Piotr PRUSKI, Stefan PASZEK

Silesian University of Technology

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Determination of Polish Power System model state matrix eigenvalues based on angular speed waveforms

***Abstract**. In the paper there are presented the calculation results of the Polish Power System (PPS) state matrix eigenvalues associated with electromechanical phenomena (i.e. electromechanical eigenvalues). These eigenvalues were calculated on the basis of the analysis of generating units angular speed waveforms simulated with the use of a 57-machine PPS model. The method for calculations of electromechanical eigenvalues consists in approximation of these waveforms by the waveforms recovered from searched eigenvalues and their participation factors.*

***Streszczenie.** W artykule przedstawiono wyniki obliczeń wartości własnych macierzy stanu Krajowego Systemu Elektroenergetycznego (KSE) związanych ze zjawiskami elektromechanicznymi (tzn. elektromechanicznych wartości własnych) na podstawie analizy przebiegów zakłóceniowych prędkości kątowej zespołów wytwórczych uzyskanych przy użyciu 57- maszynowego modelu KSE. Wykorzystana metoda obliczeń polega na apro-ksymacji przebiegów zakłóceniowych za pomocą przebiegów wyznaczonych na podstawie z poszukiwanych wartości własnych. (**Wyznaczenie wartości własnych macierzy stanu Krajowego Systemu Elektroenergetycznego na podstawie przebiegów prędkości kątowej**).*

Keywords: power system, eigenvalues associated with electromechanical phenomena, angular stability, waveform recovery.
Słowa kluczowe: system elektroenergetyczny, wartości własne związane ze zjawiskami elektromechanicznymi, stabilność kątowa, odtwarzanie przebiegów.

Introduction

Maintenance of the angular stability of a power system (PS) is one of the common conditions of its proper work. The PS angular stability can be assessed by stability factors determined based on the PS state matrix eigenvalues associated with electromechanical phenomena (i.e. electromechanical eigenvalues) [1, 2]. These eigenvalues can be calculated from the PS state equations but the calculation results depend then on the values of the system state matrix elements. The results also depend indirectly on the assumed PS models and their uncertain parameters [2, 3]. The eigenvalues can also be calculated with a good accuracy from the analysis of the actual disturbance waveforms occurring in the PS after various disturbances [2, 4].

The aim of this paper is to analyse the calculation accuracy of the state matrix electromechanical eigenvalues of the 57 – machine Polish Power System (PPS) model based on the angular speed disturbance waveforms.

Linearized model of a power system

The power system model linearized around the steady working point is described by the state and output equations [2, 5, 6]:

$$\Delta\dot{\mathbf{X}} = \mathbf{A}\Delta\mathbf{X} + \mathbf{B}\Delta\mathbf{U}, \qquad \Delta\mathbf{Y} = \mathbf{C}\Delta\mathbf{X} + \mathbf{D}\Delta\mathbf{U}, \tag{1}$$

where: $\Delta\boldsymbol{X}$, $\Delta\boldsymbol{U}$, $\Delta\boldsymbol{Y}$ - deviations of the vectors of: state variables, input and output variables, respectively. The waveforms of input quantities of the linearized system model can be calculated directly by integrating the state equation, or by using the eigenvalues and eigenvectors of the state matrix $\boldsymbol{A}$ [2, 5, 6].

The waveform of the given output value is a superposition of the modal components which depend on the eigenvalues and eigenvectors of the state matrix. For a disturbance being a Dirac pulse of the j-th input value $\Delta U_j(t) = \Delta U\delta(t-t_0)$ the i-th output value (at $\boldsymbol{D} = \mathbf{0}$ and assuming only single eigenvalues) is given by [2, 5, 6]:

$$\Delta y_i(t) = \sum_{h=1}^{n} F_{ih} e^{\lambda_h (t-t_0)} \Delta U, \quad t \ge t_0, \quad F_{ih} = \mathbf{C}_i \mathbf{V}_h \mathbf{W}_h^{\mathrm{T}} \mathbf{B}_j, \tag{2}$$

where: $\lambda_h = \alpha_h + \mathrm{j}\nu_h$ - h-th eigenvalue of the state matrix, F_{ih} - participation factor of the h-th eigenvalue in the i-th output waveform, $\boldsymbol{C}_i$ - i-th row of matrix $\boldsymbol{C}$, $\boldsymbol{V}_h$ - h-th right-side eigenvector of the state matrix, $\boldsymbol{W}_h$ - h-th left-side eigenvector of the state matrix, $\boldsymbol{B}_j$ - j-th column of matrix $\boldsymbol{B}$, n - dimension of the state matrix $\boldsymbol{A}$. The values λ_h and F_{ih} can be real or complex [2, 5, 6].

In the case of generating unit angular speed waveforms in a PS, the eigenvalues associated with motion of generating unit rotors, called *electromechanical eigenvalues* in the paper, are of decisive significance. The electromechanical eigenvalues intervene in different ways in the instantaneous power waveforms of particular generating units, which is related to the different values of their participation factors [2, 5, 6].

The method for calculations of electromechanical eigenvalues

For calculations there were used the disturbance waveforms of generating unit angular speed deviations occurring after purposeful introducing a small disturbance to the PS. The assumed disturbance was a rectangular pulse in the voltage regulator reference voltage in one of generating units. The system response to an input in the form of a short rectangular pulse with a properly selected height and length is close to the response of that system to a Dirac pulse [2, 5, 6].

Appropriate measurements of angular speed waveforms are possible with the use of the equipment developed by the Institute of Electrical Engineering and Computer Science of the Faculty of Electrical Engineering of the Silesian University of Technology.

The method for calculations of electromechanical eigenvalues used in investigations consists in approximation of angular speed waveforms in particular generating units with the use of expression (2). The electromechanical eigenvalues and their participation factors in the analysed waveform are the unknown parameters of this approximation. In the approximation process, these parameters are iteratively selected in order to minimize the value of the objective function defined as a mean square error between the approximated and approximating waveform:

$$\varepsilon_{\mathrm{w}}(\boldsymbol{\lambda}, \mathbf{F}) = \sum_{k=1}^{N} \left(\Delta\omega_{k(\mathrm{m})} - \Delta\omega_{k(\mathrm{a})}(\boldsymbol{\lambda}, \mathbf{F})\right)^2, \tag{3}$$

where: $\boldsymbol{\lambda}$ - vector of electromechanical eigenvalues, $\boldsymbol{F}$ - vector of participation factors, N - number of samples. The index m denotes the approximated waveform, while the index a the approximating waveform of the angular speed ω, calculated from the searched eigenvalues and their participation factors. The eigenvalues with small participation factor modules in the given waveform and the eigenvalues not associated with electromechanical phenomena are neglected in calculations based on this waveform.

From the performed calculations it follows that in the waveforms of angular speed deviations $\Delta\omega$ there intervene significantly not only the modal components associated with electromechanical eigenvalues but also modal components associated with other eigenvalues. To make the correct approximation of the waveforms $\Delta\omega$ possible for a pulse disturbance, it is necessary to take into account one equivalent oscillatory modal component of a relatively low frequency that represents the influence of the neglected modal components not associated with electromechanical phenomena on this waveform [5].

The objective function (3) was minimized by a hybrid optimization algorithm being a serial combination of a genetic algorithm with a gradient algorithm [2, 5, 6, 7].

Due to the existence of the objective function local minima in which the optimization algorithm may freeze, the eigenvalues were calculated repeatedly based on the same waveform. If the objective function values were higher than a certain assumed limit, the results were rejected. The final result of the calculations of real and imaginary parts of the particular eigenvalues were the arithmetic means from the real and imaginary parts of the eigenvalues obtained from the results not rejected in further calculations [2, 5, 6].

Calculations of electromechanical eigenvalues

The calculations were performed for the PPS model shown in Fig. 1. In this model there were taken into account 49 selected generating units working in high and highest voltage networks as well as 8 equivalent generating units representing the influence of PSs of neighbouring countries. The analysed PPS model was worked out in Matlab-Simulink environment. It consists of 57 models of generating units as well as the model of the network and loads [2, 6].

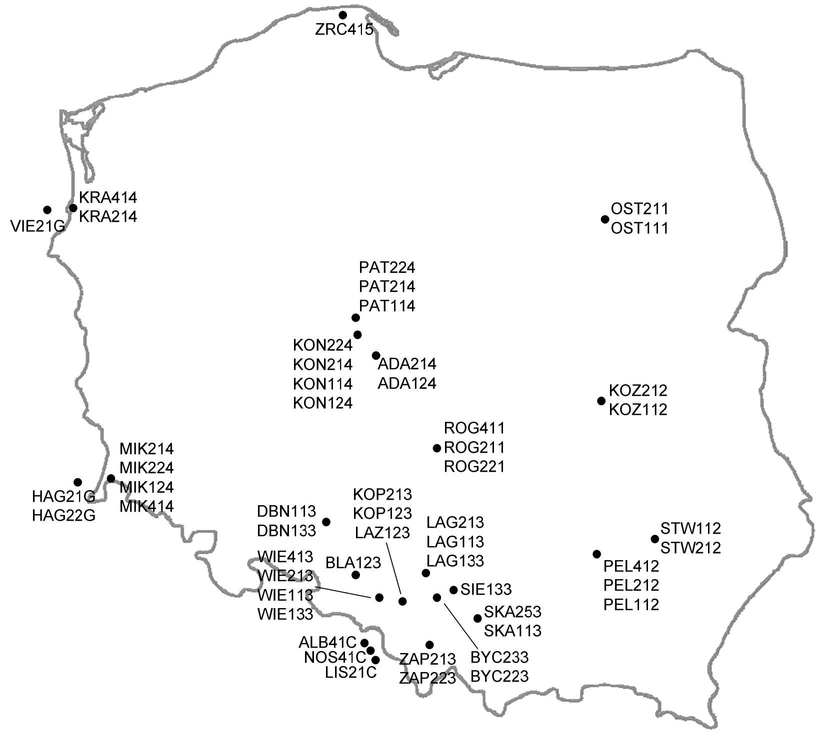

Fig.1. Generating units included in the Polish Power System model [6]

In the calculations presented in this paper there were assumed [2]: the synchronous generator GENROU model [8, 9, 10], the model of a static [10] or an electromachine [8, 9, 10] excitation system operating in the PPS, the steam turbine IEEEG1 model [8, 9, 10] or the water turbine HYGOV model [10] and, optionally, the model of a power system stabilizer PSS3B [8, 9, 10]. For the equivalent generating units representing the influence of power systems of the neighbouring countries there was used the simplified model of a synchronous generator (GENCLS [9]).

The eigenvalues (including electromechanical eigenvalues) of the system state matrix can be calculated directly on the basis of the structure and parameters of the PS model in program Matlab-Simulink. These electromechanical eigenvalues are called *original eigenvalues* further in the paper. Comparison of the eigenvalues calculated based on the minimization of the objective function (4) and the original eigenvalues is a measure of the calculation accuracy [2, 5].

The state matrix of the analysed PPS model has 56 complex electromechanical eigenvalues. They were sorted in ascending order with respect to the real parts and numbered from λ_1 to λ_{56} [2].

The selected original eigenvalues as well as the absolute errors of calculations of these eigenvalues based on angular speed waveforms are given in Tab 1.

From Tab. 1 it follows that almost all the electromechanical eigenvalues were calculated with the satisfactory accuracy. The eigenvalues λ_{50} and λ_{52} - λ_{55} were not calculated because the modal components associated with them did not influence the angular speed waveforms of any of the PPS generating units strongly enough. Because of the low values of the modules of participation factors in the waveforms of the PPS generating units, the eigenvalues λ_{50} - λ_{56} did not have a significant impact on the PPS angular stability.

Fig. 2 shows exemplary simulation waveforms of the angular speed deviation in generating units HAL113 (for a disturbance introduced to unit LAG133) and OST211 (for a disturbance introduced to unit OST111) as well as the bands of approximating waveforms corresponding to the non-rejected calculation results. The band of the approximating waveforms determines the range of the angular speed changes in which there are contained all approximating waveforms corresponding to particular calculation results.

Table 1. Selected original eigenvalues and absolute errors of calculating these eigenvalues based on angular speed waveforms

Original eigenvalues					
λ_1	-1.3099±j11.1792	λ_3	-1.2768±j10.1287	λ_6	-1.1670±j10.8599
λ_9	-1.0939±j9.8686	λ_{12}	-1.0615±j10.2550	λ_{15}	-1.0477±j10.0241
λ_{18}	-1.0087±j10.2941	λ_{21}	-0.9925±j10.1970	λ_{24}	-0.9843±j9.1122
λ_{27}	-0.8831±j9.4212	λ_{30}	-0.8660±j9.8514	λ_{33}	-0.8226±j9.1135
λ_{36}	-0.7765±j9.1363	λ_{39}	-0.7368±j9.6011	λ_{42}	-0.6372±j8.3382
λ_{45}	-0.4955±j7.3005	λ_{47}	-0.4488±j6.6540	λ_{48}	-0.4165±j8.0932
λ_{49}	-0.1710±j4.9780	λ_{51}	-0.0835±j5.6278	λ_{56}	-0.0457±j4.0116
Eigenvalues calculated based on $\Delta\omega$ waveforms					
$\Delta\lambda_1$	-0.0180±j0.1097	$\Delta\lambda_3$	-0.0430∓j0.0650	$\Delta\lambda_6$	-0.0107±j0.0038
$\Delta\lambda_9$	0.0859±j0.0072	$\Delta\lambda_{12}$	-0.0669±j0.1014	$\Delta\lambda_{15}$	-0.0279∓j0.0360
$\Delta\lambda_{18}$	-0.0159±j0.0802	$\Delta\lambda_{21}$	-0.0088±j0.1967	$\Delta\lambda_{24}$	0.0269±j0.0036
$\Delta\lambda_{27}$	-0.0217∓j0.2172	$\Delta\lambda_{30}$	-0.0781±j0.0681	$\Delta\lambda_{33}$	-0.0094±j0.0827
$\Delta\lambda_{36}$	-0.0454±j0.1293	$\Delta\lambda_{39}$	0.0466±j0.0483	$\Delta\lambda_{42}$	0.0871±j0.0341
$\Delta\lambda_{45}$	0.0270∓j0.0026	$\Delta\lambda_{47}$	-0.0060±j0.0043	$\Delta\lambda_{48}$	-0.0668∓j0.0862
$\Delta\lambda_{49}$	0.0022±j0.0043	$\Delta\lambda_{51}$	0.0749±j0.0936	$\Delta\lambda_{56}$	0.0141∓j0.1408

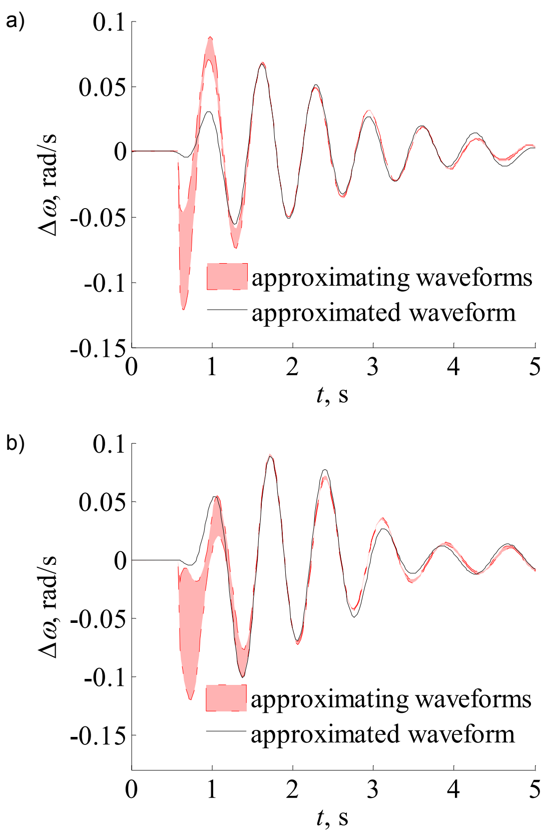

Fig. 2. Exemplary simulation waveforms of the angular speed deviation in generating units HAL113 (a) and OST211 (b)

From Fig. 2 it follows that the approximation accuracy of the analysed angular speed waveforms is satisfactory in the time interval after decay of strongly damped modal components that did not influence the calculation results. In almost all the analysed cases the bands of the approximating waveforms were very narrow.

Conclusions

From the investigations performed, one can draw the following conclusions:

– It was possible to determine electromechanical eigenvalues with the good accuracy based on the analysis of the angular speed waveforms occurring after introducing a pulse disturbance in the voltage regulation system of one of generating units. The presented method for calculations of electromechanical eigenvalues works well in the case of large power systems like the Polish Power System.

– The averaging of calculation results of particular eigenvalues based of the analysis of the angular speed waveforms of different generating units allowed increasing the calculation accuracy. When the calculation result of the given eigenvalue on the basis of one waveform differed significantly from the calculation results of that eigenvalue on the basis of other waveforms, such a result was rejected.

– Repeating calculations of the eigenvalues with use of the hybrid algorithm, at different starting points selected randomly for each calculation from the search range, eliminates the problem of algorithm freezing at local minima of the objective function.

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***Authors**: dr inż. Piotr Pruski, E-mail: Piotr.Pruski@polsl.pl, prof. dr hab. inż. Stefan Paszek, E-mail: Stefan.Paszek@polsl.pl, Politechnika Śląska, Wydział Elektryczny, Instytut Elektrotechniki i Informatyki, ul. Akademicka 10, 44-100 Gliwice.*