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Determination of the forces affecting the particles and their trajectories in the surroundings of the matrix element in a magnetic separator

***Abstract**. In the paper, the forces and trajectories of paramagnetic and ferromagnetic particles moving in the surroundings of the ferromagnetic capture element of the matrix have been determined. The influence of the flow speed of the medium, the smooth section of the collector, the value of magnetic flux density and the properties of particles on the width of the collector's particle capture zone have been analysed.*

***Streszczenie.** W pracy zostały wyznaczone siły i trajektorie paramagnetycznych oraz ferromagnetycznych cząstek poruszających się w otoczeniu ferromagnetycznego elementu wychwytującego matrycy. Analizowano wpływ prędkości przepływu medium, smukłości przekroju kolektora, wartość indukcji magnetycznej i właściwości cząstek na szerokości strefy wychwytywania cząstek. (**Wyznaczanie sił działających na cząsteczki i ich trajektorii w otoczeniu elementu matrycy separatora magnetycznego**).*

Keywords: magnetic separation, magnetic force, dynamic resistance force, simulation.
Słowa kluczowe: separacja magnetyczna, siła magnetyczna, siła oporu dynamicznego, symulacja.

Introduction

The efficiency of the separation process can be increased by enlarging the magnetic force, prolonging the duration of the action of this force on a particle and minimising the dynamic resistance force of the medium. In the first case, this amounts to producing a field with higher values of magnetic induction or the gradient of field strength in the working space of the separator. In matrix separators, the nonhomogeneity of the magnetic field is obtained by using matrices in the form of grates, grids and sievs made of flat bars forming a coil, or a large number of fine fibers of irregular shape, depending on the separation process [1, 2, 3]. To reach a relatively high value of magnetic force influencing the particle at a constant value of magnetic flux density, the defined ratio of the collector radius and the particle can be preserved. In the case of an elliptic cylinder the value of the magnetic force depends on the ratio of its semi-axis [4]. The consideration is based on the analysis of three forces: magnetic, gravitational and hydrodynamic (dynamic resistance of the medium). The simplifying assumption is that a separated particle is spherical.

Magnetic force

The force of a heterogeneous magnetic field acting on the particle should be a dominant physical quantity in the process of phase separation. It is a necessary condition of obtaining a good efficiency of the separation process. The magnetic force can be written as [5, 6]:

$$\mathbf{F}_m = (\mathbf{M}\cdot\nabla)\mathbf{B} \tag{1}$$

where: **B** - vector of the magnetic induction, **M** - vector of the effective magnetic torque of the dipole

If the medium is homogeneous, isotropic and linear, and the particle is spherical, its magnetic moment takes the form [6]:

$$\mathbf{M} = V_c \cdot \mathbf{H}\frac{\mu_c - \mu_1}{\mu_1 + D(\mu_c - \mu_1)} \tag{2}$$

where: **H** - vector of the magnetic field intensity, V_c - volume of the particle, μ_c, μ_1 - magnetic permeability of the particle and medium, D - the particle's coefficient of demagnetisation along the axis consistent with the direction of magnetic field intensity.

On the basis of equations (1) and (2) we obtain

$$\mathbf{F}_m = V_c\frac{\mu_c - \mu_1}{\mu_1 + D(\mu_c - \mu_1)}(\mathbf{H}\cdot\nabla)\mathbf{B} \tag{3}$$

We can say that the magnetic force acting on a particular particle depends on the magnetic properties of the particles and the surrounding medium, its volume and position relative to the collector, and the value of the magnetic field and its heterogeneity in the area where the particle is located. The magnetic permeability of the ferromagnetic particle is a function of magnetic field strength inside the particle μ_c, whereas it can be shown that:

$$H_{wc} = \frac{\mu_1 H_o}{\mu_1 + D(\mu_c - \mu_1)} \tag{4}$$

where: H_{wc} – the magnetic field inside the particle, H_o – intensity of the external magnetic field.

However, in the case of a paramagnetic particle, its relative permeability is constant.

To determine the relation describing the magnetic force acting on the particle which is close to the matrix element, formula (3) can be used, wherein the nonhomogeneity of the magnetic field must be determined at the point where the particle is located at a given time.

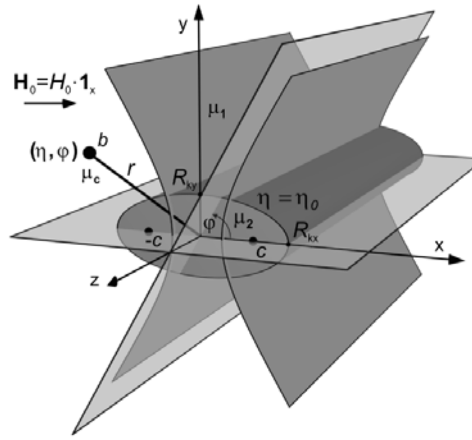

Fig.1. Elliptic cross-section collector and the particle placed in a homogeneous magnetic field

The magnetic field produced by the separator is irrotational and potential. In this case the scalar magnetic potential satisfies Laplace's equation. The ferromagnetic element (collector) considered is in the form of an infinitely long elliptic cylinder with its axis in perpendicular plane to the vector of the homogeneous flux density. The issue has been regarded as two-dimensional in the elliptic-cyllindrical system of coordinates (Fig.1).

Assuming that the axis of the collector covers the axis z, and the magnetic potential V_m is the function of coordinates η and φ, then Laplace's equation is expressed by the following relation:

$$\nabla^2 V_m = \frac{1}{c^2(\mathrm{ch}^2\eta - \cos^2\varphi)}\left(\frac{\partial^2 V_m}{\partial \eta^2} + \frac{\partial^2 V_m}{\partial \varphi^2}\right) = 0 \tag{5}$$

According to its solutions and calculating the magnetic field intensity as:

$$\mathbf{H} = -\nabla V_m = -\frac{\mathbf{1}_\eta \dfrac{\partial V_m}{\partial \eta} + \mathbf{1}_\varphi \dfrac{\partial V_m}{\partial \varphi}}{c \cdot \sqrt{(\mathrm{ch}^2\eta - \cos^2\varphi)}}, \tag{6}$$

after some manipulation we obtain the expression for the components of that field:

$$\begin{aligned} H_\eta &= \frac{-1}{c\sqrt{\mathrm{ch}^2\eta - \cos^2\varphi}}(\mathrm{C}_1\mathrm{ch}\eta + \mathrm{D}_1\mathrm{sh}\eta)\cos\varphi \\ H_\varphi &= \frac{1}{c\sqrt{\mathrm{ch}^2\eta - \cos^2\varphi}}(\mathrm{C}_1\mathrm{sh}\eta + \mathrm{D}_1\mathrm{ch}\eta)\sin\varphi \end{aligned} \tag{7}$$

where: $C_1=-cH_o K$, $D_1=cH_o(K-1)$,

$$\mathrm{K} = \frac{R_{ky}}{R_{kx}}\left(\frac{\mu_2}{\mu_1} - 1\right)\cdot\left(1 + \frac{R_{ky}}{R_{kx}}\cdot\frac{\mu_2}{\mu_1}\right)^{-1}\cdot\left(1 - \frac{R_{ky}}{R_{kx}}\right)^{-1}$$

The parameter K describes the effect of the magnetic properties of the matrix element and the shape of its cross-section on the value of the magnetic force. The magnetic permeability of the collector $\mu_2=\mu_2(H_k)$ is a function of the intensity of the magnetic field inside the collector, which is uniform and has the form:

$$H_k = H_o \frac{\mathrm{sh}\eta_o + \mathrm{ch}\eta_o}{\mathrm{ch}\eta_o + \dfrac{\mu_2}{\mu_1}\mathrm{sh}\eta_o} \tag{8}$$

where: H_k - intensity of the magnetic field inside the collector, η_o– coordinate value of η on the surface of an elliptical cylinder.

The semi-axes of the elliptical cylinder's cross-section determine the coordinate value of η_o on the cylinder's surface as follows:

$$\eta = \eta_o = 0.5\ln\frac{R_{kx} + R_{ky}}{R_{kx} - R_{ky}}, \qquad R_{kx} > R_{ky} \tag{9}$$

Thus, from (3) and (7) the magnetic force can be determined. That force, related to the volume unit, represents the product of two terms:

$$\frac{\mathbf{F}_m}{V_c} = \mathbf{f}_m = f_{m1}\cdot\mathbf{f}_{m2} \tag{10}$$

where:

$$f_{m1} = \frac{\mu_c - \mu_1}{\mu_1 + D(\mu_c - \mu_1)} \tag{11}$$

In the case of paramagnetic particles we have the inequality:

$$D(\mu_c - \mu_1) << \mu_1 \tag{12}$$

and factor f_{m1} in (11) simplifies to the form:

$$f_{m1} = \frac{\mu_c - \mu_1}{\mu_1} \tag{13}$$

Assuming that μ_1 is constant, the term $\mathbf{f}_{m2}$ depends on the components of the magnetic field strength and their derivatives with respect to the coordinates, and can be written as:

$$\begin{aligned} \mathbf{f}_{m2} = (\mathbf{H}\cdot\nabla)\mathbf{B} = \frac{1}{c\sqrt{\mathrm{ch}^2\eta - \cos^2\varphi}}\Bigg[\Bigg(&\mu_1 H_\eta \frac{\partial}{\partial\eta}H_\eta + \\ &+ \mu_1 H_\varphi \frac{\partial}{\partial\varphi}H_\eta\Bigg)\mathbf{1}_\eta + \left(\mu_1 H_\eta \frac{\partial}{\partial\eta}H_\eta + \right. \\ &\left. + \mu_1 H_\varphi \frac{\partial}{\partial\varphi}H_\varphi\right)\mathbf{1}_\varphi\Bigg] \end{aligned} \tag{14}$$

To express the magnetic force in the Cartesian coordinate system, it should be transformed according to the formula:

$$\begin{aligned} W_x &= \frac{\mathrm{sh}\eta\cdot\cos\varphi}{\sqrt{(\mathrm{ch}^2\eta - \cos^2\varphi)}}W_\eta - \frac{\mathrm{ch}\eta\cdot\sin\varphi}{\sqrt{(\mathrm{ch}^2\eta - \cos^2\varphi)}}W_\varphi \\ W_y &= \frac{\mathrm{ch}\eta\cdot\sin\varphi}{\sqrt{(\mathrm{ch}^2\eta - \cos^2\varphi)}}W_\eta + \frac{\mathrm{sh}\eta\cdot\cos\varphi}{\sqrt{(\mathrm{ch}^2\eta - \cos^2\varphi)}}W_\varphi \end{aligned} \tag{15}$$

After adequate manipulation, we obtain the expression for the magnetic force components in the relation to the volume unit:

$$\begin{aligned} f_{mx} = f_{m1}\cdot\mu_1\frac{1}{2}\frac{H_o^2}{c}\Bigg\{\Bigg[&\frac{\mathrm{sh}2\eta - 2\mathrm{e}^{-2\eta}\mathrm{K}(\mathrm{K}-1)}{(\mathrm{ch}^2\eta - \cos^2\varphi)^2} + \\ &+ \frac{\frac{1}{2}\left[\frac{1}{2}\mathrm{sh}2\eta - \mathrm{e}^{-2\eta}\mathrm{K}(\mathrm{K}-1)\right]\sin^2 2\varphi}{(\mathrm{ch}^2\eta - \cos^2\varphi)^3} + \\ &- \frac{\left[\mathrm{sh}^2\eta + \mathrm{e}^{-2\eta}\mathrm{K}(\mathrm{K}-1) + \mathrm{K}\right]\mathrm{sh}2\eta\cos^2\varphi}{(\mathrm{ch}^2\eta - \cos^2\varphi)^3}\Bigg]\mathrm{sh}\eta\cos\varphi + \\ &- \Bigg[\frac{(1-2\mathrm{K})\sin2\varphi}{(\mathrm{ch}^2\eta - \cos^2\varphi)^2} + \\ &+ \frac{\frac{1}{2}\left[\frac{1}{2}\mathrm{sh}2\eta - \mathrm{e}^{-2\eta}\mathrm{K}(\mathrm{K}-1)\right]\mathrm{sh}2\eta\sin2\varphi}{(\mathrm{ch}^2\eta - \cos^2\varphi)^3} + \\ &- \frac{\left[\mathrm{ch}^2\eta + \mathrm{e}^{-2\eta}\mathrm{K}(\mathrm{K}-1) - \mathrm{K}\right]\sin2\varphi\sin^2\varphi}{(\mathrm{ch}^2\eta - \cos^2\varphi)^3}\Bigg]\mathrm{ch}\eta\sin\varphi\Bigg\} \end{aligned} \tag{16}$$

$$f_{\mathrm{my}} = f_{\mathrm{m1}} \cdot \mu_1 \frac{1}{2} \frac{H_{\mathrm{o}}^2}{c} \left\{ \left[\frac{\mathrm{sh}2\eta - 2\mathrm{e}^{-2\eta}\mathrm{K}(\mathrm{K}-1)}{(\mathrm{ch}^2\eta - \cos^2\varphi)^2} + \right. \right.$$

$$+ \frac{\frac{1}{2}\left[\frac{1}{2}\mathrm{sh}2\eta - \mathrm{e}^{-2\eta}\mathrm{K}(\mathrm{K}-1)\right]\sin^2 2\varphi}{(\mathrm{ch}^2\eta - \cos^2\varphi)^3} +$$

$$\left. - \frac{\left[\mathrm{sh}^2\eta + \mathrm{e}^{-2\eta}\mathrm{K}(\mathrm{K}-1) + \mathrm{K}\right]\mathrm{sh}2\eta\cos^2\varphi}{(\mathrm{ch}^2\eta - \cos^2\varphi)^3} \right] \mathrm{ch}\eta\sin\varphi +$$

(17)

$$+ \left[\frac{(1-2\mathrm{K})\sin 2\varphi}{(\mathrm{ch}^2\eta - \cos^2\varphi)^2} + \right.$$

$$+ \frac{\frac{1}{2}\left[\frac{1}{2}\mathrm{sh}2\eta - \mathrm{e}^{-2\eta}\mathrm{K}(\mathrm{K}-1)\right]\mathrm{sh}2\eta\sin 2\varphi}{(\mathrm{ch}^2\eta - \cos^2\varphi)^3} +$$

$$\left. \left. - \frac{\left[\mathrm{ch}^2\eta + \mathrm{e}^{-2\eta}\mathrm{K}(\mathrm{K}-1) - \mathrm{K}\right]\sin 2\varphi\sin^2\varphi}{(\mathrm{ch}^2\eta - \cos^2\varphi)^3} \right] \mathrm{sh}\eta\cos\varphi \right\}$$

Field of medium speed and dynamic resistance force

Because the matrix element is placed in a stream of liquid, it affects the medium. Potential flows of the non-viscid and incompressible liquid can be described by the potential of speed Ψ which fulfils Laplace's equation:

(18) $$\nabla^2\Psi = \frac{1}{c^2(\mathrm{ch}^2\eta - \cos^2\varphi)} \left(\frac{\partial^2\Psi}{\partial\eta^2} + \frac{\partial^2\Psi}{\partial\varphi^2} \right) = 0$$

In the case considered the potential of medium speed is the periodical and even the function of φ. The vector of medium speed is described by the gradient of the potential $\boldsymbol{\vartheta} = \nabla\psi$. Since the particles of liquid flow around the matrix element, the vector component of speed which is perpendicular to the collector's surface is equal to zero. In some considerable distance from the matrix element the stream of liquid is homogeneous and its speed is $\boldsymbol{\vartheta} = \vartheta_{\mathrm{o}} \cdot \mathbf{1}_{\mathrm{x}}$. It can be shown that the components of the speed vector outside the matrix element converted to the rectangular coordinates have the form:

$$\vartheta_{\mathrm{x}} = \frac{\vartheta_{\mathrm{o}}\,\mathrm{sh}\eta\cos^2\varphi\left(\mathrm{ch}\eta_{\mathrm{o}}\mathrm{sh}\eta - \mathrm{sh}\eta_{\mathrm{o}}\mathrm{ch}\eta\right)}{\exp(-\eta_{\mathrm{o}})(\mathrm{ch}^2\eta - \cos^2\varphi)} + $$

$$+ \frac{\vartheta_{\mathrm{o}}\,\mathrm{ch}\eta\sin^2\varphi\left(\mathrm{ch}\eta_{\mathrm{o}}\mathrm{ch}\eta - \mathrm{sh}\eta_{\mathrm{o}}\mathrm{sh}\eta\right)}{\exp(-\eta_{\mathrm{o}})(\mathrm{ch}^2\eta - \cos^2\varphi)},$$

(19)

$$\vartheta_{\mathrm{y}} = \frac{\vartheta_{\mathrm{o}}\,\mathrm{ch}\eta\sin 2\varphi\left(\mathrm{ch}\eta_{\mathrm{o}}\mathrm{sh}\eta - \mathrm{sh}\eta_{\mathrm{o}}\mathrm{ch}\eta\right)}{2\exp(-\eta_{\mathrm{o}})(\mathrm{ch}^2\eta - \cos^2\varphi)} - $$

$$- \frac{\vartheta_{\mathrm{o}}\,\mathrm{sh}\eta\sin 2\varphi\left(\mathrm{ch}\eta_{\mathrm{o}}\mathrm{ch}\eta - \mathrm{sh}\eta_{\mathrm{o}}\mathrm{sh}\eta\right)}{2\exp(-\eta_{\mathrm{o}})(\mathrm{ch}^2\eta - \cos^2\varphi)}$$

While considering the movement of small particles in the separation processes the force of dynamic resistance has been described by Stoke's equation:

(20) $$\mathbf{F}_{\mathrm{d}} = 6\ \pi\ \zeta\ b\left(\vartheta - \vartheta_{\mathrm{c}}\right)$$

where: ζ - dynamic coefficient of medium viscosity, $\boldsymbol{\vartheta}$ - vector of medium speed, $\boldsymbol{\vartheta}_{\mathbf{c}}$ - vector of particle speed.

The gravitational force is expressed by the following dependence:

(21) $$\mathbf{F}_{\mathrm{g}} = (\rho_{\mathrm{c}} - \rho_{\mathrm{o}})\,V\,\mathbf{g}$$

where: ρ_{c} - particle density, ρ_{o} - medium's density, **g** - gravitational acceleration.

Equations for particle movement

To determine the trajectory of the particle the following system of equations should be solved:

(22) $$\frac{\mathrm{d}\,\vartheta_{\mathrm{c}}}{\mathrm{d}t} = \frac{\mathbf{F}_{\mathrm{m}} + \mathbf{F}_{\mathrm{g}} + \mathbf{F}_{\mathrm{d}}}{m}, \quad \frac{\mathrm{d}\,\mathbf{s}}{\mathrm{d}t} = \vartheta_{\mathrm{c}}$$

where: **s** - distance covered by a particle; *m* - particle mass.

The Runge-Kutty method of the 4th rank with the automatic selection of the integration step has been used to solve the system of equations numerically. The following input data that characterise the separation process have been assumed: ϑ_{o} = 0.05 m/s, ρ_{c} = 5000 kg/m^3, ρ_{o} = 1000 kg/m^3, radius of particle b = 7.5·10^{-6} m, magnetic susceptibility of paramagnetic particle χ_{c} = 0.007 and the medium χ_{o} = 0, ζ = 0.001 N/m·s, g = 9.81 m/s^2. The magnetic properties of the collector and the ferromagnetic particle are specified by the characteristics of the magnetisation process B=f(H) (Fig.2). Selected results of the calculations are shown in Figures 3,4,5,6.

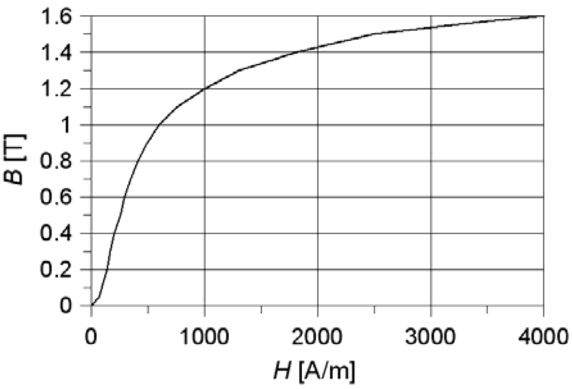

Fig.2. Magnetisation curve of the collector and particles

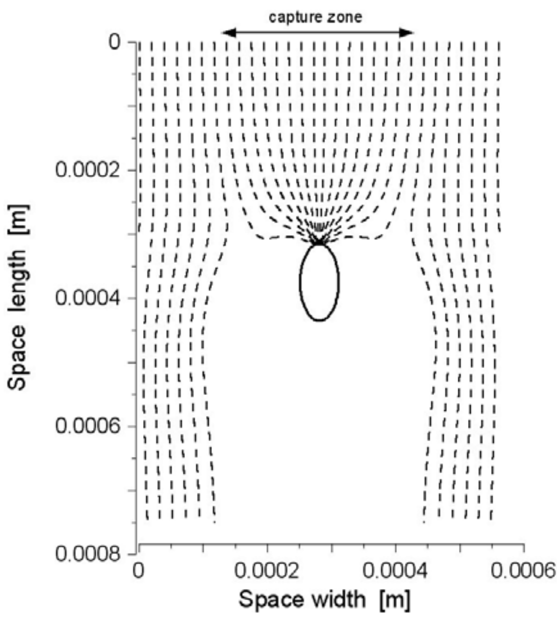

Fig.3. Trajectories of paramagnetic particles at magnetic induction B = 0.8 T, radii of collector's section R_{kx}= 6·10^{-5} m, R_{ky} =3·10^{-5} m

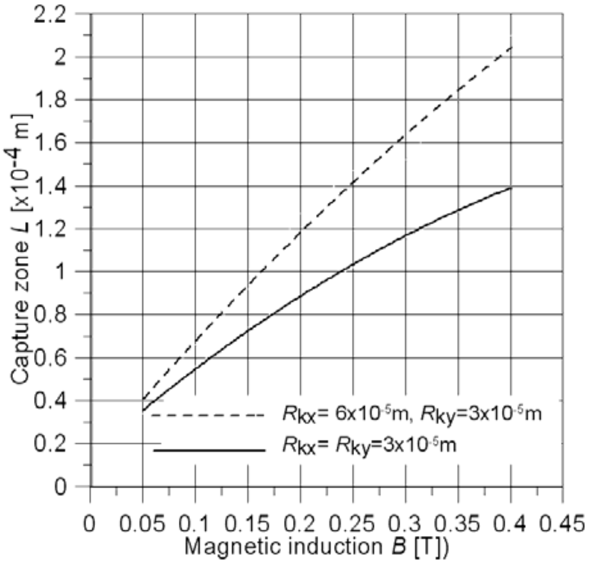

Fig.4. Width of the capture zone of the circular and elliptic collector's fibre versus the magnetic induction for ferromagnetic particles at ϑ_o = 0.05 m/s

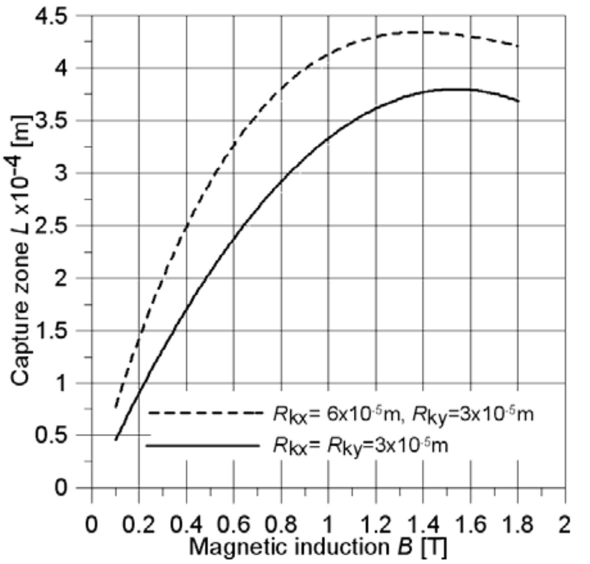

Fig.5. Width of the capture zone of the circular and elliptic collector's fibre versus the magnetic induction for paramagnetic particles at ϑ_o = 0.05 m/s

The width of the capture zone changes linearly along with the increase of the induction while the magnetic permeability of the collector and the particle are constant values. In case of the ferromagnetic particles and elliptic collector (R_{kx}/R_{ky} = 2) analysed it is 5 times higher, and for the circular collector almost four times higher, at induction values ranging from 0.05 T to 0.4 T.

The growth of induction and the saturation of the collector clearly influences the width of the capture zone. At the assumed parameters of paramagnetic particles' separation the width rise of the capture zone is small in the field of induction value over 1.2 T.

The width of the capture zone reaches its maximum value of $3.78 \cdot 10^{-4}$ m at the induction value of 1.5 T for the circular section of the matrix element. Similarly, for the elliptic section collector (R_{kx}/R_{ky} = 2) the maximum width of the capture zone is $4.34 \cdot 10^{-4}$ m at the induction of 1.4 T.

The width of the capture zone decreases with increasing flow velocity. For the paramagnetic particles placed in a magnetic field of 0.5 T, the change in flow speed from 0.01 m/s to 0.1 m/s reduces the width of the capture zone 2.3 times for the circular section collector.

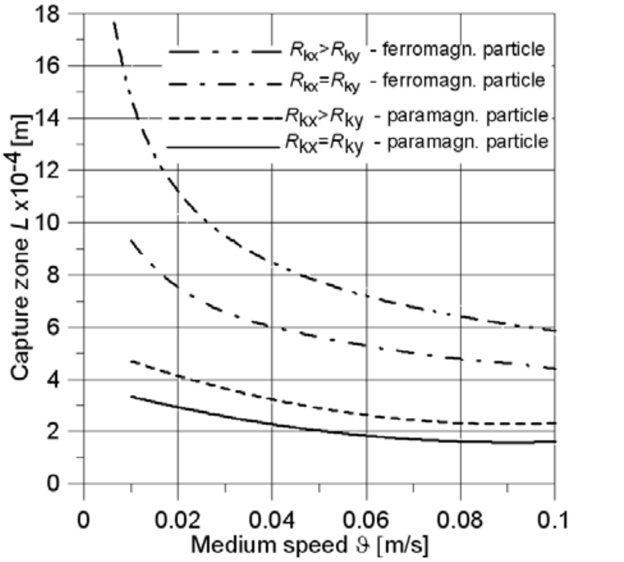

Fig.6. The dependence of the collector's capture zone on the speed of the medium flow carrying paramagnetic particles at the induction value B=0.5 T, and ferromagnetic particles at the induction value B=0.1 T for R_{kx} = R_{ky} = 3x10^{-5}m and R_{kx} = 6x10^{-5}m, R_{ky} = 3x10^{-5}m

It is similar for the collector with an elliptic cross section (R_{kx}/R_{ky} = 2); the capture zone width is reduced in this case over 2.2 times. During the separation of ferromagnetic particles in a magnetic field of 0.1 T and at the assumed change in flow velocity, there is a reduction in the width of the capture zone 2.1 and 2.8 times for a circular section collector and an elliptic section one (R_{kx}/R_{ky} = 2), respectively.

Conclusion

The mathematical model presented in the paper enables us to analyse the magnetic separation process more precisely and assess its efficiency in relation to various parameters, such as: magnetic flux density, velocity of medium flow, dimensions of the semi-axis of the matrix element, and properties of the particles and their surroundings.

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