# Calculation of local force acting on stator tooth in permanent magnet motor

Abstract. The paper presents the procedure of local force calculation in permanent magnet motor. The author developed computer software for electromagnetic field and force distribution computations. In the software the finite element (FE) method is applied. The FE mesh consist of homogeneous regular elements. In the paper force distribution on an inner surface of stator tooth has been analyzed. The results of local force calculation using the proposed method have been compared with the results obtained in Maxwell software.

**Streszczenie**. W artykule przedstawiono procedurę wyznaczania rozkładu sił lokalnych w silniku magnetoelektrycznym. Autor stworzył specjalny program komputerowy do obliczania rozkładu pola elektromagnetycznego oraz sił lokalnych. W oprogramowaniu tym zastosowano metodę elementów skończonych (MES). Siatka modelu MES składa się z jednorodnych regularnych elementów. W pracy analizowano rozkład sił na wewnętrznej powierzchni zęba stojana. Rozkłady sił wyznaczone we własnym oprogramowaniu zostały porównane z wynikami uzyskanymi z programu Maxwell. **Obliczanie rozkładu sił lokalnych działających na ząb stojana silnika magnetoelektrycznego**.

Keywords: magnetic force, permanent magnet motor, finite element method. Słowa kluczowe: siły pochodzenia magnetycznego, silnik magnetoelektryczny, metoda elementów skończonych.

#### Introduction

In the design process of electrical machines it is important, among other things, to know the distribution of electromagnetic force density. The force may cause structural deformation. This deformation brings about such problems as mechanical vibrations and noises. For the calculation of the local magnetic force (distribution of force) different methods are used. These techniques are derived from methods for the total magnetic force computation e.g., methods based on the virtual work principle, the Maxwell stress tensor, equivalent magnetization models [1, 2].

In the paper the procedure of local force calculation in permanent magnet motor is presented. In this approach the Maxwell stress tensor method was implemented. For the calculation of magnetic field and local force distributions, the author has elaborated a special computer program. In the elaborated software the finite element method and **A** formulation have been applied. In the software it is possible to calculate a force for each finite element.

Efficacy of proposed method was verified by the estimation of force distribution on a stator tooth surface in a electrical machine with surface mounted permanent magnets. Calculations have been performed in elaborated software and in professional electromagnetic FEM software - Maxwell.

## Procedure of local force calculation

According to planar symmetry of many permanent magnet motors in the presented approach the field problem was reduced to 2D case. In order to obtain field distributions the edge element method (EEM) has been used. The equations of this method represent the loop equations of 2D facet network (FN). The vector of edge values of **A** represents the loop fluxes **\varphi** in the loops of FN. The loop equations that represent EEM equations for magnetic field can be written as follows:

$$\boldsymbol{k}_{m}^{T}\boldsymbol{R}_{\mathrm{u}}\boldsymbol{k}_{m}\boldsymbol{\varphi} = \boldsymbol{k}_{m}^{T}\boldsymbol{\theta}_{b}$$

(1)

where:  $\mathbf{R}_{\mu}$  is the matrix of branch reluctances,  $\mathbf{k}_{m}$  is the loop matrix of FN,  $\mathbf{\theta}_{b}$  is the vector of branch magnetomotive force (*mmfs*). In studied problem the  $\mathbf{\theta}_{b}$  is the sum of two factors: (a)  $\mathbf{\theta}_{m}$  that represents branch *mmfs* in area of permanent magnets and (b)  $\mathbf{\theta}_{0}$  that represents branch *mmfs* in area of stranded coils.

In order to calculate magnetic field source in permanent magnet region the electrical vector potential  $T_m$  has been used [3]. The values of this vector are determined by demagnetization curve of the permanent magnet material. The vector of branch *mmfs* in the winding areas  $\theta_0$  has been computed applying the edge values of vector potential  $T_0$ . The vector  $T_0$  values represents loop currents around edges of elements intersecting surfaces created by surrounding turns.

According to the Maxwell stress, the radial force densities for the finite element (in air region) can be expressed as:

(2) 
$$f_{r,i} = \frac{B_{r,i}^2 - B_{t,i}^2}{2\mu_0}$$

Here,  $B_{r,i}$ ,  $B_{t,i}$  are the radial and tangential components of magnetic field density for the finite element and  $\mu_0$  is the magnetic permeability of free space.

In elaborated software the mesh is composed from 4-nodal elements. Element edges are parallel to axis of cylindrical coordinate system. The mesh is a regular in relation to the angular coordinate  $\alpha$ . The part of finite element mesh has been shown in fig. 1. For such a mesh  $B_r$  and  $B_t$  for the edges *i*,*i*+1 and *i*,*j* are described as follows:

(3) 
$$B_{r,i,i+1} = \frac{\varphi_{i+1} - \varphi_i}{e_{i,i+1} \cdot l}$$

(4) 
$$B_{t,i,j} = \frac{\varphi_i - \varphi_j}{e_{i,j} \cdot l}$$

where: *e* is the length of a finite element edge, *l* is the length of the model.

There are several formulas of determining the mean value of the  $B_r^2$  [4, 5]. In this research the author employ the method presented in [5], where the authors calculated a global force utilizing a regular FE mesh. For the *i*-th element  $B_{r,i}^2$  is equal to product of  $B_{r,i,i+1}$  and  $B_{r,jj+1}$ . The  $B_{t,i}^2$ , for the *i*-th element, is equal to 0.5 ( $B_{t,i,j}^2 + B_{t,i+1,j+1}^2$ ).

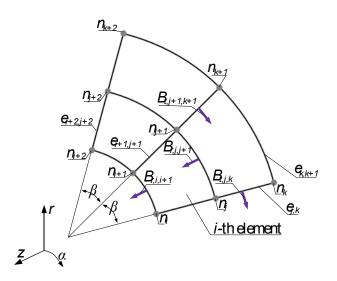


Fig.1. Part of the mesh with 4-nodal elements regular in  $\alpha$  direction

#### Force distribution on a stator tooth surface

The presented method was used to compute local force distribution in permanent magnet motor. The structure of magnetic circuit of considered motor is shown in fig. 2. The torque and electromotive force characteristics of the studied motor were presented in [6]. At first the analysis was performed for no-load operation mode of the machine (force is generated only by permanent magnets – the current in the winding equals zero). The previous experiences of the author shows that in such motor, local force is concentrated on an inner surface of the stator's teeth [7]. A dominant part of this force is radial component. Therefore in the paper the radial component of local force distribution on the inner surface of the stator teeth has been analyzed.

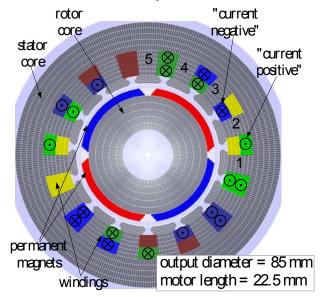


Fig.2. Structure of analysed permanent magnet motor

In fig.3 the force density distribution for the teeth number 1, 2, 3 and 5 is presented. An angle  $0^{\circ}$  indicate the left side of the tooth, what is shown in fig. 4. The negative values of the force mean that the sense of the force vector is from stator to rotor. The radial force was calculated in the finite elements of air region which are placed next to the inner edge of the tooth. In order to achieve good accuracy a density of the mesh in this region was increased.

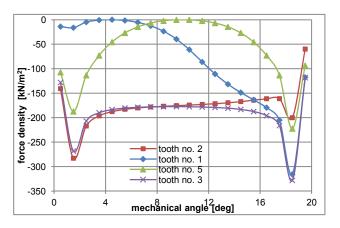


Fig.3. Radial component of force density acting on an inner surface of stator teeth

We can observe that value of force is directly related to the location of the stator tooth relative to the magnet location. It is important to notice that there are some part of teeth for which the local force is equal to zero.

The force distribution was also calculated in the Maxwell software. The obtained results are presented in fig. 5. The values in this characteristic are the values of quantity named: *edge force density* (*EFD*) [7]. In the Maxwell software, for 2D analysis, the *EFD* is calculated for finite element edges which are part of the edge of an object surrounded by air.

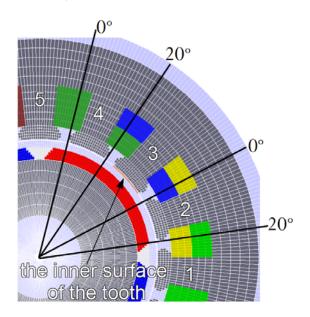


Fig.4. Part of FE model of considered motor

We can observe that the obtained characteristics are very similar. This indicate that proposed method is proper regard to local force calculation. The biggest differences, on those curves, are in a middle of the tooth, for angles of about 7 and 13 deg. For the Maxwell's characteristic there are peaks (two for tooth no. 2 and one for tooth no. 1) which aren't physical justification. The reason of this anomaly might be the improper discretization of stator. In the Maxwell software the mesh of finite elements in this region is unnecessarily dense. Nevertheless the impact of this atypical force distribution will be probably negligible in regard to mechanical calculation.

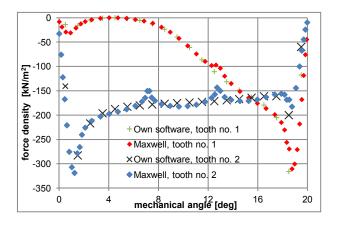


Fig.5. Comparison of radial component of force density calculated in own software and in Maxwell software

At the second part of the research the analysis for rated load operation mode was performed. In this case the motor is supplied in the BLDC mode, this means that current flows only through two phases while remaining phase is left open. (in fig. 2 a current direction and conducted phases are indicated). A comparison of force distribution at no-load and rated load state is presented in fig. 6. We can note that for some teeth the force density is bigger at rated load operation mode. This is due to the fact that for these teeth the magnetic field from current flow increases permanent magnet's field in radial direction. For a few other teeth, the situation is reversed. There are also five teeth for which occurrence of current almost not change the force density in radial direction. The average value of force density has increased from 142 kN/m<sup>2</sup> for no-load state to 172,9 kN/m<sup>2</sup> for rated load state.

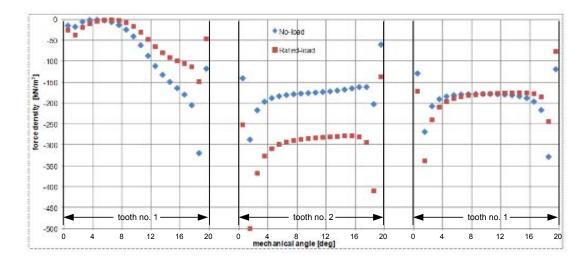


Fig.6. Force density acting on an inner surface of stator teeth for no-load and rated load state

# Conclusion

The paper presents the method of local force calculation, in the field finite element model with regular mesh. The use of homogeneous FE network simplify a process of a force computation. The results of sample calculations show that proposed procedure is efficient with respect to local force density estimation. The interesting finding based on the obtained result is that radial force density for rated load state is smaller, for some parts of stator teeth, than for no-load state.

In FEM mechanical solvers the load (e.g. force) is inserted at the edge of an object or on the points at this edge. These points usually have to be arrange in some kind of regular pattern. Because of that, a force distribution determined in elaborated software can be directly used as a source for mechanical calculations. Any additional transformations aren't necessary.

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