P^s(X,Y,Z)$ and the observation point $P(x,y,z)$ and μ_0 is the magnetic permeability of the vacuum.

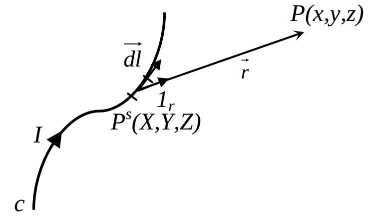

Fig.2. Current path generating the magnetic field

For calculation purposes, the current path is divided into small straight-line segments. For simplicity consider only the i-th segment of the current path. It is convenient to define two different Cartesian reference systems: the first one x, y, z is a reference system (external reference system), the second one x', y', z' is referred to the i-th segment, Figure 3. It should be noted, that the reference coordinate system can be arbitrary located in the space, it is however reasonable to locate the xy plane on the earth surface.

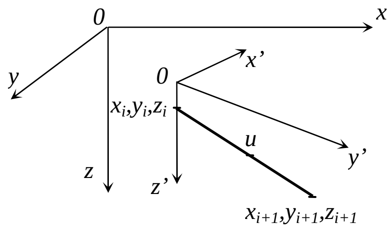

Fig.3. Reference systems and the i-th segment of the current path

The terminating points of the i-th segment have in the external (unprimed) reference system the coordinates (x_i,y_i,z_i) and $(x_{i+1},y_{i+1},z_{i+1})$ respectively. The segment lies in the $y'z'$ plane of the second (primed) coordinate system, so that $x'=0$ and $z'=z$. The segment can be generally described by the straight-line equation in the "slope-intercept" form

(2) $$z' = m_i y' + c_i$$

where

(3) $$m_i = \frac{z'_{i+1} - z'_i}{y'_{i+1} - y'_i}$$

(4) $$c_i = -\frac{z'_{i+1} - z'_i}{y'_{i+1} - y'_i} y'_i + z'_i$$

Now we express (2) in parametric form with respect to the parameter u (progressive along the i-th segment) indicated on Figure 3

(5) $$\begin{aligned} X'(u) &= 0 \\ Y'(u) &= u \qquad 0 \le u \le L_i \\ Z'(u) &= m_i u + c_i \end{aligned}$$

where L_i

(6) $$L_i = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}$$

To obtain (5) in the reference coordinates system, the roto-translation formulas in the tridimensional space [27] should be applied. Thus:

(7) $$\begin{bmatrix} X(u) \\ Y(u) \\ Z(u) \end{bmatrix} = \begin{bmatrix} \beta_i & \alpha_i & 0 \\ -\alpha_i & \beta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X'(u) \\ Y'(u) \\ Z'(u) \end{bmatrix} + \begin{bmatrix} x_i \\ y_i \\ 0 \end{bmatrix}$$

where the coefficients α_i and β_i (direction cosines) are described as:

(8) $$\alpha_i = \frac{x_{i+1} - x_i}{\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}}$$

(9) $$\beta_i = \frac{y_{i+1} - y_i}{\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}}$$

From (7) and (5) it follows that

(10) $$\begin{aligned} X(u) &= \alpha_i u + x_i \\ Y(u) &= \beta_i u + y_i \\ Z(u) &= m_i u + c_i \end{aligned}$$

In order to apply the formula (1), we have to find suitable expressions $1_r(u)$ and $\vec{dl}(u)$. By looking at Figure 2., if $(X(u), Y(u), Z(u))$ are the coordinates of the generic element $\vec{dl}(u)$, we have

(11) $$1_r(u) = \frac{(x - X(u))1_x + (y - Y(u))1_y + (z - Z(u))1_z}{\sqrt{(x - X(u))^2 + (y - Y(u))^2 + (z - Z(u))^2}}$$

and taking into account (10)

(12) $$\vec{dl}(u) = \alpha_i du 1_x + \beta_i du 1_y + m_i du 1_z$$

Remembering that

(13) $$r^2(u) = (x - X(u))^2 + (y - Y(u))^2 + (z - Z(u))^2$$

and inserting (11), (12) and (13) into (1) we obtain the three components of the magnetic flux density field

(14) $$B_{ix}(x,y,z) = \frac{\mu_0 I}{4\pi} \int_0^{L_i} \frac{\beta_i(z - m_i u - c_i) - m_i(y - \beta_i u - y_i)}{\left[(x - \alpha_i u - x_i)^2 + (y - \beta_i u - y_i)^2 + (z - m_i u - c_i)^2\right]^{\frac{3}{2}}} du$$

(15) $$B_{iy}(x,y,z) = -\frac{\mu_0 I}{4\pi} \int_0^{L_i} \frac{\alpha_i(z - m_i u - c_i) - m_i(x - \alpha_i u - x_i)}{\left[(x - \alpha_i u - x_i)^2 + (y - \beta_i u - y_i)^2 + (z - m_i u - c_i)^2\right]^{\frac{3}{2}}} du$$

(16) $$B_{iz}(x,y,z)=\frac{\mu_0 I}{4\pi}\int_0^{L_i}\frac{\alpha_i(y-\beta_i u-y_i)-\beta_i(x-\alpha_i u-x_i)}{\left[(x-\alpha_i u-x_i)^2+(y-\beta_i u-y_i)^2+(z-m_i u-c_i)^2\right]^{\frac{3}{2}}}du$$

The integrals in formulas (14) – (16) have to be solved numerically.

Taking into account, that the magnetic field can be produced by more conductors in complex configuration, the total magnetic field can be obtained by superposition of the contributions produced by each segment. If N_c denotes the number of conductors and N_s is a number of segments considered, from equations (14) – (16) we have:

(17) $$B_x(x,y,z)=\sum_{i=1}^{N_s}\sum_{j=1}^{N_c}\frac{\mu_0 I_j}{4\pi}\int_0^{L_{ij}}\frac{\beta_{ij}(z-m_{ij}u-c_{ij})-m_{ij}(y-\beta_{ij}u-y_{ij})}{\left[(x-\alpha_{ij}u-x_{ij})^2+(y-\beta_{ij}u-y_{ij})^2+(z-m_{ij}u-c_{ij})^2\right]^{\frac{3}{2}}}du$$

(18) $$B_y(x,y,z)=-\sum_{i=1}^{N_s}\sum_{j=1}^{N_c}\frac{\mu_0 I_j}{4\pi}\int_0^{L_{ij}}\frac{\alpha_{ij}(z-m_{ij}u-c_{ij})-m_{ij}(x-\alpha_{ij}u-x_{ij})}{\left[(x-\alpha_{ij}u-x_{ij})^2+(y-\beta_{ij}u-y_{ij})^2+(z-m_{ij}u-c_{ij})^2\right]^{\frac{3}{2}}}du$$

(19) $$B_z(x,y,z)=\sum_{i=1}^{N_s}\sum_{j=1}^{N_c}\frac{\mu_0 I_j}{4\pi}\int_0^{L_{ij}}\frac{\alpha_{ij}(y-\beta_{ij}u-y_{ij})-\beta_{ij}(x-\alpha_{ij}u-x_{ij})}{\left[(x-\alpha_{ij}u-x_{ij})^2+(y-\beta_{ij}u-y_{ij})^2+(z-m_{ij}u-c_{ij})^2\right]^{\frac{3}{2}}}du$$

Finally, the modulus of the magnetic flux density field can be obtained from the formula:

(20) $$\left|B(x,y,z)\right|=\sqrt{\left|B_x(x,y,z)\right|^2+\left|B_y(x,y,z)\right|^2+\left|B_z(x,y,z)\right|^2}$$

It should be noted, that the above considerations are valid for segments, which are not parallel to the $0z$ axis.

Consider the case shown in Figure 4.

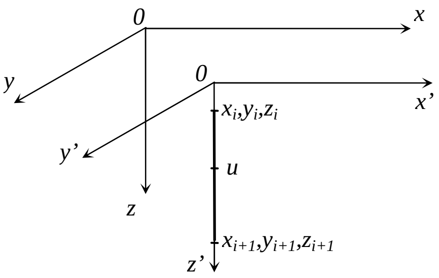

Fig.4. Reference systems and the i-th segment of the current path parallel to the $0z$ axis

Let us denote again ($X(u)$, $Y(u)$, $Z(u)$) the coordinates of the generic element $\vec{dl}(u)$, where the parameter u (progressive along the i-th segment) is indicated on Figure 4. In the primed coordinate system one can write

(21) $$X'(u)=0 \qquad Y'(u)=0 \qquad Z'(u)=u \qquad 0\le u\le L_i$$

where L_i denotes the length of the segment considered and

(22) $$L_i=z_{i+1}-z_i$$

Regarding the translation of the coordinates we obtain:

(23) $$X(u)=x_i \qquad Y(u)=y_i \qquad Z(u)=u$$

In order to apply the formula (1), we have to find suitable expressions $1_r(u)$ and $\vec{dl}(u)$. By looking at Figure 2 and taking into account that ($X(u)$, $Y(u)$, $Z(u)$) are the coordinates of the generic element $\vec{dl}(u)$, we have

(24) $$1_r(u)=\frac{(x-x_i)1_x+(y-y_i)1_y+(z-u)1_z}{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-u)^2}}$$

and

(25) $$\vec{dl}(u)=du1_z$$

Remembering the formula (13) and inserting (24) and (25) into (1) we obtain the two components of the magnetic flux density field:

(26) $$B_{ix}(x,y,z)=-\frac{\mu_0 I}{4\pi}\int_0^{L_i}\frac{y-y_i}{\left[(x-x_i)^2+(y-y_i)^2+(z-u)^2\right]^{\frac{3}{2}}}du$$

(27) $$B_{iy}(x,y,z)=\frac{\mu_0 I}{4\pi}\int_0^{L_i}\frac{x-x_i}{\left[(x-x_i)^2+(y-y_i)^2+(z-u)^2\right]^{\frac{3}{2}}}du$$

If N_c denotes the number of conductors and N_{sv} is a number of vertical segments considered, from equations (26) and (27) we have:

$$B_x(x,y,z) = -\sum_{i=1}^{N_{sv}}\sum_{j=1}^{N_c}\frac{\mu_0 I_j}{4\pi}\int_0^{L_{ij}}\frac{y-y_{ij}}{\left[\begin{matrix}(x-x_{ij})^2+(y-y_{ij})^2+\\+(z-u)^2\end{matrix}\right]^{\frac{3}{2}}}du \quad (28)$$

$$B_y(x,y,z) = \sum_{i=1}^{N_{sv}}\sum_{j=1}^{N_c}\frac{\mu_0 I_j}{4\pi}\int_0^{L_{ij}}\frac{x-x_{ij}}{\left[\begin{matrix}(x-x_{ij})^2+(y-y_{ij})^2+\\+(z-u)^2\end{matrix}\right]^{\frac{3}{2}}}du \quad (29)$$

It should be noticed, that if the current I flows opposite to the $0z$ axis

$$\overrightarrow{dl}(u) = -du1_z \quad (30)$$

and the components of the magnetic flux density field change their signs.

The resultant modulus of the magnetic flux density field produced by arbitrary geometrical configuration of current carrying underground conductors can be obtained using the formula (20).

Example

Consider the underground 3-phase cable line 3xYAKXS 1x35 mm^2 0,6/1 kV buried at depth 0,7 m in flat configuration; the distance between the cables is 70 mm. The route of the cable line is shown in Figure 5. Each cable of the line is divided into 5 straight-line segments. The terminating points of the segments e.g. of the middle cable are: P_1(-10; 5; 0), P_2(-10; 5; 0,7), P_3(0; 5; 0,7), P_4(0; -5; 0,7), P_5(10; -5; 0,7) and P_6(10; -5; 0). It is assumed that the currents in the particular cable are: I_1=164exp(0) A, I_2=164exp(-j120º) A, I_3=164exp(j120º) A and the earth permeability is μ_0.

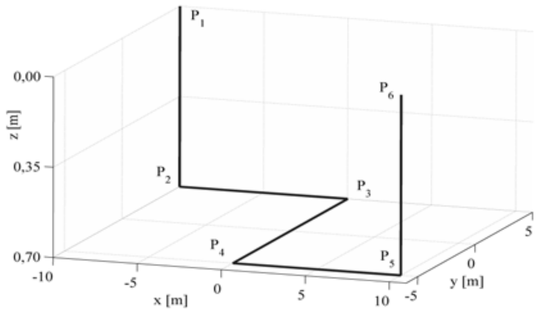

Fig. 5. Route of underground cable line with complex geometry

In Figure 5 the 3-dimensional magnetic flux density distribution is ploted using the MatLab. The calculations are curried out for the plane located 1 m above the earth surface.

Final remarks

The formulas derived allows to manage cases with any complex geometry of the cable such as changes of direction of a cable line, changes of burial depth of the line and crossing between two cable lines as well. They can be used in practical cases, when there are often several cables nearby, and the fields interact with each other. The necessary data for magnetic field calculations are: the number of conductors, the current in each conductor of the cable lines, the x, y, z coordinates of the observation point P, the number of segments the cable is divided into, and the coordinates (x_i,y_i,z_i) and $(x_{i+1},y_{i+1},z_{i+1})$ of terminating points of each segment. It should be noted that all coordinates refer to the reference system, which can be arbitrary located in the space.

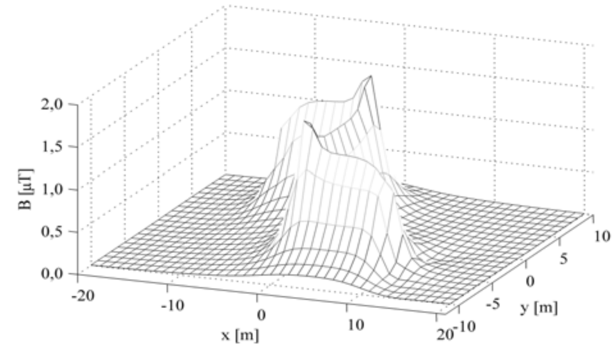

Fig. 6. Module of the magnetic flux density

The method assumes that the cable segment is described by the straight-line equation in the "slope-intercept" form, effects of earth currents onto magnetic field are negligible and that power-line currents have prescribed values. The formulas obtained in the paper require numerical integration which can be performed by the use of freely available tools.

It should be pointed out, that the capability of the method developed is greater then other analytical methods presented in the literature and the direct analytical approach used, as opposed to the commercialized 3D simulators, enables any physical interpretation of the phenomena being simulated.

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