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Fractional models of selected combustion engine ignition systems

Abstract. *This paper attempts to introduce a quasi-inductive element into ignition system and describes it by fractional order equation. Two typical systems have been studied and numerical analysis has been conducted.*

Streszczenie. *W pracy podjęto próbę wprowadzenia elementów quasi-indukcyjnych, aby opisać równaniem ułamkowego rzędu układy zapłonowe. Dwa typowe systemy zostały zbadane, po czym przeprowadzono dla nich analizę numeryczną. (**Ułamkowe modele wybranych układów zapłonowych silników spalinowych**)*

Keywords: ignition system, fractional order derivatives, transient states.
Słowa kluczowe: układ zapłonowy, pochodne ułamkowego rządu, stany przejściowe.

Introduction

Ignition systems of modern vehicles are modeled by electrical circuits whose mathematical description is given by nonlinear equations [1-3]. Studies on the dynamics of ignition systems are hard and results of analysis and digital simulation differ from the experimental ones. In our paper an attempt has been made to introduce quasi-inductive element L^{α} (described by the equation of fractional order α) into a model of the ignition system. Ignition systems are magnetically coupled primary and secondary circuits. The object of our research is an electrical circuit modeling the primary side of the ignition system. The paper attempts to answer the question whether it is possible to model nonlinearity and losses in actual systems by an induction element of fractional order.

Fractional models of ignition systems

Generally, ignition systems can be represented as systems with energy storage in inductance and the ones with energy storage in capacitance [4, 5, 9, 11]. Replacing the inductive element (ignition coil) with L^{α} element we obtain two models shown in Figure 1.

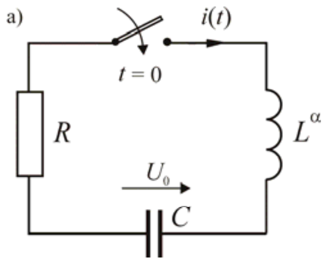

Fig. 1.a) model of a system with energy stored in capacitance

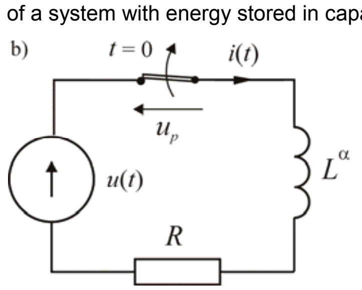

Fig. 1.b) model of a system with energy stored in inductance

Analysis of transient state and digital simulations

Two systems presented in Fig.1 are analyzed. System 1a can be written as:

(1) $$u(t) + Ri(t) + L\,{}_0^C D_t^{\alpha} i(t) = 0; \quad u(t) = \frac{1}{C}\int i(t)dt + U_0$$

where: $u(t)$ capacitor voltage, ${}_0^C D_t^{\alpha} f(t)$ - derivative of non-integer order α, according to Caputo definition [4, 5, 7].

Reducing the set into one equation with respect to $i(t)$:

$$\frac{1}{C}\int i(t)dt + U_0 + Ri(t) + L\,{}_0^C D_t^{\alpha} i(t) = 0$$

and using Laplace transformation [4]:

$$\frac{1}{sC} I(s) + \frac{U_0}{s} + RI(s) + s^{\alpha} LI(s) = 0$$

we obtained current transform:

(2) $$I(s) = -CU_0 / [Cs(Ls^{\alpha} + R) + 1] \, .$$

To determine inverse transform a continued fraction expansion (CFE) metod [6, 8,11,12] was applied. Accordlingly, for fifth-order approximation (5A):

(3) $$s^{\alpha} = \frac{N(\alpha)}{D(\alpha)} = \frac{P_{50}s^5 + P_{51}s^4 + P_{52}s^3 + P_{53}s^2 + P_{54}s + P_{55}}{Q_{50}s^5 + Q_{51}s^4 + Q_{52}s^3 + Q_{53}s^2 + Q_{54}s + Q_{55}}$$

where:

$$P_{50} = Q_{55} = -a^5 - 15a^4 - 85a^3 - 225a^2 - 274a - 120$$
$$P_{55} = Q_{50} = a^5 - 15a^4 + 85a^3 - 225a^2 + 274a - 120$$
$$P_{51} = Q_{54} = 5a^5 + 45a^4 + 5a^3 - 1005a^2 - 3250a - 3000$$
$$P_{54} = Q_{51} = -5a^5 + 45a^4 - 5a^3 - 1005a^2 + 3250a - 3000$$
$$P_{52} = Q_{53} = -10a^5 - 30a^4 + 410a^3 + 1230a^2 - 4000a - 12000$$
$$P_{52} = Q_{53} = 10a^5 - 30a^4 - 410a^3 + 1230a^2 + 4000a - 12000$$

we obtained current series for α = 0,9; 0,8; 0,5 and compared them to the ones in classical circuit RLC.

The system presented in Fig. 1b models transient state for a switch-off state of RL^{α} circuit. As it is well known there is no classical solution in this case as commutation laws are not satisfied. For an ideal open switch and time t>0 we got the equation:

(4) $$L\,{}_0^C D_t^{\alpha} i(t) + u_p(t) = U$$

where: $u_p(t)$ is a voltage on an open switch.
Current equals step function: $i(t) = -U \cdot \mathbf{1}(t)/R$ hence, equation (4) for $0 \le \alpha \le 1$ takes the form:

$$-Ls^{\alpha}\frac{U}{sR} + Ls^{\alpha-1}i(0) + Ls^{\alpha-2}i^{(1)}(0) + U_p(s) = \frac{U}{s} \tag{5}$$

where: $i(0^+) = 0$ and

$$i^{(1)}(0^+) = \left(-\frac{U}{R}\right)\frac{d}{dt}\mathbf{1}(0^+) = \left(-\frac{U}{R}\right)\delta(0^+) = 0 \quad [8].$$

so voltage transform on switch contact is:

$$U_p(s) = \frac{U}{s} - s^{\alpha}\cdot\frac{U \cdot L}{s \cdot R} \tag{6}$$

By determining the inverse transform as described above voltage series shown in Fig.3 were obtained.

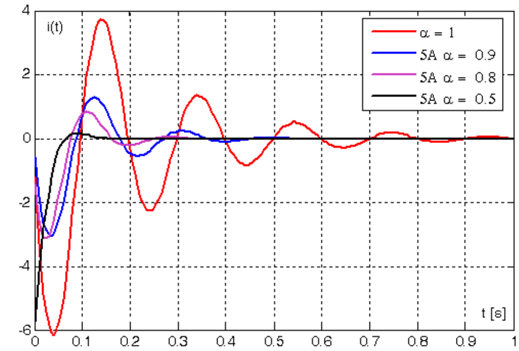

Fig.2. Current series for RCL^{α} for a switch-on state of a circuit.

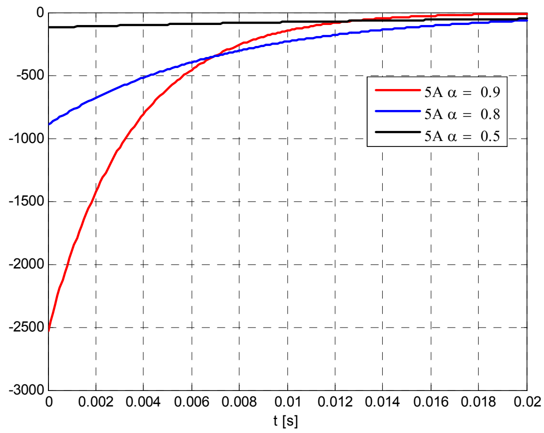

Fig.3. Voltage series for derivatives α = 0.9 , 0.8 , 0.5.

Model of the studied ignition system

For the experiments the model of the ignition system presented in Figure 4 was used.

The equation of state in a classical form can be written as follows:

$$\begin{bmatrix} \dfrac{di_L(t)}{dt} \\ \dfrac{du_C(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_1}{L} & \dfrac{1}{L} \\ -\dfrac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_L(t) \\ u_C(t) \end{bmatrix} \tag{7}$$

The solution of the equation will be given with regard to the results of the equation of non-integer order.

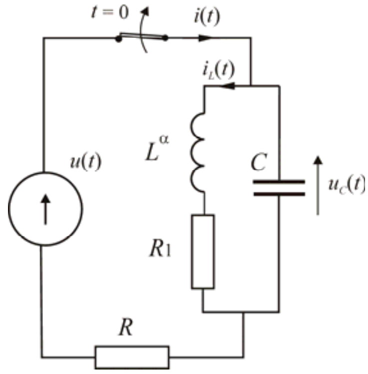

Fig.4. A model of the studied ignition system (E = 12 V, R = 2.8 Ω, R_1 = 0.56 Ω, L = 0.01 H, C = 0.25 μF).

Substituting the derivative of the first order in equation (7) by the derivative of non-integral order α that fulfils inequity $0 < \alpha < 1$ we obtain the equation of state (8).

$$\begin{bmatrix} {}_0^C D_t^{\alpha} i_L(t) \\ \dfrac{du_C(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_1}{L} & \dfrac{1}{L} \\ -\dfrac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_L(t) \\ u_C(t) \end{bmatrix} \tag{8}$$

After the Laplace transform with given initial conditions:

$$i_L(0) = \frac{E}{R+R_1} = I_0, \quad u_C(0) = \frac{ER_1}{R+R_1} = U_0, \tag{9}$$

we obtain:

$$\begin{bmatrix} s^{\alpha}I_L(s) - s^{\alpha-1}I_0 \\ sU_C(s) - U_0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{bmatrix} \begin{bmatrix} I_L(s) \\ U_C(s) \end{bmatrix} \tag{10}$$

where: $A_{11} = -\frac{R_1}{L}$ $A_{12} = \frac{1}{L}$ $A_{21} = -\frac{1}{C}$.

Then solving the equation (10) with regard to transforms of current in the coil and capacitor voltage we get:

$$\mathrm{I}_L(s) = \frac{\left(s^{\alpha}CLI_0 + CU_0\right)}{sCR_1 + s^{1+\alpha}CL + 1} \tag{11}$$

$$\mathrm{U}_C(s) = \frac{s^{\alpha}CLU_0 - s^{\alpha-1}LI_0 - CR_1U_0}{sCR_1 + s^{1+\alpha}CL + 1} \tag{12}$$

To determine the reverse transform the CFE method was used for the fifth-order approximation which allowed us to approximate s^{α} factor with the quotient of polynomials of integer degrees [7, 9].
Finally the transforms of current in the coil and capacitor voltage assumed the form:

$$\mathrm{I}_L(s) = \frac{(N(\alpha)CLI_0 + CU_0D(\alpha))}{sCR_1D(\alpha) + sN(\alpha)CL + D(\alpha)} \tag{13}$$

$$\mathrm{U}_C(s) = \frac{sN(\alpha)CLU_0 - N(\alpha)LI_0 - sD(\alpha)CR_1U_0}{s(sCR_1D(\alpha) + sN(\alpha)CL + D(\alpha))} \tag{14}$$

In both cases the poles were single which made it easier to determine the reverse transforms and obtain time series of current in a coil and time series of capacitor voltage.

Fig. 5 and Fig. 6 present the comparison of the results obtained for the derivatives of order α= 0.9999 and α= 0.99 with classical ones (Fig. 7 and 8).

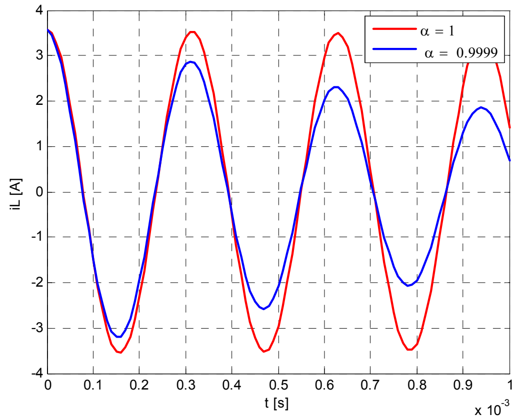

Fig. 5. Current series for the order α = 1 and α = 0.9999.

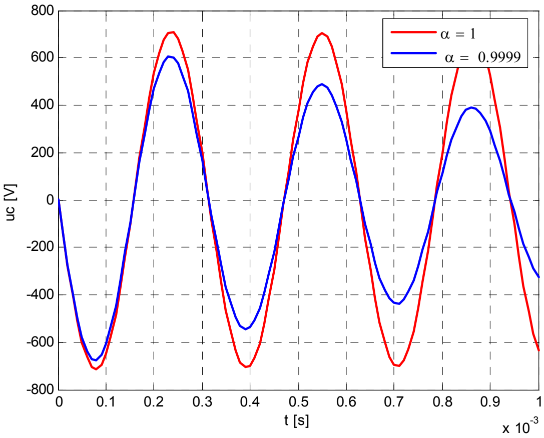

Fig. 6. Capacitor voltage series for the order α = 1 and α= 0.9999.

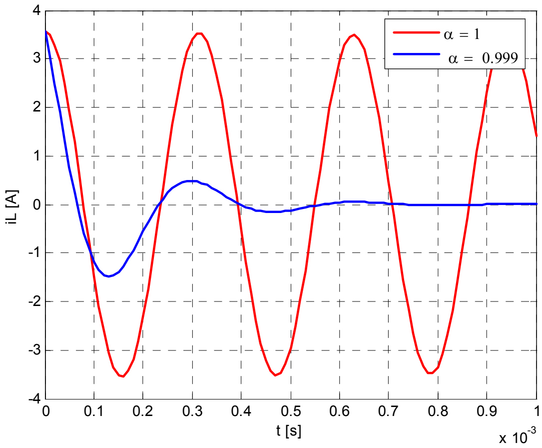

Fig. 7. Current series for the order α = 1 and α = 0.999.

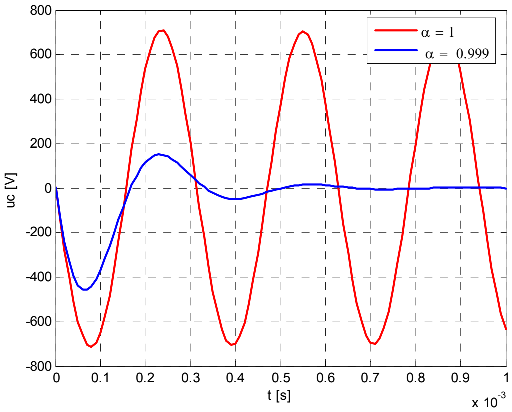

Fig. 8. Capacitor voltage series for the order α=1 and α=0.999

As it is seen in the figures, the system is very sensitive to the first order of the equation (8) – the reduction of order by 0.001 results in significant attenuation of oscillation of both the current and capacitor voltage.

The results of physical experiment shown in Fig. 10. (blue line) were observed for Opel Astra ignition system on a specially constructed test presented in Fig. 9. Time series of current and voltage were obtained by means of digital oscilloscope (Tectrinic type: DPO 4104).

Fig.9. Measurement set.

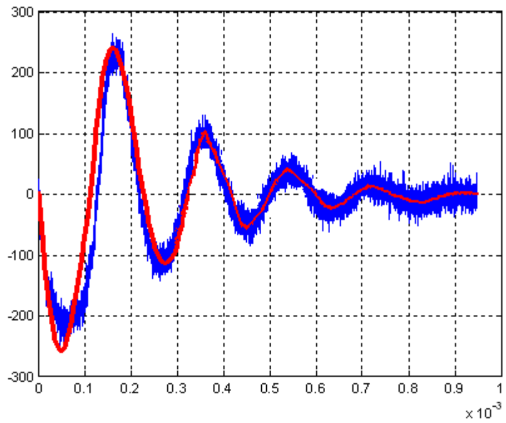

Fig. 10. Solution of the fractional system of order α = 0.993.

Analyzing the results obtained in the physical experiment and the results of numerical simulation for different fractional orders, it was found that the numerical solution for the order α = 0,993 best matches the experimental results. The results of numerical simulation for integer order system (classical solution) significantly diverge from the measurements.

Conclusion

The analysis shows that fractional systems can be used to model real physical processes occurring in the ignition systems of combustion engines with spark ignition.

Conventional (classical) approach to the analysis of electrical systems and their modeling usually ignores the effects resulting from imperfections of elements (losses, non-linearity). Such an approach does not always allow one to obtain reliable models and sometimes causes design errors. The use of derivatives of fractional order makes it possible to compensate for the omitted phenomena.

The application of CFE method to determine the inverse transform, and thus solve differential equation of fractional order appears to be very useful.

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REFERENCES

[1] Wang, I.Q. et al.: Circuit model and parasitic parameter extraction of the spark plug in the ignition system, In: *Turk J Elec Eng & Comp Sci*, Vol. 20, No.5, 2012
[2] Soldera, F.A., Mucklich, F.T., Hrastnik, Kaiser, K., T.: Description of the discharge process in spark plugs and its correlation with the electrode erosion patterns, In: *IEEE Transactions on Vehicular Technology*, Vol. 53, pp. 1257-1265, 2004
[3] Różowicz S., Analysis of the impact battery state of charge as a parameter on the primary side of the ignition system. *Przegląd Elektrotechniczny*, no.12, pp 72-75 (2014)
[4] Ezekoye D., Hall M., Matthews R., Railplug Ignition System for Enhanced Engine Performance and Reduced Maintenance, USA, 2005.
[5] Kaczorek T., Selected problems of fractional systems theory,. *Springer-Verlag,* Berlin-Heidelberg, (2011)
[6] Włodarczyk M., Zawadzki A., Connecting a Capacitor to Direct Voltage in Aspect of Fractional Degree Derivatives. *Przegląd Elektrotechniczny*, R. 85 NR 10/2009, str.: 120-122
[7] Krishna B.T., Studies on fractional order differentiators and integrators: A survey, *Signal Processing* 91, 2011, pp. 386–426,.
[8] Kaczorek T., Fractional positive linear system and electrical circuits. *Przegląd Elektrotechniczny*, no. 9, (2008), pp.135-141
[9] Krishna B.T., Reddy K.V.V.S., Active and passive realization of fractance device of order ½. *Journal of Activeand Passive Electronic Components* 5, (2008) doi: 10.1155/2008/369421
[10] Zawadzki A., Applying derivatives of the fractional order for modeling transient states in electrical circuits containing inductance, *Przegląd Elektrotechniczny*, no. 4, (2013), pp.92-94
[11] Sugi M., Hirano Y., Miura Y. F., and Saito K., Simulation of fractal immittance by analog circuits: An approach to the optimized circuits. *IEICE Trans. Fundam. Electron. Commun. Comput. Sci.*, vol. E82, (1999) pp. 1627–1634
[12] Maundy B., Elwakil A. S., and Gift S., On a multivibrator that employs a fractional capacitor. J. Analog Integr. *Circuits Signal Process.*, vol. 62, (2010), pp. 99–103

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