Yuriy SHAPOVALOV[1], Bohdan MANDZIY[1], Dariya BACHYK[1]

Lviv Polytechnic National University(1)

doi:10.15199/48.2017.10.27

The Effectiveness of Multivariate Computation of the Energy Characteristics of Linear Periodically-Time-Variable Circuits in the environment of MAOPCs

***Abstract**. The paper presents two new software functions of the system UDF MAOPCs. The first software function is designed to determine the instantaneous power in the branch of linear periodically-time-variable circuit and the second software function is designed to determine the transfer coefficient of average power from input range to the output. The paper also presents computer experiments of the usage of these functions. Also it presents advantages of solving the multivariate problems of design of linear periodically-time-variable circuit in the environment of UDF MAOPCs.*

***Streszczenie.** W artykule zaprezentowano dwie nowe funkcje systemu UDF MAOPCs. Pierwsza z nich umożliwia określanie mocy chwilowej w obwodzie liniowym o okresowo zmiennych parametrach. Drugie oprogramowanie umożliwia określenie współczynnika przesyłania średniej mocy z wejścia do wyjścia obwodu. **Skuteczność określania charakterystyk energetycznych obwodu liniowego z okresowo zmiennymi parametrami za pomocą środowiska MAOPCs**.*

Keywords: frequency symbolic method, system UDF functions MAOPCs, linear periodically-time-variable circuits.
Słowa kluczowe: środowisko MAOPCs, parametry przesyłu mocy

Introduction

The ability of linear periodically-time-variable (LPTV) circuits to convert spectrum signals and perform parametric amplification of signals at low noise levels led to their important role in modern electronics. Over the past decade, the interest in LPTV circuits has significantly increased in connection with appearance of high-temperature superconductors and opportunities for their use in parametric amplifiers, high-Q resonators, filters, delay lines, etc. [1,2,3,4]. Modern software for circuit simulation of electronic circuits makes it possible to simulate the behavior of LPTV circuits usually in time domain, but does not provide a calculation of the functional characteristics of parametric nodes in steady state (frequency analysis). The most practical applicable method for LPTV circuits analysis in frequency domain, in our opinion, is the frequency symbolic method (FS-method) [5]. This method is based on using transfer functions of circuits that are represented in symbolic form, and allows us: a) calculate conjugate and normal parametric transfer functions [5,6] as approximate analytical expressions, the first of which connects the input and the output signals, second - contains information about the stability of the circuit; b)set the parameters of arbitrary elements circuit as symbols and calculate transfer functions of circuits in symbolic form; c) multiple times assign to those symbols the different values and explore their influence on the characteristics of the circuit. The confirmation of the foregoing is the system user-defined functions (UDF) MAOPCs (Multivariate Analysis and Optimization of the Parametric Circuits) [7], which are based on FS- method that is realized in a Matlab environment [8] and is intended for modeling, multivariate analysis and optimization of LPTV circuits in the frequency and time domains. By the FS-method, parametric transfer functions (conjugate and normal) of the circuit define approximate polynomials of Fourier in complex or trigonometric form, and, as was noted above, can contain a series of parameters that are defined as symbols. Along with it, in the system UDF MAOPCs there are present functions of the sensitivity coefficients definition, the analysis of tolerances for the elements parameters, optimization and more. Since the architecture of the system UDF MAOPCs is based on the principles of Matlab, therefore it foresees the creation of user-defined functions. MAOPCs is an open system, which if it necessary, it can completed by new functions.

The task of calculating power in the branches of circuit is actual and practically important task of analysis of LPTV circuits, because there is the transformation of energy source of pumping in the energy of signal in these circuits. Currently, there are absent calculation functions of energy characteristics of circuits, including determination of the instantaneous power in the branches of the circuit and the transfer coefficients of average power from input circuit to it's output. This limits the possibilities of research, for example, the temperature schedule of the scheme work and also analysis of schemes in a range of high and ultra-high frequency, where it is more convenient to operate with powers and not with voltages or currents.

In this paper:

- are described new functions of the system UDF MAOPCs, which expand its functional capabilities in the part of calculation of energy characteristics of LPTV circuits;
- are presented the ways of increasing the solution action in the system UDF MAOPCs of the multivariate problems, which significantly increase their effectiveness;
- are presented the results of computer experiments.

Main part

Computation of the Energy Characteristics. To achieve this goal we need to input into in the system UDF MAOPCs: a) the function «PowerInTheBranch()», designed for determination of the instantaneous power in the selected (by inverse Fourier or Laplace transform) branch of the circuit, representing the multiplication of previously identified time dependencies of voltage and current of this branch; b) the function «TransferCoefficientOfPower()», which is designed for determining the transfer coefficient of medium power for the period of the input signal of the circuit to its output in steady state.

For the correct execution of the function «Power-InTheBranch ()», previously we need to follow these steps.

1. By double execution of the function «TrFunc ()» [7] we are forming two parametric transfer function of the signal from the entrance to the voltage on the selected branch of circuit and from the entrance to the current on the same branch. Every such parametric transfer function $W(s,t)$ in the form of polynomial $W(s,t)$ of Fourier or in the complex form

$$\hat{W}(s,t) = W_{\pm 0}(s) + \sum_{i=1}^{k} \begin{bmatrix} W_{-i}(s) \cdot e^{(-ji\Omega t)} + \\ +W_{+i}(s) \cdot e^{(+ji\Omega t)} \end{bmatrix}, \tag{1}$$

where $W_{\pm 0}(s), W_{-i}(s), W_{+i}(s),$ - are the time-independent t fractional-rational functions of a complex variable s, k - is the number of harmonics in the approximation, $\Omega = 2\pi/T$, T - the period of the parameter change of the parametric element of the circle under the action of the pump signal.

2. By double execution of the function «OutVar()» we are determining images of current and voltage of selected branch of the circuit in the form

(2) $$Y(s,t) = W(s,t) \cdot X(s),$$

where $X(s)$ and $Y(s,t)$ - are images of input and output signals, respectively.

3. By double execution of the function «Laplace-Transform()» we are determining the current and voltage of the branch of the circuit in the time domain. These values as a functions of time $y(t)$ we are determining by inverse Fourier transform for steady mode

(3) $$y(t) = \frac{1}{2 \cdot \pi} \cdot \int_{-\infty}^{\infty} W(j\omega,t) \cdot X(j\omega) e^{j\omega \cdot t} d\,\omega,$$

or by Laplace transform for transient mode

(4) $$y(t) = \frac{1}{2 \cdot \pi \cdot j} \cdot \int_{c-j\cdot\infty}^{c+j\cdot\infty} W(s,t) \cdot X(s) e^{s \cdot t} ds,$$

where $X(j\omega)$ - is the image of the input signal $x(t)$, that acts on the circuit.

Only now, performing the function «PowerInTheBranch ()», we calculate the instantaneous power of the selected branch circle by multiplying the time dependence of voltage and current in it.

The function "TransferCoefficientOfPower ()" defines the transfer coefficient of average power from the period of the input signal of the circuit to its output in steady state as follows.

1. After performing the function «PowerInTheBranch ()» we are forming instantaneous powers in the input and output branches of circuit.

2. Performing the function «TransferCoefficientOfPower ()» leads us to determine the transfer coefficient of average power K_P in the form

$$K_P = P_{out}/P_{in}$$

where $P_{in} = (1/T) \cdot \int_0^T [i_{in}(t) \cdot u_{in}(t)] dt$,

$$P_{out} = (1/T) \cdot \int_0^T [i_{out}(t) \cdot u_{out}(t)] dt, \quad T = 2 \cdot \pi/\omega.$$

Computer experiment 1. For single-circuit parametric amplifier from the fig. 1. in steady state in the system UDF MAOPCs: a) check balance of the instantaneous powers of the branches of circuit by the Tellegen's theorem;
b) determine the transfer coefficients of average power from its input to the output.

For the purpose of such verification for the given circuit, we determine the sum $P(t)$ of the instantaneous powers of each branch of this circuit in steady state:

(5) $$P(t) = i_1(t) \cdot u_1(t) + i_1(t) \cdot u_{12}(t) + i_2(t) \cdot u_2(t) + \\ + i_3(t) \cdot u_2(t) + i_4(t) \cdot u_2(t).$$

We verify (1) for equality to zero, performing in the system UDF MAOPCs the sequence of functions, that is presented in [9]. The results of such verification are given in Table 1, with considering 4 harmonic components in the transfer functions [5]. The obtained instantaneous powers of the branches (lines 1-5, Table 1) and the values of the sum (5) (line 6, Table 1) are shown in the table, considering 5 decimal places.

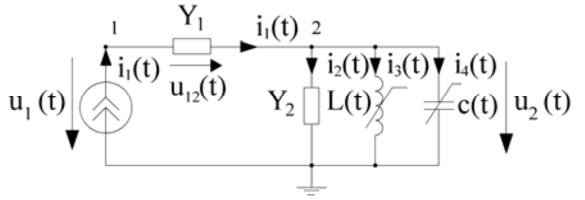

Fig.1. Single circuit parametric amplifier,
$i_1(t) = cos(\omega_s t - \pi/4)$,
$c(t) = c_0 \cdot (10 \cdot 10^{-12} + 0.05 \cdot cos(4\pi 10^8 t + 0)), \omega_s = 2\pi 10^8$
$L(t) = L_0 \cdot (0.2533 \cdot 10^{-6} + 0.05 \cdot cos(4\pi 10^8 t + \pi))$
$Y_1 = 0.25S, Y_2 = 0.0004S$

The function «TransferCoefficientOfPower()» defines the transfer coefficient of average power from the amplifier input to its output $K_P = P_{out}/P_{in} = 21.75$ by the expressions: $P_{in} = (1/T) \cdot \int_0^T [i_1(t) \cdot u_1(t)] dt = 1.25e-5$,
$P_{out} = (1/T) \cdot \int_0^T [i_2(t) \cdot u_2(t)] dt = 2.71e-4$,
where $T = 2 \cdot \pi / (2 \cdot \pi \cdot 10^8) = 10^{-8}$ s.

Table 1

№	t, s	0.801e-6	0.802e-6	0.803e-6	0.804e-6
1	$i_1 \cdot u_{12}, W$	0.3902 e-7	0.3176 e-7	0.0824 e-7	0.0098 e-7
2	$i_2 \cdot u_2, W$	0.4969 e-3	0.4681 e-3	0.0929 e-3	0.0276 e-3
3	$i_3 \cdot u_2, W$	-0.0012	0.0037	0.0032	-0.0019
4	$i_4 \cdot u_2, W$	0.0008	-0.0041	-0.0033	0.0018
5	$i_1 \cdot u_1, W$	-0.1130e-3	-0.9391e-4	-0.2318e-4	-0.3298e-5
6	$P(t), W$	0	0	0	0

Zero values $P(t)$ for different time points from Table 1. are convincing us in the adequacy of calculations that were performed for the single-circuit amplifier in the environment of the system UDF MAOPCs. At the fig. 2, there is built dependence of the transfer coefficients of average power P_{out}/P_{in} from change m_c of parametric capacity $c(t)$ and change m_L of parametric inductivity.

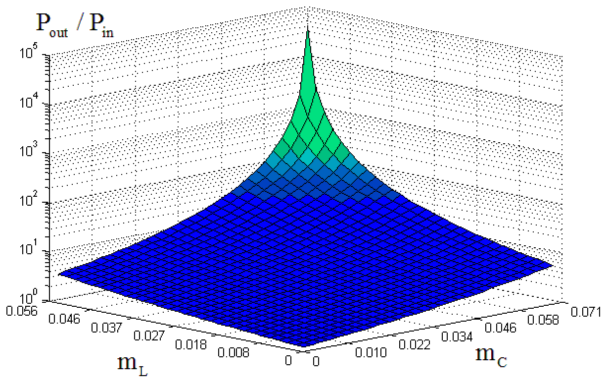

Fig. 2. Dependence of the transfer coefficient of average power P_{out}/P_{in} from change of the modulation depth m_c and m_L of the single circuit parametric amplifier

Fig. 3 shows the dependence that reflects the expenditure of time for the multivariate analysis in the system UDF MAOPCs and in program Micro-Cap.

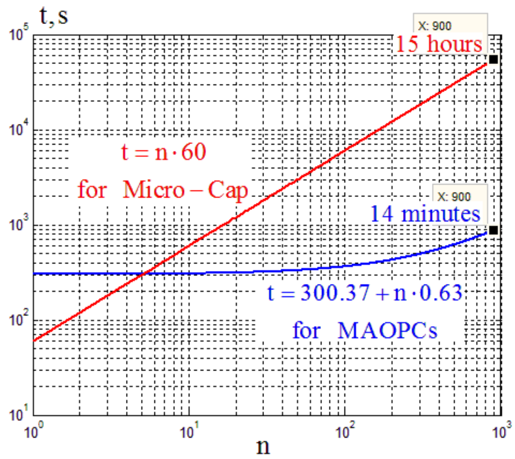

Fig. 3. The assessment of expenditure of time for the multivariate analysis in the system UDF MAOPCs and in the program Micro-Cap; 300.37 s - the computation time of the transfer function with symbolic parameters t, mc, mL in the system UDF MAOPCs; the computation time for the one pair of values mc and mL: a) the transfer coefficient of average power - 0.63s; (in the system UDF MAOPCs); b) the time dependence (0-70 ms) of instantaneous power-60 s . (Micro-Cap); n- the number of pairs values mc and mL

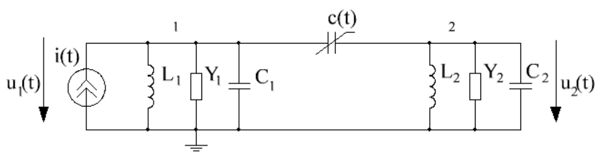

Fig.4. Double-circuit parametric amplifier

$$i(t) = 0.1 \cdot 10^{-3} \cdot cos(\omega_{signal} \cdot t + \pi / 4), C_1 = C_2 = 68 pF,$$

$$L_1 = 36.70795 nH, L_2 = 9.312609 nH, Y_1 = Y_2 = 0.0001 S,$$

$$c(t) = 1 \cdot 10^{-12} \cdot (1 + m_c \cdot cos(\Omega \cdot t)), \Omega = 2 \cdot \pi \cdot 2.98573 \cdot 10^8,$$

$$\omega_{signal} = 2 \cdot \pi \cdot 10^8 \, rad / s, \omega_{idle} = 2 \cdot \pi \cdot 1.98573 \cdot 10^8 \, rad / s$$

Computer experiment 2. For double-circuit parametric amplifier from the fig. 4. in steady state in the system UDF MAOPCs determine the transfer coefficients of average power from its input to the signal P_{out1}/P_{in} and to the idle P_{out2}/P_{in} contours, in dependence on the modulation depth m_c of the parametric element *c(t)*.

The execution results of this experiment are shown in Table 2 and Table 3, with considering 2 harmonic components in the transfer functions [5]. Table. 2 shows the instantaneous powers at the input of the circle (line 1, Table 2), at the signal contour (line 2, Table. 2) and at the idle contour (line 3 of Table 2).

Table 2

№	t, нс	70000	70001	70002	70003
1	$i \cdot u_1$, W	6.17e-5	2.81e-6	2.59e-5	9.72e-5
2	$(u_1)^2 \cdot Y_1$, W	7.61e-5	3.23e-6	3.26e-5	1.19e-4
3	$(u_2)^2 \cdot Y_2$, W	4.19e-7	4.85e-5	2.55e-5	1.40e-5

Table. 3 shows the obtained values of average power $P_{in} = (1/T) \cdot \int_0^T [i(t) \cdot u_1(t)] dt$, $P_{out1} = (1/T) \cdot \int_0^T \left[(u_1)^2 \cdot Y_1 \right] dt$, where $T = 2\pi / \omega_{signal}$, $P_{out2} = (1/T) \cdot \int_0^T \left[(u_2)^2 \cdot Y_2 \right] dt$, where $T = 2\pi / \omega_{idle}$, at different values of the modulation depth m_c of the parametric element $c(t)$ within the limits $0 - 0.225$. The limit 0.225 of the value m_c is selected from those considerations that at $m_c \geq 0.235$ the amplifier becomes unstable.

Increase of the action of multivariate analysis of lptv circuits by the FS-method

Symbolic methods, which also include the FS-method, advantageously differ from numerical methods by these facts:

- Symbolic methods based on the determination of transfer functions, which allows not to solve repeatedly the differential equation that describes the circuit by changing the input signal, but quickly multiply images of these signals to the ready transfer functions;
- In the transfer functions beside with independent symbolic variables s, t are present symbolic parameters of arbitrary elements of the circle, allowing you quickly generate transfer functions for a large number of numerical values for such variables and parameters.

Table 3

m_c	**0**	**0.01**	**0.05**	**0.1**	**0.125**	**0.15**	**0.163**	**0.175**	**0.2**	**0.213**	**0.225**
P_{in}, W	5.00e-5	5.00e-5	5.00e-5	5.00e-5	5.00e-5	5.00e-5	5.00e-5	5.00e-5	5.00e-5	5.00e-5	5.00e-5
P_{outU1}, W	4.99e–5	5.01e–5	5.51e–5	7.62e–5	1.01e–4	1.52e–4	2.043 e–4	2.86e–4	8.74e-4	0.0026	0.0363
P_{outU2}, W	0.12e-8	1.97e–7	5.23e–6	2.88e–5	5.98e–5	1.30e–4	2.054 e–4	3.32e–4	13.2e-4	0.0045	0.0695
P_{out1}/P_{in}	0.9999	1.003	1.102	1.52	2.02	3.05	4.08	5.73	17.48	53.28	7.26e+2
P_{out2}/P_{in}	0.242e-4	0.0039	0.105	0.58	1.20	2.60	4.10	6.64	26.45	91.41	1.39e+3

We know that numerical methods in comparison with symbolic in one-variant analysis are faster. But in multivariate analysis for the described above features of symbolic methods, the advantages in action of the numerical methods are not as obvious. Thus, Figure 6 shows the dependence that reflects expenditure of time for computing the energy characteristics in the system UDF MAOPCs and in program Micro-Cap from the number n of pairs of values of the depth modulation m_c and m_L parametric elements $c(t)$ and $L(t)$. By $n < 5$ the action of Micro-Cap prevails. But by $n > 5$ the system UDF MAOPCs, based on the FS method becomes a high-speed. And the more the n is, the advantage of the system UDF MAOPCs becomes bigger. For $n = 900$ (surface of Figure 2 has built by such number of pairs of values) such time advantage is $15 \times 60 / 14 = 64{,}29$ times.

In addition to these obvious advantages of the FS method, it has one significant advantage, which we will look after consideration the following comments.

Remark 1. Differential equations that had composed relative to the input signal and other variable of circuit significantly are different in bulkiness. Thus, in the figure 5 is shown bulkiness of differential equations, relative to the input signal and another variable of double-circuit parametric amplifier from the figure 4. From Figure 5 it follows that, for example, bulkiness of differential equation, which had formed relatively pair $i(t), i_{c1}(t)$ and which in the computer memory occupies 16735 bytes prevails bulkiness of differential equation, which had formed relatively pair $i(t), i_{L1}(t)$ and which occupies 2723 bytes, in 6,14 times.

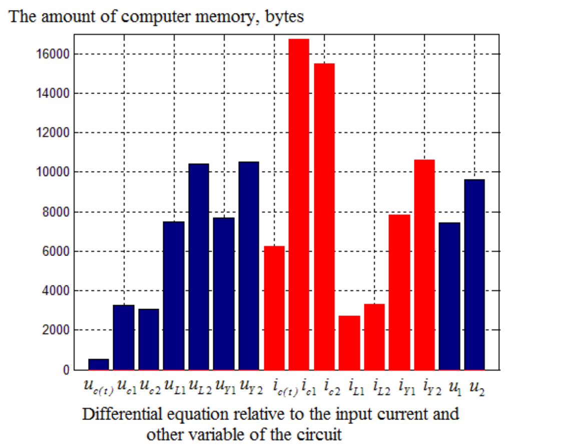

Fig. 5. The comparative histogram of volumes of differential equations in symbolic form generated from mathematical model of amplifier from figure 4

Remark 2. The differential equation which had formed relative to the input signal and variable of parametric element (the current if the parametric element is inductance, or voltage, if the parametric element is capacity) is the least bulky. From figure 5 it follows that equation which had formed relative to $i(t), u_c(t)$ for the amplifier from the figure 4 is only 517 bytes and is less bulky compared with the equation which had formed relative to $i(t), i_{c1}(t)$, in 32.3 times.

Remark 3. The formation of the transfer function by a bulkier differential equation takes more computer time and requires more memory. With the growth this bulkiness appropriate time and memory grow dramatically.

In connection with the following remarks, we suggest the following algorithm of formation of necessary transfer function.

Step 1. In time domain we are forming differential equation relative to input variable and variable of parametric element (regardless of which variable of circuit is considered to be output).

Step 2. By this equation by FS method we are forming transfer function of signal from the input to the parametric element.

Step 3. We are determining the image of signal on the parametric element by the transfer function which had formed and by given the image of the input signal.

Step 4. By the substitution theorem [10], in the circuit we replace parametric element by signal source which equal to the determined signal on the parametric element. Such substitution, without changing the voltages and currents in a circuit, transform it into a circle with constant parameters (if in the circuit is present parametric element) and two known sources.

Step 5. By known methods of modeling of linear circuits with constant parameters in the frequency domain we determine the image of the sought output signal.

As practice has shown, the usage of the system UDF MAOPCs presented algorithm that significantly reduces necessary computer time and memory for determination of the transfer function of circuit from input to its output. Note that the presented algorithm : a) spreads easily in case there are several parametric elements in a circuit and several entrances or exits; b) cannot be used in programs of numerical analysis of LPTV circuits (including Micro-Cap), because they don't have calculations in the frequency domain, and so the user has to solve a differential equation formed directly relative to input and output; c) provides a significant reduction in necessary time and computer memory, and so for a number of design problems the FS method can deal considerably more quick than traditional numerical methods. The last conclusion is important in practical and theoretical ways. We illustrate it by the computer experiment for the circuit from figure 4.

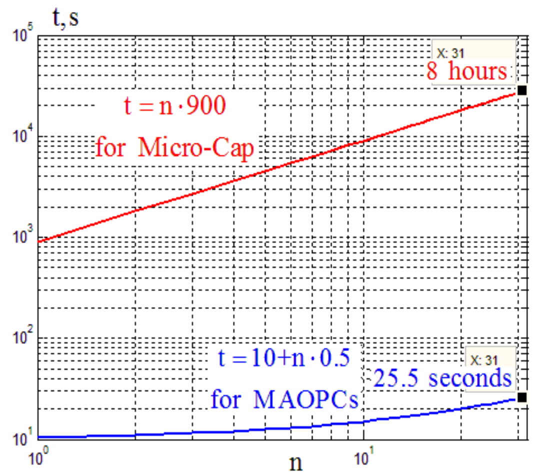

Fig. 6. The assessment of expenditure of time for the multivariate analysis in the system UDF MAOPCs and in the program Micro-Cap; 10 s - the computation time of the transfer function with symbolic parameters t, mc in the system UDF MAOPCs; the computation time for one value mc: a) the transfer coefficient of average power - 0.5s; (in the system UDF MAOPCs); b) the time dependence (0-80 μs) of instantaneous power-15minutes (Micro-Cap); n- the number of values mc

Computer experiment 3. For circuit from figure 4 investigate the time dependence of the transfer coefficient of average power from the number values of the depth modulation m_c of parametric capacity $c(t)$ in the environment of the system UDF MAOPCs and determine the time dependence (0-80 ms) of instantaneous power from the number of values of the depth modulation m_c of parametric capacity $c(t)$ in the environment of Micro-Cap.

Results of the research of the circuit from figure 4 are shown in figure 6 from which follows: a) the system UDF MAOPCs is more high-speed than the Micro-Cap, for any of the following values n; b) the winning of time is bigger, than for the bigger number n of values m_c necessary to perform the calculations; c) for the number $n = 31$ of values m_c the winning of time is 1130 times.

Conclusions

1)Software functions «PowerInTheBranch()» and «TransferCoefficientOfPower()» that were developed in the system UDF MAOPCs, allow to calculate the instantaneous powers, average power and transfer coefficients of average power.

2) Due to symbolic presentation of parametric transfer functions in the system UDF MAOPCs, we can achieve high action of calculations, which in most cases is higher or much higher than programs can provide by the numerical methods. Therefore, in a number of design of the multivariate tasks, the system UDF MAOPCs can become one of the few, if not the only one, that can ensure the effective multivariate calculation of transfer functions of LPTV circuit at change of parameters of its elements.

3)For most of linear periodically-time-variable circuits the determination of one symbolic transfer function of circuit takes up to 7 minutes (MATLAB 7.6.0 on a computer with a processor Intel Core i5-3317U CPU @ 1.70 GHz and operative memory 8.00 Gb.) and the number of harmonic components in the transfer functions that provides the necessary accuracy is small. After the formation of symbolic parametric transfer functions, the multivariate calculation of instantaneous powers of linear periodically-time-variable circuit in the system UDF MAOPCs occurs in seconds.

Authors: DSc, Prof. eng. Yuriy Shapovalov, E-mail: shapov@polynet.lviv.ua; DSc., Prof.,eng. Bohdan Mandziy, E-mail: bmandziy@polynet.lviv.ua; Ph.D. Dariya Bachyk, E-mail: dariya.bachyk.smal@gmail.com all: Lviv Polytechnic National University,12,Stepana Bandery Street, Lviv, 19013, Ukraine..

REFERENCES

[1] Tholen, E.A. et al. Nonlinearities and parametric amplification in superconducting coplanar waveguide resonanors. *Appl. Phys. Lett.*90, 253509(2007).
[2] Yamamoto,T.et al . Flux-driven Josephson parametric amplifier. *Appl.Phys. Lett.*93, 042510 (2008).
[3] Byeong Ho Eom, Peter K. Day, Henry G. LeDuc and Jonas Zmuidzinas. A wideband, low-noise superconducting amplifier with high dynamic range. *Nature Physics, Vol.8, August 2012.*
[4] Vanassche P. Systematic Modeling and Analysis of Telecom Frontends and their Building Blocks/ P. Vanassche, G. Gielen, W. Sansen //Springer; 2005. – 230 p.
[5] Shapovalov Yu. Symbolic analysis of linear electrical circuits in the frequency domain. Fixed and variable parameters. Lviv, Lviv Polytechnic National University publication, 2014 – 324 p.
[6] Yu. Shapovalov, B. Mandziy, and S. Mankovsky, «The peculiarities of analysis of linear parametric circuit performed by frequency-symbolic method», Przeglad Elektrotechniczny, vol.86, no 1, pp. 158-160, 2010.
[7] Yu. Shapovalov, B. Mandziy and D.Bachyk «The system functions MAOPCs for analysis and optimization of linear periodically time-variable circuits based on the frequency symbolic method», Przeglad Elektrotechniczny, vol.91, no 7, pp. 39-42, 2014, http://pe.org.pl/articles/2015/7/13.pdf .
[8] https://www.mathworks.com/
[9] Shapovalov Yu. The investigation of the energy characteristics of linear periodically-time-variable circuits by the frequency symbolic method / Yu. Shapovalov, B. Mandziy, D.Bachyk // Computational Problems of Electrical Engineering (CPEE 2016): Int. Conf., September 14 – 17, 2016 : Proc. – Sandomierz, Poland. – P. __.
[10] C. A. Desoer and E. S. Kuh, Basic Circuit Theory, New York, NY: McGraw-Hill, 1969