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Reduction of impedance matrices of power busducts

***Abstract**. A general method of reduction of self and mutual impedance matrix of a busduct system with busbars in parallel or series is described. The method allows us finding the impedance matrix of a reduced system when given the impedance matrix of the busbars or their fragments. It can be also used in optimization of certain features of impedance matrix of the reduced system.*

***Streszczenie.** Rozważono ogólną metodę redukcji macierzy impedancji własnych i wzajemnych układu szynoprzewodów połączonych równolegle lub szeregowo. Pozwala ona znaleźć macierz impedancji układu zredukowanego dla zadanej macierzy impedancji szynoprzewodów cząstkowych. Może być także stosowana do optymalizacji pewnych cech macierzy impedancji. (**Redukcja macierzy impedancji torów prądowych**).*

Keywords: impedance matrix, power busducts, impedance reduction.
Słowa kluczowe: macierz impedancji, tory wielkoprądowe, redukcja impedancji.

Introduction

There are situations when an existing power busduct has to be expanded or rearranged due to modernization of receivers or changes in supply system. When the busduct consists of many busbars that can be connected in various ways it is worth to select the best one with respect to certain criterion. Some possible criterions are the lowest power losses in the busduct at a given transferred power, the lowest voltage drops, and so on. In a single phase systems the obvious criterion is the smallest module of the busduct impedance. It is desirable to be able to recalculate quickly various variants of busbar arrangement to select the most appropriate one. A similar situation takes place for a newly designed busduct with many busbars. The method described here uses the general approach to calculation the impedance matrix of a reduced system when the full impedance matrix is given.

The self and mutual impedance matrix

Let us consider a system of n busbars that can be regarded as magnetically coupled two-terminals (Fig. 1).

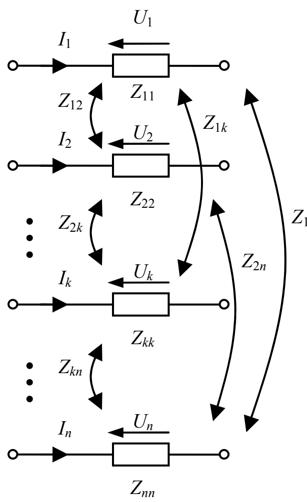


Fig.1. A system of n magnetically coupled two-terminals

If they carry currents whose phasors are I_1, I_2, …, I_N, then the voltage between the terminals of i-th busbar equals

(1) $$U_i = \sum_{i=1}^{n} Z_{ij} I_j, \qquad i = 1, 2, ..., n,$$

where Z_{ij} is the mutual impedance between two-terminals i and j (when $j \neq i$) and the self impedance of two- terminal i (when $j = i$). The above equation system can be written in matrix form as follows:

(2) $$\mathbf{U} = \mathbf{ZI},$$

where $\mathbf{U} = \{U_i\}_n$ and $\mathbf{I} = \{I_i\}_n$ are the vectors of voltages and currents, respectively, connected with each two-terminal, and $\mathbf{Z} = [Z_{ij}]_{n\times n}$ is the self and mutual impedance matrix of the system. The impedance matrix can be determined in measurements or via theoretical considerations (e.g. [1-10]). The admittance matrix is defined as $\mathbf{Y} = \mathbf{Z}^{-1}$.

System reduction

Suppose we know matrix $\mathbf{Z}$. Let the two-terminals be grouped in m groups, and connected either in series or in parallel, but in the same way throughout the whole system (Fig. 2). In this manner, we obtain a reduced system of m two-terminals. Our goal is to determine $\mathbf{Z}' = [Z'_{kl}]_{m\times m}$ – the equivalent matrix of self and mutual impedances of the reduced system. Notation $i\in k$ will be used to denote that component i belongs to group k.

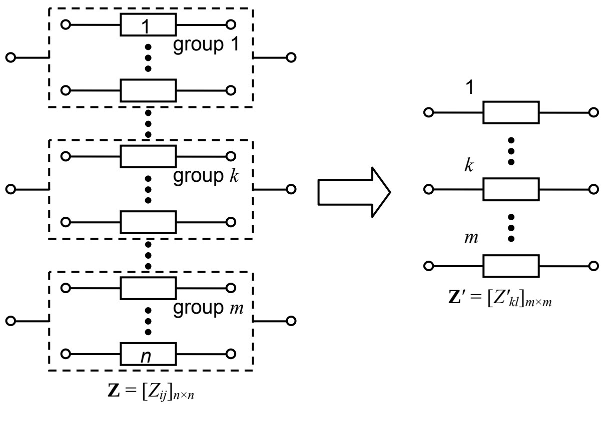


Fig.2. Reduction of n two-terminals system into m two-terminals one

Reverse connection

Suppose we know matrix $\mathbf{Z}$ of an n two-terminals system and consider the same system in which k-th two-terminal is connected in a reverse way (Fig. 3).

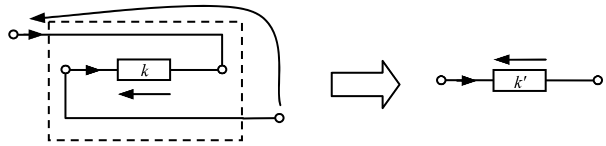


Fig.3. Reverse connection of a two-terminal

Obviously, in the new system $U'_k = -U_k$ and $I'_k = -I_k$, and the remaining voltages and currents are the same in both systems. Therefore, we have

$$\mathbf{U}' = \mathbf{R}(k)\mathbf{U}, \qquad \mathbf{I}' = \mathbf{R}(k)\mathbf{I}, \tag{3}$$

where $\mathbf{R}(k)$ is a diagonal matrix with –1 on k-th position and +1 otherwise. Since $\mathbf{R}(k)^{-1} = \mathbf{R}(k)$, we have $\mathbf{I} = \mathbf{R}(k)\mathbf{I}'$, and therefore $\mathbf{U}' = \mathbf{R}(k)\mathbf{Z}\mathbf{I} = \mathbf{R}(k)\mathbf{Z}\mathbf{R}(k)\mathbf{I}'$. Hence,

$$\mathbf{Z}' = \mathbf{R}(k)\mathbf{Z}\mathbf{R}(k). \tag{4}$$

Matrix $\mathbf{Z}'$ has the same elements as $\mathbf{Z}$ except for k-th column and k-th row, in which the elements have opposite signs with exclusion of the diagonal, which is the same as in the original matrix $\mathbf{Z}$. In general, if there are several reverse connections in the system, the resulting impedance matrix can be obtained as follows: start with the original matrix $\mathbf{Z}$ and for each reversely connected two-terminal, change the sign of impedances in the corresponding row and then in the corresponding column (so the diagonal will remain unchanged).

As an example, consider three coupled windings (e.g. in an air transformer) of impedance matrix $\mathbf{Z} = [Z_{ij}]_{3\times3}$. If we interchange the terminals in the first winding, the impedance matrix will be as follows:

$$\mathbf{Z}' = \left[\begin{array}{c|cc} Z_{11} & -Z_{12} & -Z_{13} \\ \hline -Z_{21} & Z_{22} & Z_{23} \\ -Z_{31} & Z_{32} & Z_{33} \end{array}\right]. \tag{5}$$

Series connection

Let the components in each group be connected in series (Fig. 4). The voltage on group k equals the sum of voltages on components belonging to the group, whereas the current in k-th group is the same in all components. It is worth to take into considerations the possibility that a two-terminal can be connected into the group in a regular or reverse way (Fig. 5). Then the voltages on particular components should be taken with sign plus (regular connection) or minus (reverse connection). Notation $i\in+k$ and $i\in-k$ will be used for regular and reverse membership of i-th two-terminal in group k, respectively. Let us introduce connection matrix $\mathbf{C} = [C_{ki}]_{m\times n}$, the elements of which are

$$\mathbf{C} = \begin{cases} +1 & \text{for } i \in +k, \\ -1 & \text{for } i \in -k, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

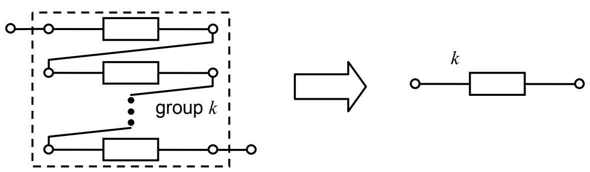


Fig.4. Group k connected in series

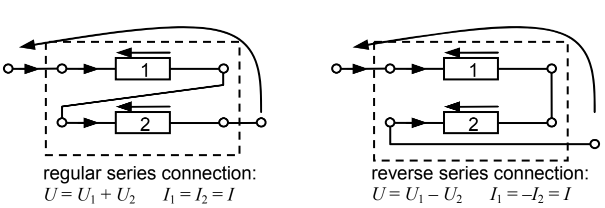


Fig.5. Regular and reverse series connection

Then the relationships between the voltage and current vectors of the original and resulting system can be expressed as

$$\mathbf{U}' = \mathbf{C}\mathbf{U}, \qquad \mathbf{I} = \mathbf{C}^{\mathrm{T}}\mathbf{I}'. \tag{7}$$

Using the above relationships, we have $\mathbf{U}' = \mathbf{C}\mathbf{U} = \mathbf{C}\mathbf{Z}\mathbf{I} = \mathbf{C}\mathbf{Z}\mathbf{C}^{\mathrm{T}}\mathbf{I}'$, and hence

$$\mathbf{Z}' = \mathbf{C}\mathbf{Z}\mathbf{C}^{\mathrm{T}}. \tag{8}$$

In scalar form we have

$$Z'_{kl} = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ki}Z_{ij}C_{jl} = \sum_{i\in k}\sum_{j\in l}(\pm Z_{ij}). \tag{9}$$

The second form is a result of the fact that $C_{ki}C_{jl}$ only takes a non-zero value when component i belongs to group k and component j belongs to group l at the same time. Moreover, $C_{ki}C_{jl}$ equals then either +1 or –1, depending on whether components i and j are connected in a similar way (either regular or reverse) or different way (one regular and the other reverse), respectively.

As an example, consider two coils in series connected in similar or dissimilar way. We have $\mathbf{C} = [1 \;\; \pm1]$, and hence

$$\mathbf{Z}' = \begin{bmatrix} 1 & \pm1 \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix} \begin{bmatrix} 1 \\ \pm1 \end{bmatrix} = \begin{bmatrix} Z_{11} + Z_{22} \pm 2Z_{12} \end{bmatrix}. \tag{10}$$

This stays in agreement with the well-known formula for impedance of two magnetically coupled coils.

Parallel connection

Consider the case in which the components of each group are connected in parallel (Fig. 6). The total current of group k equals the sum of currents in particular two-terminals belonging to the group. The currents of particular components should be taken with sign plus for regular connection or minus for reverse connection (see Fig. 7).

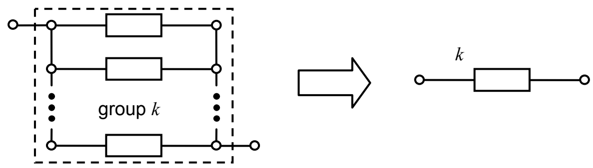


Fig.6. Group k connected in parallel

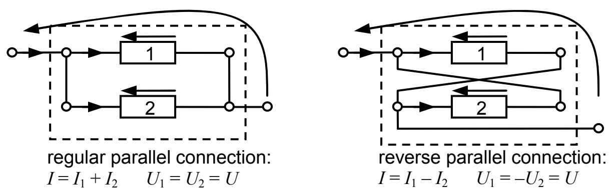


Fig.7. Regular and reverse parallel connection

Observe that the following relationships hold in this case:

$$\mathbf{I}' = \mathbf{C}\mathbf{I}, \qquad \mathbf{U} = \mathbf{C}^{\mathrm{T}}\mathbf{U}' \tag{11}$$

so that we obtain $\mathbf{I}' = \mathbf{C}\mathbf{I} = \mathbf{C}\mathbf{Y}\mathbf{U} = \mathbf{C}\mathbf{Y}\mathbf{C}^{\mathrm{T}}\mathbf{U}'$, and therefore,

$$\mathbf{Y}' = \mathbf{C}\mathbf{Y}\mathbf{C}^{\mathrm{T}}. \tag{12}$$

Hence, the impedance matrix equals

$$\mathbf{Z}' = \left(\mathbf{C}\mathbf{Z}^{-1}\mathbf{C}^{\mathrm{T}}\right)^{-1}. \tag{13}$$

It should be noted that the admittance matrix of the reduced parallel system is related with the admittance matrix of the full system in the same way as the corresponding impedance matrices for series connections – see Eqs. (8) and (12).

As an example, consider two magnetically coupled coils in parallel. They can be connected in a similar or dissimilar way, i.e. $\mathbf{C} = [1 \ \ \pm 1]$, and therefore,

$$\mathbf{Y}' = \begin{bmatrix} 1 & \pm 1 \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \pm 1 \end{bmatrix} =$$

$$(14) \qquad = \begin{bmatrix} 1 & \pm 1 \end{bmatrix} \frac{1}{Z_{11}Z_{22} - Z_{12}^2} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{12} & Z_{11} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \pm 1 \end{bmatrix} =$$

$$= \left[\frac{Z_{11} + Z_{22} \mp 2Z_{12}}{Z_{11}Z_{22} - Z_{12}^2} \right].$$

This result stays in agreement with the well-known formula for equivalent impedance of two coupled coils in parallel.

Example of application

The considered method uses extensively matrix operations; therefore, larger systems usually require appropriate computer software. To demonstrate the method, we consider below a relatively simple system. It consists of a single phase power busduct with 4 busbars supplying certain load. The busbars are arranged in two buses, each with 2 busbars (Fig. 8). Our goal is to find out the reduced impedance of the busduct, Z_B.

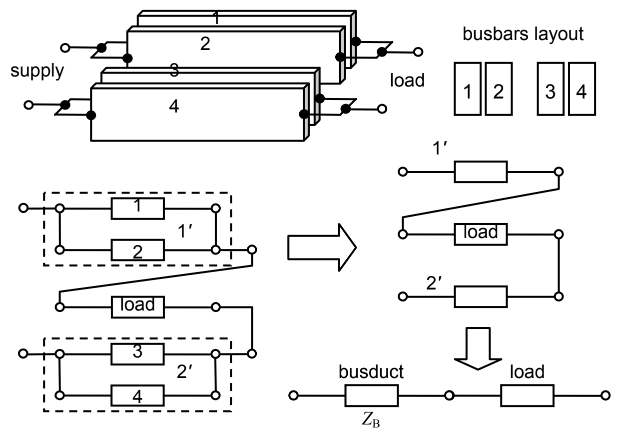


Fig.8. Single phase busduct with two busbars per bus supplying a load and its two-terminal representation (full and reduced)

Let the impedance matrix of the four busbars alone be $\mathbf{Z} = [Z_{ij}]_{4\times4}$. This matrix can be found via various computational methods, e.g. [1,3,9,10]. Matrix $\mathbf{Z}$ is symmetrical, therefore the admittance matrix, $\mathbf{Y} = [Y_{ij}]_{4\times4} = \mathbf{Z}^{-1}$, is also symmetrical. Denote Y_{ij} as $z_{ij}/|\mathbf{Z}|$, where z_{ij} is cofactor of element Z_{ij} in matrix $\mathbf{Z}$. Let busbars 1 and 2 compose group 1 and busbars 3 and 4 – group 2. The busbars are connected in parallel in each group, and the connection is regular. Therefore, the connection matrix has the following form:

$$(15) \qquad \mathbf{C} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

This yields

$$(16) \qquad \mathbf{Y}' = \mathbf{C}\mathbf{Z}^{-1}\mathbf{C}^{\mathrm{T}} = \frac{1}{|\mathbf{Z}|} \begin{bmatrix} y'_{11} & y'_{12} \\ y'_{12} & y'_{22} \end{bmatrix},$$

where

$$(17) \qquad y'_{11} = z_{11} + z_{22} + 2z_{12}, \quad y'_{22} = z_{33} + z_{44} + 2z_{34}, \quad y'_{12} = z_{13} + z_{14} + z_{23} + z_{24}.$$

Then the impedance matrix of the reduced busduct equals

$$(18) \qquad \mathbf{Z}' = \frac{|\mathbf{Z}|}{y'_{11}y'_{22} - y'^2_{12}} \begin{bmatrix} y'_{22} & -y'_{12} \\ -y'_{12} & y'_{11} \end{bmatrix}.$$

We have now the two groups connected in series as a higher order group. This time the connection is reverse, therefore the connection matrix is $\mathbf{C}' = [1 \ \ -1]$, and $\mathbf{Z}'' = \mathbf{C}'\mathbf{Z}'\mathbf{C}'^{\mathrm{T}}$. Matrix $\mathbf{Z}''$ has only one element equal to

$$(19) \qquad Z''_{11} = |\mathbf{Z}| \frac{y'_{11} + y'_{22} + 2y'_{12}}{y'_{11}y'_{22} - y'^2_{12}} = Z_{\mathrm{B}}.$$

This formula simplifies in case of certain symmetry. If we assume symmetrical buses ($Z_{11} = Z_{44}$, $Z_{22} = Z_{33}$, $Z_{34} = Z_{12}$ and $Z_{24} = Z_{13}$), then the busduct impedance will be

$$(20) \qquad Z_{\mathrm{B}} = 2\frac{(Z_{11} - Z_{14})(Z_{22} - Z_{23}) - (Z_{12} - Z_{13})^2}{(Z_{11} - Z_{14}) + (Z_{22} - Z_{23}) - 2(Z_{12} - Z_{13})}.$$

If in addition $Z_{22} = Z_{11}$ and $Z_{23} = Z_{14}$, then

$$(21) \qquad Z_{\mathrm{B}} = Z_{11} + Z_{12} - Z_{13} - Z_{14}.$$

Formula (20) is very similar to the one for two coupled coils in parallel. In fact, the original electric diagram can be reduced to such equivalent parallel impedances via the method of coupling elimination together with symmetry assumptions (see Fig. 9). The original connection is shown briefly in diagram 1. It is redrawn with the load shifted outside the busduct in diagram 2, and a balanced bridge (due to symmetry) is obtained. This fact is used to remove the dashed branch. Then couplings 1-3 are replaced with equivalent negative ones (diagram 3). In diagram 4, couplings 1-4 and 2-3 are eliminated, and a shunt is inserted (dashed line), which does not affect current flow due to symmetry. We have then two identical segments, each of them being a parallel connection of coupled impedances $Z_{11} - Z_{14}$ and $Z_{22} - Z_{23}$ with mutual impedance $Z_{12} - Z_{13}$. This stays in agreement with Eq. (20).

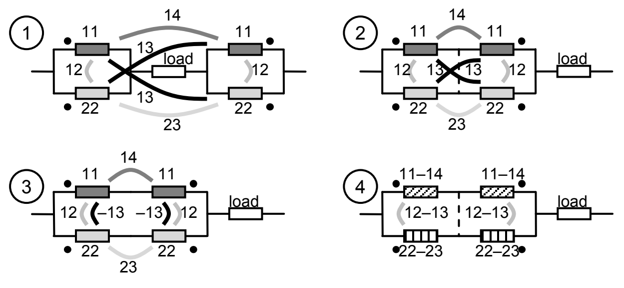


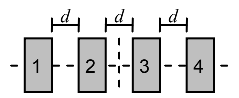
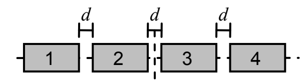
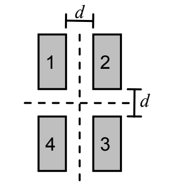
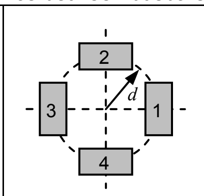
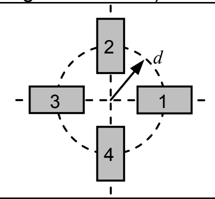
Fig.9. Reducing the considered busbar system via coupling elimination in case of symmetrical buses

Eq. (20) allows us determining how to connect the busbars (within given geometry) to obtain the impedance module $|Z_{\mathrm{B}}|$ as low as possible. Mutual impedances usually are nearly inductive; therefore,

$$(22) \qquad Z_{12} \approx \mathrm{j}X_{12}, \ Z_{13} \approx \mathrm{j}X_{13}, \ Z_{12} - Z_{13} \approx \mathrm{j}(X_{12} - X_{13}).$$

If coupling 1-2 (and the same 3-4) is stronger than coupling 1-3 (and the same 2-4), the equivalent coupling 1-2 and 1-3 (impedance $Z_{12} - Z_{13}$) is positive, otherwise it is negative. Since two coupled coils in parallel have lower impedance for negative coupling, we expect that $|Z_B|$ would be lower if $X_{13} > X_{12}$, i.e. if the coupling between the busbars belonging to one bus is weaker than the coupling between busbars from different buses. This result is confirmed directly via field calculations performed with FEMM software [11]. The first row of Table 1 shows some exemplary configurations of 4 busbars of rectangular cross sections. The busbars can be arranged in pairs 12-34, 13-24 and 14-23. The full impedance matrix was calculated with FEMM. The reduced impedance matrix was calculated both with FEMM as well as via system reduction, and full agreement was achieved. Small discrepancies appear in further significant figures (not given in Table 1) due to numerical errors introduced by the finite elements method used in FEMM software. We assumed the busbars are made of copper (conductivity 58 MS/m) and the cross section dimensions are 4 cm × 1 cm. They are placed in air and frequency is assumed to be 50 Hz. Distance between the busbars, d, is given in Table 1 for each configuration.

Table 1. Impedances of selected configurations of 4 identical busbars (each busbar is made of copper and has a rectangular cross-section 4 cm × 1 cm; impedances are calculated for a frequency of 50 Hz and expressed in μΩ/m; distance between busbars is given in row 2)

Configuration					
d	1 cm	1 cm	1 cm	4 cm	4 cm
The impedance matrix of 4 busbars calculated with FEMM					
$Z_{11} = Z_{44}$	47.33 + j398.6	50.72 + j395.7	51.61 + j394.2	48.18 + j397.1	49.04 + j396.2
$Z_{22} = Z_{33}$	48.37 + j398.3	53.79 + j393.1	51.61 + j394.2	48.18 + j397.1	49.04 + j396.2
$Z_{12} = Z_{34}$	2.70 + j350.1	1.57 + j308.7	6.33 + j345.8	0.18 + j298.7	2.00 + j296.8
$Z_{13} = Z_{24}$	0.59 + j315.7	–2.76 + j265.6	–1.33 + j303.9	–1.93 + j277.4	1.54 + j276.4
Z_{14}	–0.51 + j292.8	–4.00 + j240.1	–1.14 + j310.3	0.18 + j298.7	2.00 + j296.8
Z_{23}	3.11 + j350.0	2.79 + j307.8	–1.14 + j310.3	0.18 + j298.7	2.00 + j296.8
Impedance Z_B for arrangement 12-34 calculated with FEMM and Eqs. (19), (20), (21)					
$Z_{B12\text{-}34}$ FEMM	57.5 + j101.3	63.2 + j151.4	60.4 + j125.7	50.1 + j119.7	47.5 + j119.8
$Z_{B12\text{-}34}$ Eq.(19)	57.5 + j101.3	63.2 + j151.4	60.4 + j125.7	50.1 + j119.7	47.5 + j119.8
$Z_{B12\text{-}34}$ Eq.(20)	57.5 + j101.3	63.2 + j151.4	60.4 + j125.7	50.1 + j119.7	47.5 + j119.8
$Z_{B12\text{-}34}$ Eq.(21)	not applicable	not applicable	60.4 + j125.7	50.1 + j119.7	47.5 + j119.8
Impedance Z_B for arrangement 13-24 calculated with FEMM and Eqs. (19), (20), (21)					
$Z_{B13\text{-}24}$ FEMM	46.6 + j36.2	50.2 + j70.3	45.1 + j42.0	45.9 + j77.2	46.6 + j79.0
$Z_{B13\text{-}24}$ Eq.(19)	46.6 + j36.2	50.2 + j70.3	45.1 + j42.0	45.9 + j77.2	46.6 + j79.0
$Z_{B13\text{-}24}$ Eq.(20)	46.6 + j36.2	50.2 + j70.3	45.1 + j42.0	45.9 + j77.2	46.6 + j79.0
$Z_{B13\text{-}24}$ Eq.(21)	not applicable	not applicable	45.1 + j42.0	45.9 + j77.1	46.6 + j79.0
Impedance Z_B for arrangement 14-23 calculated with FEMM and Eqs. (19), (20), (21)					
$Z_{B14\text{-}23}$ FEMM	45.9 + j54.0	52.8 + j94.1	45.5 + j54.7	50.1 + j119.7	47.5 + j119.8
$Z_{B14\text{-}23}$ Eq.(19)	45.9 + j54.0	52.8 + j94.1	45.5 + j54.7	50.1 + j119.7	47.5 + j119.8
$Z_{B14\text{-}23}$ Eq.(20)	not applicable	not applicable	45.5 + j54.7	50.1 + j119.7	47.5 + j119.8
$Z_{B14\text{-}23}$ Eq.(21)	not applicable	not applicable	45.5 + j54.7	50.1 + j119.7	47.5 + j119.8

Conclusions

A systematic method of obtaining impedance matrices of reduced systems was described. The method uses matrix operations; therefore, it works best on numerical values and with computer software. It is helpful in finding the impedance matrices for various arrangements of busbars. It can be used in optimization of certain features of impedance matrices for a given geometry. It is also used implicitly in certain integral methods for numerical determination of the impedance matrix, e.g. [1].

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