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Differential Group Delay estimation error resulting from the use of the first order PMD model

Abstract. *The case is considered in which momentary DGD is estimated from waveforms of a signal transmitted in a fiber while a PMD model is used that ignores PMD frequency dependence, a situation which can lead to erroneous estimation. Due to random nature of momentary parameters of PMD it is proposed to evaluate the errors with the use of confidence intervals. With this tool it is shown that the considered simplification of the PMD model can result in errors of unacceptable level.*

Streszczenie. *Rozpatrzono przypadek w którym chwilowa wartość DGD jest estymowana na podstawie przebiegów sygnału przesyłanego w światłowodzie, przy czym użyto modelu PMD, który zaniedbuje zależność PMD od częstotliwości, sytuację, która może prowadzić do błędnej estymacji. Ze względu na losową naturę chwilowych wartości parametrów PMD zaproponowano oceniać błędy z użyciem przedziałów ufności. Przy pomocy tego narzędzia wykazano, że rozważane uproszczenie modelu PMD może skutkować błędami na nieakceptowalnym poziomie.* ***(Błąd estymacji różnicowego opóźnienia grupowego wynikający z użycia modelu PMD pierwszego rzędu)***

Keywords: optical fiber, polarization mode dispersion, monitoring differential group delay.
Słowa kluczowe: światłowód, dyspersja polaryzacyjna, monitorowanie różnicowego opóźnienia grupowego.

Introduction

Modern fiber optic direct detection communication networks are using more and more faster transmission speeds up to the limit at which polarization mode dispersion (PMD) effects cannot be neglected. PMD causes distortion to the signal that carries data, consequence of which can be an inacceptable level of transmission errors. PMD is inherent to optical fibers. It is caused by random birefringence resulting from imperfect circularity of a fiber cross section originated during manufacturing or induced due to mechanical stresses exerted on a fiber in exploitation. Due to randomness of PMD any static characterization of a fiber can rely only on statistical parameters, like PMD coefficient which is a mean (average) of differential group delay (DGD) of a fiber per a kilometer length. An average DGD of a fiber is the PMD coefficient times square root of a fiber length. Temporal behavior of PMD however, can manifest in momentary DGD that can be well below or above the average DGD for a time scale which depends on the environment in which the fiber cable is laid. DGD dynamics expressed in terms of time constants can extend from months, for undersea cables, to seconds and even milliseconds, for aerial cables [1]. In exploitation, knowledge on temporal DGD (in some cases, more generally, all parameters of PMD that describe the PMD behavior sufficiently for a given application) can be of great practical importance, because at moments of intense PMD the transmission channel suffers outage, a situation that should be avoided e.g. through switching a data stream to an another channel. Values of PMD parameters learnt during channel's work provide sufficient acumen for such events could be properly managed. Among many methods for "in-work" DGD monitoring the one that extracts PMD parameter values from transmitted and received electric waveforms can be particularly advantageous for it eliminates the requirement for a dedicated measurement equipment at the cost of the use of signal sampling and computation of signal models, the latter believed to be far less expensive in massive implementation. On the other side, the amount of computation required by this method is it's apparent disadvantage. This aspect may be of utmost importance in practical implementations of the considered concept of monitoring because monitored data should be acquired regularly every relatively short time interval which dictates the maximum time (1 second, at maximum for PMD) for data processing by the implemented algorithm. When PMD parameters are estimated from waveforms the main computation workload results from global optimization being performed for each estimation step in order to inverse a suitably parametrized "forward model" of a PMD affected received signal. The model shall be as simple as possible, in particular shall use minimum parameters, in favor of reducing the optimization work, which in general has O^N complexity for N dimensional parameter space. The simplest description of momentary PMD ignores PMD dependence on frequency. It is the first order PMD model for which PMD effects on a transmitted signal in direct detection system are governed by two parameters: DGD and power split ratio between polarization modes. The model extension that explains PMD dependence on the first power of frequency is the second order PMD. It adds extra two parameters. Higher order PMD require even more. In the context of DGD estimation, staying with the first order PMD seems beneficial in terms of workload however, may be detrimental to estimation accuracy for this simple model imperfectly mimics reality. Then, question arises how large systematic error may result when first order PMD parameters, particularly DGD, are estimated with the use of the minimum model, i.e. the first order PMD, that neglects the frequency dependent effects.

In the paper the question is answered by presenting the method for assessment of the estimation errors and the answer is illustrated with a suitable example. The analysis is scoped to the systematic errors of DGD alone, so the results are applicable to the worst-case channel quality assessment, time-domain measurement of average DGD of a fiber and, others that do not require knowledge on the other first order PMD parameter. However here, this particular application scenario is considered in which DGD estimates are also used for calculation of PMD induced signal distortions, hence they are obtained through joint estimation with power split ratio. Further, the analysis is limited to the case of (a) single mode direct detection systems with on-off signaling, (b) optical fiber operation in the linear regime and, (c) negligible influence on PMD from the third and higher order PMD.

The paper is organized as follows. It starts from presentation of the first and the second order PMD models, then explains the considered concept for estimation of PMD parameters from transmitted waveforms. Next, a proposal for characterization of the estimation error is described. The proposed tool is used to evaluate estimation errors which is presented in the subsequent section. The paper ends with conclusions summarizing the key findings.

PMD models up to the second order

In the linear operation regime an optical fiber can be modeled by a four-port device in which input-output pairs serve individual orthogonal polarization modes. Accordingly, optical field signal in a fiber shall be described by a two component vector eg. $\boldsymbol{S}(t)=[s_a(t),s_b(t)]^T$ describing the two modes [1]. As the second and higher order PMD effects depend on deviation of optical frequency from its central value, it is natural to present PMD effects in the frequency domain. For the first order PMD model one writes in the frequency domain [2]:

(1) $$\boldsymbol{Y}(\omega)=\boldsymbol{R}_\tau \boldsymbol{R}_\gamma \boldsymbol{S}(\omega)$$

where: τ is the momentary DGD, γ is power split ratio, $\boldsymbol{S}(\omega)$ is spectrum of the optical field injected to the optical fiber by a transmitter, $\boldsymbol{Y}(\omega)$ is spectrum of the optical field incident at a photodiode, $\boldsymbol{R}_\tau$ and $\boldsymbol{R}_\gamma$ are rotation matrices defined as follows:

(2) $$\boldsymbol{R}_\tau=\begin{bmatrix}\exp(j0.5\omega\tau) & 0\\ 0 & \exp(-j0.5\omega\tau)\end{bmatrix}$$
$$\boldsymbol{R}_\gamma=\begin{bmatrix}\cos(0.5\alpha) & -\sin(0.5\alpha)\\ \sin(0.5\alpha) & \cos(0.5\alpha)\end{bmatrix}$$

In (2) α denotes the angle between $\boldsymbol{S}(\omega)$ polarization plane and the slow axis of the optical fiber, $cos^2(\alpha/2)=\gamma$. In (1) a scaling factor resulting from transmission loss to optical signal is omitted for simplicity. Possible phase delay was dropped because it has no significance in direct detection systems.

The analytic formula for the second order PMD does not exist, hence various approximations were proposed. Among them the Bruyere-Kogelnik and Orlandini-Vincetti models are recognized as the most accurate [3]. From the two, the former has the advantage of being an extension to the first order PMD model and allows to express to the total effects easily [3]:

(3) $$\boldsymbol{Y}(\omega)=\boldsymbol{R}_p^{-1}\boldsymbol{R}_\tau \boldsymbol{R}_p \boldsymbol{R}_\gamma \boldsymbol{S}(\omega)$$

where:

(4) $$\boldsymbol{R}_\tau=\begin{bmatrix}\exp(j\varphi(\omega)) & 0\\ 0 & \exp(-j\varphi(\omega))\end{bmatrix}$$
$$\boldsymbol{R}_p=\begin{bmatrix}\cos(0.25p\omega) & -\sin(0.25p\omega)\\ \sin(0.25p\omega) & \cos(0.25p\omega)\end{bmatrix}$$
$$\varphi(\omega)=0.5\left(\omega\tau+\omega^2\tau_\omega\right)$$

with two extra parameters: τ_ω defined as polarization dependent chromatic dispersion (PCD) and p defined as depolarization rate of principal states of polarization, related to commonly used polarization state dispersion (PSD) parameter. If this parameter value is denoted by psd, it is related to τ and p by the simple formula: $psd=\tau\cdot p$.

DGD estimation with the use of the first order PMD model

The first order PMD model can be transformed to the time domain. After conversion to optical power it can be deduced that the two independent polarization components with $\pm 0.5\tau$ time shifts combine their powers to form power of the received optical signal. This reads as follows:

(5) $$\xi_y(t)=\gamma\xi_s\left(t+\frac{\tau}{2}\right)+(1-\gamma)\xi_s\left(t-\frac{\tau}{2}\right)$$

where: $\xi_y(t)$ and $\xi_s(t)$ are optical power of output and input optical fields, respectively. In the monitoring systems considered in the paper a momentary DGD value (i.e. τ) is being learnt through recording the photo-converted versions of waveforms being transmitted via the monitored optical channel at both ends and then by processing the samples to effect the joint τ and γ estimate. The processing algorithm makes use of the mathematical model (5) with continuous time replaced by discrete sampling time instances. Number of samples is finite and say equals N. The $[\tau,\gamma]^T$ joint parameter value pair which provides the best, in probabilistic sense, fit between the signal model and the measurement data is the estimate searched for. Denoting $\xi_y=\boldsymbol{m}(\xi_s,\tau,\gamma)$ the model that relates ξ_y vector of output power samples with ξ_s vector of input power samples and the first order PMD parameters, the formula for the joint $[\tau,\gamma]^T$ maximum likelihood (ML) estimate is expressed as [4]:

(6) $$[\tau,\gamma]^T=f(\xi_s,\xi_y)=\underset{\tau\geq 0,1\geq\gamma\geq 0}{\arg\max}\ \rho_y\left(\xi_y-\boldsymbol{m}(\xi_s,\tau,\gamma)\right)$$

In (6) $\rho_y(\nu)$ is the probability density function of the noise ν with which the output signal samples ξ_y were acquired. For Gaussian noise, which will be the assumption hold through the subsequent text, the optimization in (6) can be simplified to a minimization of a quadratic norm $||\xi_y-\boldsymbol{m}(\xi_s,\tau,\gamma)||$.

Please note, that regardless of the form of (6) exact values of the estimate can be obtained only if a global optimizer is used. Such optimizers exist for certain classes of objective functions, including those functions which have finite number of local extrema in a bounded parameter space. If it is the case one may turn to complete search algorithms [5]. The $||\xi_y-\boldsymbol{m}(\xi_s,\tau,\gamma)||$ falls into this class. From (5) it flows that if $\xi_s(t)$ is a lowpass signal, which is a justified assumption in telecomm applications, in the domain of τ the objective function can be developed in a Fourier series with finite number of terms. Hence, τ coordinates of possible local minima cannot be spaced closer than a reciprocal of the highest frequency in the Fourier series. Additionally, the quadratic dependence of the objective function on γ parameter eliminates other options than a single minimum in the γ domain, for any τ. For such class of objective functions a complete search algorithm is capable of providing a sure result (in contrast to non-complete search options) in finite time [5].

A method for evaluation of systematic errors of momentary DGD estimates due to the negligence of higher order PMD

The momentary DGD to be estimated as well as the remaining PMD parameters, that can affect the DGD estimate obtained according to (6), are random and, generally, can take any value within domains of their variability. Assessment of an estimation error of a momentary DGD shall use maximization of a difference between outputs of the models (1) and (3) (or their time domain equivalents) over the entire space of PMD parameters engaged in the models: momentary DGD, power split factor between polarization modes, PCD and PSD. One may expect an easy result of such an assessment: large, or even tending to infinity, estimation errors returned when the true description of PMD involves large values of PCD and PSD parameters. However, this can rarely happen. Momentary values of DGD, PCD and PSD parameters have certain statistical distribution. They show correlation and their behavior is to some extent scaled by the average value of DGD in a fiber, or a chain of fibers that form an optical communication line [7].

Fortunately, the average value of DGD can be easily assessed. In a typical situation optical fibers have documented PMD coefficients and lengths. Using this data it is easy to assess the average DGD, denoted here by τ_{av}, that shall characterize the fiber chain [6]:

(7) $$\tau_{av} = \sqrt{\sum_{k=1}^{K} pmd_k^2 L_k}$$

where: pmd_k and L_k are PMD coefficients and lengths of fiber segments respectively. Taking it into account it is proposed to characterize the estimation error with the use of a confidence calculus and to do it in the context of an average DGD of a fiber under monitoring. The appropriate measure of the error of interest can be an allowance for an absolute value of a DGD estimation error which may not be exceeded with some prescribed probability, while estimation is based on measurements in a fiber with arbitrary average DGD. Such an error allowance is equivalent to a two sided confidence interval. The proposed measure can characterize capability of a DGD monitoring system to make adequately accurate measurements in certain fibers through giving answers to questions for what fibers and for what total lengths the monitoring system fits precision requirements of a monitoring application. For any τ_{av} a characteristics that tells how estimation errors relate to τ, i.e. momentary DGD, being estimated will complete the description of DGD estimation errors.

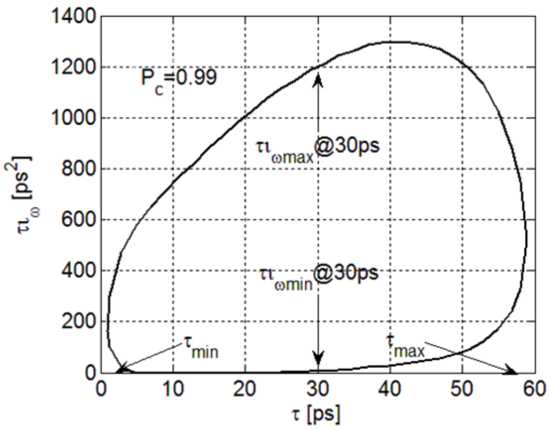

Fig.1. The contour delimiting the area with an arbitrary probability of $[\tau,\tau\iota_\omega]^T$ occurrence in a fiber with τ_{av}=25ps.

Confidence intervals for the DGD estimation error can be evaluated through maximization of the absolute estimation error while PMD parameters: τ, γ, τ_ω and p (ref. eq. 4) are sampled from the highest probability density (HPD) space which hosts the values with prescribed confidence. Unfortunately, the joint probability density function for τ, γ, τ_ω and p, required for this method, has been awaiting formulation, yet. Considering this deficiency it is proposed to continue with the suggested approach however, with the use of another result, published in [7], for the joint pdf for DGD and the compound second order PMD (SOPMD) parameter. This parameter, named here $\tau\iota_\omega$, is related to τ_ω and p by:

(8) $$\tau\iota_\omega = \sqrt{{\tau_\omega}^2 + (p\tau)^2}$$

The joint DGD and SOPMD parameters probability distribution function has no analytic formula, hence results are available only in the graphical form of contour plots defining areas within which τ (momentary DGD) and $\tau\iota_\omega$ (SOPMD) values occur with some probability P_c. Here, a contour for P_e=0.99 taken from [7] was approximated and the corresponding graph is depicted in the Figure 1. Although the plots in [7] were calculated for an isotropic birefringent fiber with a fixed τ_{av}=25ps it is scalable to any τ_{av} value, according to the general statistical properties of PCD and PSD. The τ axis scales proportionally with τ_{av}, while the $\tau\iota_\omega$ axis scales with the square of τ_{av} [7]. From the contours in the Figure 1 the minimum and maximum values of τ for a given τ_{av} can be read. Within this range the minimum and maximum value of $\tau\iota_\omega$ can be deduced for any τ. With this results a corresponding $\tau\times\tau_\omega\times p\times\gamma$ subspace is defined by conjunction of three following conditions:

(9) $$\begin{aligned} &\tau\iota_{\omega\min} \le \sqrt{(\tau_\omega)^2 + (p\tau)^2} \le \tau\iota_{\omega\max} \\ &\tau_{\min} \le \tau \le \tau_{\max} \\ &0 \le \gamma \le 1 \end{aligned}$$

where: $\tau\iota_{\omega min}$, $\tau\iota_{\omega max}$, τ_{min} and τ_{max} are found for a given τ.

The most straightforward way to calculate a τ estimation error is to generate a signal affected by the first and second order PMD, with PMD parameters being samples from the $\tau\times\tau_\omega\times p\times\gamma$ subspace, find the estimate using (6) and then calculate an absolute value of a difference between the resulted estimate of τ and the τ value used for signal generation. Signal generation involves the $\xi_y=\boldsymbol{M}(\xi_s,\tau,\gamma,\tau_\omega,p)$ which relates the ξ_y and ξ_s vectors. It uses the time domain equivalent of the formula (3) to find $y(t)$ and $s(t)$ fields from which the corresponding power signals $\xi_y(t)$ and $\xi_s(t)$ can be easily obtained.

Results and discussion

The method presented in the preceding section for calculation of confidence intervals for DGD estimation error is strictly procedural and no way was found for analytic formulation. Therefore one shall resort to numerical calculations in order to illustrate behavior of the confidence intervals versus average DGD and other involved parameters. This forces to make assumptions regarding those other parameters which are not of direct interest however, influence results. It concerns selection of a shape of the transmitted signal, transmitted data sequence, bit signaling interval length, sampling interval, optical and electrical filtering and, confidence level to mention the most influential.

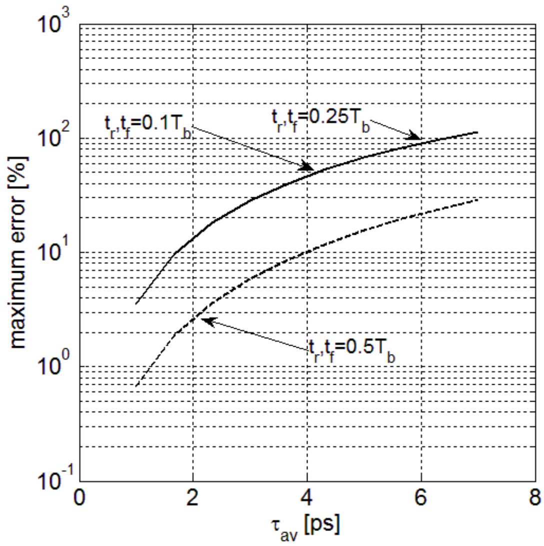

Fig.2. Confidence interval widths for errors of the DGD estimator versus average DGD in a fiber.

Here, these questions were answered through making commonly agreed choices or a selection of possibly enough representative example. The transmitted signal was assumed the 10Gbps non-return to zero (NRZ) with pseudorandom PRBS-7 data sequence, which is typical for testing in data communication. Semi-rectangular signal shape was selected with rise/fall times options equal to 0.1, 0.25 and 0.5 of the bit period (in the following denoted by T_b) achieved by Gaussian filtering. No other optical or electrical filtering was applied except that related to the PMD phenomenon. The PMD effects were simulated using the model (3). Optical signal was converted to its electrical equivalent in an ideal photodiode, then sampled every 6.25ps to provide 1024 samples. Signal generation, PMD emulation and photo-conversion were implemented in the VPITransmissionMaker simulation tool.

The DGD estimator along with the model (1) and confidence interval evaluation were implemented in Matlab. For finding the maximum widths of confidence intervals a genetic search algorithm was adopted for the global optimization task. There were 50 genes in each generation. The crossing rule, mutation probability and mutation strength parameters were fixed through all generations. Each final solution was obtained after 10 generations and the result was selected from 50 candidates based on maximum value of estimation errors effected in the search. Genetic algorithms provide approximate results. However, it is believed that the obtained maxima are close to the true ones for an increase of the mutation intensity parameter altered results insignificantly. The confidence level was P_c*=0.99* i.e. it met the data presented in the Fig 1. The τ_{av} (average DGD that permanently characterizes a fiber of given length) range for which calculations were performed is 1.0-7.0 ps. It covers practically all transmission distances that shall call for PMD monitoring.

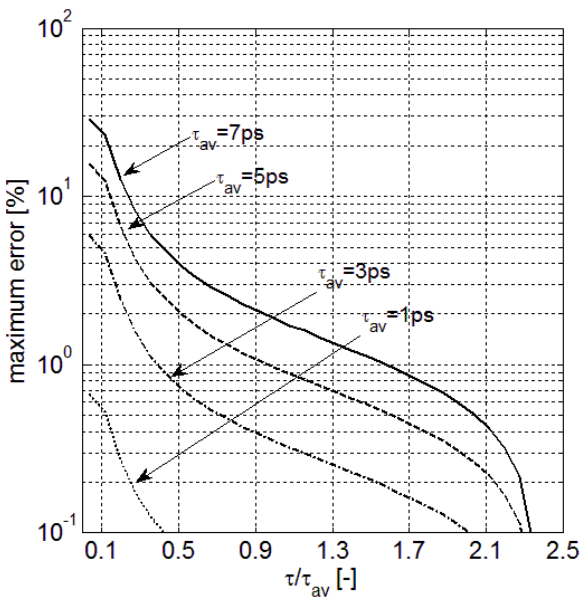

Fig.3. Confidence interval widths for errors of the DGD estimator versus ratio of momentary to average DGD.

In the Fig 2 the dependence of maximum relative widths of confidence intervals of momentary DGD estimates (maximum error axis), with respect to the true momentary DGD, versus the τ_{av} is depicted. The plots are given for three sets of rise (t_r) and fall (t_r) time parameters of the transmitter signal. From the graph it is evident that, due to the negligence of the second order PMD, the widths of confidence intervals are unacceptably large. For almost entire analyzed range of the τ_{av} the maximum errors, given in terms of the relative widths of confidence intervals maximized over the entire $\tau\times\tau_\omega\times p\times\gamma$ subspace at P_c=0.99, exceed 10% of the nominal τ value and reach 100% for the upper τ_{av} range limit. An exception from this general behavior happens for a signal with rise/fall time equal 50% of the bit period. Only in this case maximum estimation errors at 0.99 confidence level are below 30% within the whole analyzed τ_{av} range. Closer look at the behavior of momentary DGD estimates in this particular case provides the Figure 3 where confidence interval widths are related to τ, i.e. the momentary DGD, while the τ_{av} is the plot parameter. It can be learnt that the high values of confidence interval widths concentrate around low τ values, irrespectively what value τ_{av} takes. The intervals shrink quite rapidly when τ departs from zero, particularly for low τ_{av}. If there is an application for which the lowest τ, say approx. 1.5ps, represents negligible informative value, it can enjoy relatively low maximum estimation errors in the remaining τ range. However, this observation is not reproduced for the two other shapes of a transmitted signal.

Conclusions

In view of random character of the phenomena governing the DGD estimation error the use of confidence intervals can be a tool to characterize such errors. Negligence of higher order PMD effect in the PMD model that is used to estimate DGD from transmitted on-off waveforms results in large confidence intervals. The confidence intervals at 99% confidence can be as wide as 100% of the estimated DGD. It is relaxed for the case of a slow rise/fall pulses. Nevertheless, the maximum errors can be 30%. This a negative recommendation for a simplification of the PMD model by ignoring PMD dependence on frequency in an attempt to reduce the computation workload required to calculate an estimate.

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