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Currents' Physical Components (CPC) of the Supply Current of Unbalanced LTI Loads at Asymmetrical and Nonsinusoidal Voltage

***Abstract**. Power properties of unbalanced linear time-invariant (LTI) loads in three-phase three-wire circuits at asymmetrical and nonsinusoidal supply voltages are investigated in this paper. It is demonstrated that the supply current of such loads can be decomposed into Currents' Physical Components (CPC), associated with distinctive physical phenomena in the load. The power equation is developed and it is also demonstrated how the current components can be expressed in terms of the supply voltage and the equivalent parameters of the load.*

Streszczenie.** W artykule analizowane są właściwości energetyczne niezrównoważonych, czasowo-niezmienniczych (LTI) odbiorników trójfazowych, zasilanych napięciem niesymetrycznym i niesinusoidalnym w systemach trójprzewodowych. Pokazano, że prądy zasilania w takich warunkach mogą być rozłożone na składowe stowarzyszone z konkretnymi zjawiskami fizycznymi w odbiorniku, to jest na Składowe Fizyczne. Pokazano jak Składowe Fizyczne Prądów mogą być obliczane na podstawie znajomości napięć zasilania i równoważnych parametrów odbiornika. **(Składowe Fizyczne Prądów (CPC) zasilania liniowych, czasowo niezmienniczych (LTI) odbiorników trójfazowych zasilanych napięciem niesymetrycznym i niesinusoidalnym).

Keywords: power definitions, current decomposition, scattered current, scattered power, unbalanced current, unbalanced power.
Słowa kluczowe: definicje mocy, rozkład prądu, prąd rozrzutu, moc rozrzutu, prąd niezrównoważenia, moc niezrównoważenia.

Introduction

This paper extends the Currents' Physical Components (CPC)-based power theory of three-phase circuits with unbalanced linear time-invariant (LTI) loads, supplied in a three-wire configuration with a sinusoidal, but asymmetrical voltage, which was developed in paper [19], to similar loads supplied with a nonsinusoidal voltage. In general, this paper presents some new results obtained in investigations on the power theory development. These studies initiated in 1892 by Steinmetz [1], were continued for the whole XX century [2-12] and even now. The most important in these studies, due to amount of energy transferred, are power properties of three-phase circuits. Unfortunately, it was not possible to explain and describe power properties of such circuits in a right way before power properties of single-phase circuits were not explained. Moreover, studies on power properties of three-phase systems were substantially hampered by a wrong definition of the apparent power S introduced [3, 6] to electrical engineering by the American Institute of Electrical Engineers (AIEE) and supported by IEEE Standard Dictionary of Electrical and Electronic Terms [15].

Power properties of single-phase circuits with LTI loads and nonsinusoidal voltage were at last explained [13] in the frame of the CPC in 1984. A right definition of the apparent power S for three-phase circuits was selected in [16].

Results presented in [13] and on [16] have created conditions for the development of the power theory of three-phase circuits. First results were obtained in 1988 [14], but still the development of the power theory of three-phase circuits is delayed with respect to practical situations.

The current and consequently, also the supply voltage asymmetry in distribution systems is mainly caused by aggregates of single-phase loads of different power, which form three-phase unbalanced loads, as it is shown in Fig. 1. Single-phase loads are mainly composed of fluorescent lumps, video and computer-like appliances or microwave ovens. Such devices are classified as harmonics generating loads (HGL) and cause the current and consequently, the voltage distortion in the distribution system. These could be residential distribution systems or commercial buildings with particular floors supplied from different phases. The voltage asymmetry in distribution systems can also be caused by high power three-phase loads that draw current from only one or two lines, such as for example, traction loads, or AC arc furnaces with an extinct arc [17, 18].

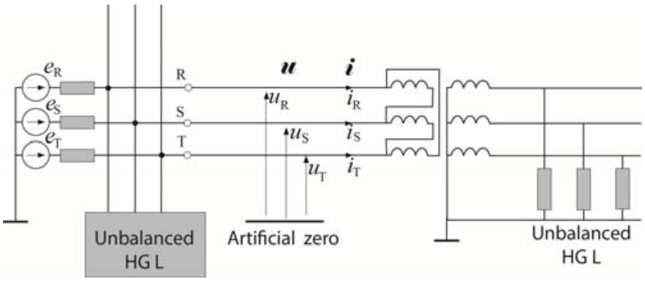

Fig. 1. Three-phase, three-wire system with aggregates of single-phase loads

Power properties of three-phase LTI unbalanced loads in three-phase circuits with asymmetrical, but sinusoidal supply voltage were described, using the CPC concept, in [19]. Power properties of such LTI loads with asymmetrical, but nonsinusoidal voltage are the subject of this paper. It means that harmonics generating loads are approximated in this paper by LTI loads, i.e., it is assumed that the load current distortion is caused only by harmonics present in the supply voltage.

This paper can be regarded as a continuation of studies presented in the paper [19]. The approach to analysis of the power properties in terms of the CPC, as well as main symbols remain the same. Consequently, it would be highly recommended that the reader of this paper is acquainted with paper [19].

The symbols in [19] were used for description of three-phase circuits with sinusoidal voltages and currents. These symbols have to be first modified to make possible of using them for description of similar circuits with nonsinusoidal voltages and currents.

Symbols

The internal voltage of the distribution system, expressed in the form of a three-phase vector

$$\boldsymbol{e} = [\boldsymbol{e}_{\mathrm{R}}, \boldsymbol{e}_{\mathrm{S}}, \boldsymbol{e}_{\mathrm{T}}]^{\mathrm{T}}$$

can be asymmetrical and distorted. The line voltages, arranged in a vector

$$\boldsymbol{u} = [u_{\mathrm{R}}, u_{\mathrm{S}}, u_{\mathrm{T}}]^{\mathrm{T}}$$

are referenced to an artificial zero, as it is shown in Fig. 2, so that they do not contain symmetrical component of the zero sequence $\boldsymbol{e}^{\mathrm{z}}$. The voltage at the load terminals R, S and T contains only the positive $\boldsymbol{u}^{\mathrm{p}}$ and the negative $\boldsymbol{u}^{\mathrm{n}}$ sequence components.

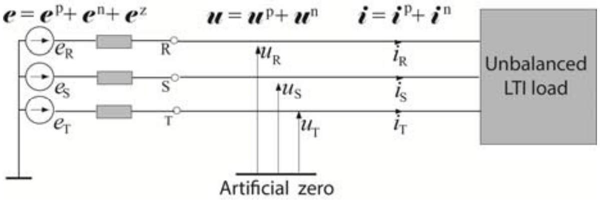

Fig. 2. LTI load supplied with a voltage referenced to artificial zero

We can assume that all voltages and currents, denoted generally by $x(t)$, are periodic and can be expressed in terms of their harmonics $x_n(t)$ and presented in a complex form of the Fourier series, namely, as

$$x(t) = \sum_{n\in N} x_n(t) = X_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{X}_n\, e^{jn\omega_1 t} \tag{1}$$

where N denotes the set of harmonic orders n and

$$\boldsymbol{X}_n = X_n\, e^{j\alpha_n} = \frac{\sqrt{2}}{T}\int_0^T x(t)\, e^{-jn\omega_1 t} dt \tag{2}$$

is the complex rms (crms) value of the n^{th} order harmonic. Modern measurement instruments calculate these values digitally. Instead of using (2), they process samples of voltages or currents x_k, provided by voltage or current sensors and analog-to-digital (A/D) converters. Such digital instruments can calculate the crms values $\boldsymbol{X}_n$ using the Discrete Fourier Transform (DFT), namely

$$\boldsymbol{X}_n = \frac{\sqrt{2}}{K}\sum_{k=0}^{k=K-1} x_k\, e^{-j\frac{2\pi n}{K}k} \tag{3}$$

where K denotes the number of samples in one period T of the supply voltage variability. To avoid the spectrum aliasing, the number of samples per period should be, according to the Nyquist Criterion, higher than the double value of the highest order harmonic of the sampled quantity. Moreover, to enable reduction of calculation with the Fast Fourier Transform (FFT), the number of samples K per period T should be an integer power of 2.

At such assumption, the voltage vector can be expressed as

$$\boldsymbol{u}(t) \stackrel{\mathrm{df}}{=} \begin{bmatrix} u_{\mathrm{R}}(t) \\ u_{\mathrm{S}}(t) \\ u_{\mathrm{T}}(t) \end{bmatrix} = \sum_{n\in N} \boldsymbol{u}_n(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{4}$$

and similarly, the vector of line currents

$$\boldsymbol{i}(t) \stackrel{\mathrm{df}}{=} \begin{bmatrix} i_{\mathrm{R}}(t) \\ i_{\mathrm{S}}(t) \\ i_{\mathrm{T}}(t) \end{bmatrix} = \sum_{n\in N} \boldsymbol{i}_n(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \begin{bmatrix} \boldsymbol{I}_{\mathrm{R}n} \\ \boldsymbol{I}_{\mathrm{S}n} \\ \boldsymbol{I}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_n\, e^{jn\omega_1 t}. \tag{5}$$

The vector of the supply voltage $\boldsymbol{u}(t)$ as referenced to an artificial zero of the circuit can be decomposed into symmetrical components of the positive and the negative sequence as follows

$$\begin{aligned} \boldsymbol{u}(t) &= \sum_{n\in N} \boldsymbol{u}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n\, e^{jn\omega_1 t} \\ &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} (\boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{U}_n^{\mathrm{n}})\, e^{jn\omega_1 t} \\ &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} (1^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}} + 1^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}})\, e^{jn\omega_1 t} = \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}. \end{aligned} \tag{6}$$

Symbol 1^{p} denotes a symmetrical unit vector of the positive sequence, while 1^{n} denotes a symmetrical unit vector of the negative sequence, defined as

$$1^{\mathrm{p}} \stackrel{\mathrm{df}}{=} \begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{-j2\pi/3} \\ 1e^{j2\pi/3} \end{bmatrix}, \quad 1^{\mathrm{n}} \stackrel{\mathrm{df}}{=} \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{j2\pi/3} \\ 1e^{-j2\pi/3} \end{bmatrix} \tag{7}$$

and shown in Fig. 3.

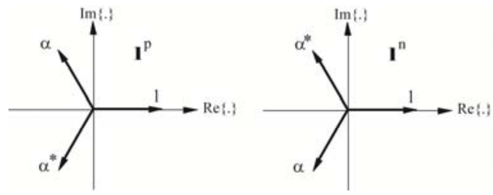

Fig. 3 Three-phase symmetrical unit vectors 1^{p} and 1^{n}

Symbols $\boldsymbol{U}_n^{\mathrm{p}}, \boldsymbol{U}_n^{\mathrm{n}}$ in (6) denote the crms values of symmetrical components of the supply voltage harmonic of the n^{th} order. They are equal to

$$\begin{bmatrix} \boldsymbol{U}_n^{\mathrm{p}} \\ \boldsymbol{U}_n^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix}. \tag{8}$$

When the supply voltage is symmetrical, then its harmonics are also symmetrical and they have the sequence dependent on the harmonic order n. Harmonics of the order $n = (3k+1)$ are of the positive sequence; harmonics of the order $n = (3k-1)$ are of the negative sequence and harmonics of the order $n = 3k$ are of the zero sequence. These last harmonics of the zero sequence are not visible in line-to-line voltages of the load. They are visible in the line-to-ground voltage of the supply source, however, affecting the three-phase rms value of that voltage $||\boldsymbol{e}||$. To avoid the effect of such zero sequence harmonics on the power factor, the zero sequence has to be eliminated from the load line voltages by referencing them to the artificial zero, as this is illustrated in Fig. 2.

When the supply voltage is asymmetrical, then the voltage harmonics can contain symmetrical components of all orders, however. In particular, the third order harmonic can exist both in the load voltage and its current, because when the supply voltage is asymmetrical, the third order harmonic is not exclusively of the zero sequence. It can have symmetrical components of the positive and the negative sequence as well.

Currents' Physical Components (CPC)

The load in Fig. 4(a) is equivalent with respect to active power P to a balanced resistive load, shown in Fig. 4(b), of conductance

(9) $$G_{\rm b} = \frac{P}{||\boldsymbol{u}||^2}$$

where $||\boldsymbol{u}||$ denotes three-phase rms value of the supply voltage, equal to the root of sum of squares of line voltages rms values, namely

(10) $$||\boldsymbol{u}|| = \sqrt{||u_{\rm R}||^2 + ||u_{\rm S}||^2 + ||u_{\rm T}||^2}\ .$$

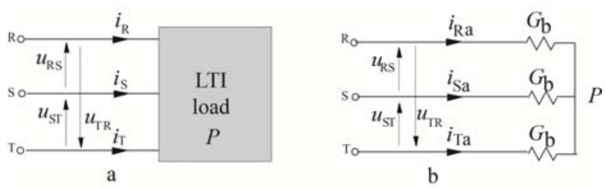

Fig. 4. A three-phase load and a balanced resistive load equivalent with respect to active power P

The current of such an equivalent load is

(11) $$\boldsymbol{i}_{\rm a} = G_{\rm b}\,\boldsymbol{u} = \sqrt{2}\,{\rm Re}\sum_{n\in N} G_{\rm b}\,\boldsymbol{U}_n\, e^{jn\omega_1 t} = \sqrt{2}\,{\rm Re}\sum_{n\in N} G_{\rm b}\,(\mathbf{1}^{\rm p}\boldsymbol{U}_n^{\rm p} + \mathbf{1}^{\rm n}\boldsymbol{U}_n^{\rm n})e^{jn\omega_1 t}$$

and will be referred to as ***the active current*** of the load.

According to the assumption in this paper, the load is linear and time-invariant, so that the current response of the load to this voltage can be calculated harmonic-by-harmonic.

The load at each harmonic frequency has the active and reactive powers. For a harmonic of the $n^{\rm th}$ order

(12) $$P_n = {\rm Re}\{\boldsymbol{U}_n^{\rm T}\boldsymbol{I}_n^*\},\qquad Q_n = {\rm Im}\{\boldsymbol{U}_n^{\rm T}\boldsymbol{I}_n^*\}\ .$$

The load can be unbalanced for the $n^{\rm th}$ order harmonic, but with respect to the active and reactive powers P_n and Q_n at voltage $\boldsymbol{u}_n$ such a load is equivalent to a balanced load of the phase admittance

(13) $$\boldsymbol{Y}_{n\rm b} = G_{n\rm b} + jB_{n\rm b} = \frac{P_n - jQ_n}{||\boldsymbol{u}_n||^2} = \frac{\boldsymbol{C}_n^*}{||\boldsymbol{u}_n||^2}$$

where $||\boldsymbol{u}_n||$ denotes the three-phase rms value of the $n^{\rm th}$ order voltage harmonic, equal to

(14) $$||\boldsymbol{u}_n|| = \sqrt{U_{{\rm R}n}^2 + U_{{\rm S}n}^2 + U_{{\rm T}n}^2}\ .$$

The symbol „$\boldsymbol{C}$" instead of a common symbol „$\boldsymbol{S}$" is used in definition (13) to avoid a confusion of the complex power P_n+jQ_n with the apparent power S_n which can contain also components other than only the active and reactive powers.

A balanced load which is equivalent to the original one for the $n^{\rm th}$ order harmonic with respect to active and reactive powers P_n and Q_n is shown in Fig. 5.

The supply current of such an equivalent load is composed of the ***active current***

(15) $$\boldsymbol{i}_{n\rm a} = G_{n\rm b}\,\boldsymbol{u}_n = \sqrt{2}\,{\rm Re}\{G_{n\rm b}\,(\boldsymbol{U}_n^{\rm p} + \boldsymbol{U}_n^{\rm n})e^{jn\omega_1 t}\} = \sqrt{2}\,{\rm Re}\{G_{n\rm b}\,(\mathbf{1}^{\rm p}\boldsymbol{U}_n^{\rm p} + \mathbf{1}^{\rm n}\boldsymbol{U}_n^{\rm n})e^{jn\omega_1 t}\}$$

and the ***reactive current***

(16) $$\boldsymbol{i}_{n\rm r} = B_{n\rm b}\,\boldsymbol{u}_n(t + \frac{T}{4n}) = \sqrt{2}\,{\rm Re}\{jB_{n\rm b}\,(\boldsymbol{U}_n^{\rm p} + \boldsymbol{U}_n^{\rm n})e^{jn\omega_1 t}\} = \sqrt{2}\,{\rm Re}\{jB_{n\rm b}\,(\mathbf{1}^{\rm p}\boldsymbol{U}_n^{\rm p} + \mathbf{1}^{\rm n}\boldsymbol{U}_n^{\rm n})e^{jn\omega_1 t}\}.$$

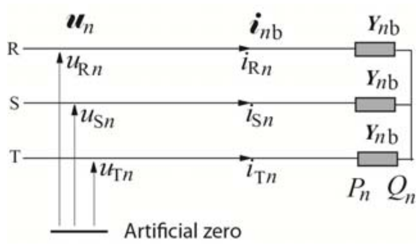

Fig. 5. A balanced load equivalent to the original one with respect to P_n and Q_n powers of the $n^{\rm th}$ order harmonic

Admittance $\boldsymbol{Y}_{n\rm b}$ is admittance of an equivalent balanced load for the $n^{\rm th}$ order harmonic, while for such a harmonic the load can be unbalanced. Consequently, the $n^{\rm th}$ order harmonic of the load current $\boldsymbol{i}_n$ can contain the ***unbalanced current***

(17) $$\boldsymbol{i}_{n\rm u} = \boldsymbol{i}_n - \boldsymbol{i}_{n\rm b} = \boldsymbol{i}_n - (\boldsymbol{i}_{n\rm a} + \boldsymbol{i}_{n\rm r}) = \sqrt{2}\,{\rm Re}\{\boldsymbol{I}_n - \boldsymbol{Y}_{n\rm b}\,(\mathbf{1}^{\rm p}\boldsymbol{U}_n^{\rm p} + \mathbf{1}^{\rm n}\boldsymbol{U}_n^{\rm n})e^{jn\omega_1 t}\}.$$

It means that each current harmonic $\boldsymbol{i}_n$ can be regarded as a sum of three components

(18) $$\boldsymbol{i}_n = \boldsymbol{i}_{n\rm a} + \boldsymbol{i}_{n\rm r} + \boldsymbol{i}_{n\rm u}$$

and consequently, the load current (6) is equal to

(19) $$\boldsymbol{i}(t) = \begin{bmatrix} i_{\rm R}(t) \\ i_{\rm S}(t) \\ i_{\rm T}(t) \end{bmatrix} = \sum_{n\in N}\boldsymbol{i}_n(t) = \sum_{n\in N}(\boldsymbol{i}_{n\rm a} + \boldsymbol{i}_{n\rm r} + \boldsymbol{i}_{n\rm u}).$$

The current

(20) $$\sum_{n\in N}\boldsymbol{i}_{n\rm r} \stackrel{\rm df}{=} \boldsymbol{i}_{\rm r} = \sqrt{2}\,{\rm Re}\sum_{n\in N} jB_{n\rm b}\,(\mathbf{1}^{\rm p}\boldsymbol{U}_n^{\rm p} + \mathbf{1}^{\rm n}\boldsymbol{U}_n^{\rm n})e^{jn\omega_1 t}$$

occurs in the load current because of the phase-shift of the load current harmonics with respect to the supply voltage harmonics. Therefore it can be regarded as a reactive current of the load.

The current

(21) $$\sum_{n\in N}\boldsymbol{i}_{n\rm u} \stackrel{\rm df}{=} \boldsymbol{i}_{\rm u}$$

occurs in the load current because of the load imbalance for harmonic frequencies.

The current

(22) $$\sum_{n\in N}\boldsymbol{i}_{n\rm a} = \sqrt{2}\,{\rm Re}\sum_{n\in N} G_{n\rm b}\,(\mathbf{1}^{\rm p}\boldsymbol{U}_n^{\rm p} + \mathbf{1}^{\rm n}\boldsymbol{U}_n^{\rm n})e^{jn\omega_1 t}$$

is not the active current $\boldsymbol{i}_{\rm a}$ of the load, however. These two currents differ by

(23) $$\sum_{n\in N}\boldsymbol{i}_{n\rm a} - \boldsymbol{i}_{\rm a} = \sqrt{2}\,{\rm Re}\sum_{n\in N}(G_{n\rm b} - G_{\rm b})\,\boldsymbol{U}_n\, e^{jn\omega_1 t} = \sqrt{2}\,{\rm Re}\sum_{n\in N}(G_{n\rm b} - G_{\rm b})(\mathbf{1}^{\rm p}\boldsymbol{U}_n^{\rm p} + \mathbf{1}^{\rm n}\boldsymbol{U}_n^{\rm n})e^{jn\omega_1 t} \stackrel{\rm df}{=} \boldsymbol{i}_{\rm s}.$$

This difference can have a non-zero value only when the conductances $G_{n\rm b}$ for harmonic frequencies are different than the conductance G_b of the equivalent balanced load. Since conductances for harmonic frequencies G_{nb} are usually scattered around G_b, therefore, the current $\boldsymbol{i}_s$ which is an effect of this scatter, will be referred to as a ***scattered current***.

Combining (11) – (23), the load current can be expressed as

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_s + \boldsymbol{i}_r + \boldsymbol{i}_u \,. \tag{24}$$

Each of these four currents is associated with a different physical phenomenon in the load. The active current $\boldsymbol{i}_a$ is associated with the phenomenon of permanent energy delivery, with active power P, to the load. Any other current component does not contribute to this transfer. The scattered current $\boldsymbol{i}_s$ is associated with the phenomenon of a change of the load conductance G_{nb} with harmonic order n. The reactive current $\boldsymbol{i}_r$ is associated with the phenomenon of a phase-shift of the load current harmonics with respect to the supply voltage harmonics. The unbalanced current $\boldsymbol{i}_u$ is associated with the load imbalance for harmonic frequencies. Because of this association of the load currents components $\boldsymbol{i}_a, \boldsymbol{i}_s, \boldsymbol{i}_r$ and $\boldsymbol{i}_u$ with physical phenomena in the load, these currents are referred to as the Currents' Physical Components (CPC). It does not mean that these currents do exist physically, however. They are mathematical, rather than physical entities. Nonetheless, if any of above described physical phenomenon exists in the load, then the load current contains a component associated with this phenomenon.

Orthogonality of CPC

Current components in decomposition (24) affect the three-phase rms value $||\boldsymbol{i}||$ of the load current independently of each other on the condition that they are mutually orthogonal, meaning that their scalar product

$$(\boldsymbol{i}_x, \boldsymbol{i}_y) = \frac{1}{T}\int_0^T \boldsymbol{i}_x^{\mathrm{T}}(t)\, \boldsymbol{i}_y(t)\, dt \tag{25}$$

is equal to zero.

Harmonics of different order n are mutually orthogonal. Therefore, calculation of the three-phase rms value of quantities that are a sum of harmonics is straightforward. In particular, the three-phase rms value of the reactive, scattered and the unbalanced currents are equal to

$$||\boldsymbol{i}_r|| = \sqrt{\sum_{n\in N} ||\boldsymbol{i}_{nr}||^2} = \sqrt{\sum_{n\in N} B_{nb}^2\, ||\boldsymbol{u}_n||^2} \tag{26}$$

$$||\boldsymbol{i}_s|| = \sqrt{\sum_{n\in N} ||\boldsymbol{i}_{ns}||^2} = \sqrt{\sum_{n\in N} (G_{nb} - G_b)^2\, ||\boldsymbol{u}_n||^2} \tag{27}$$

$$||\boldsymbol{i}_u|| = \sqrt{\sum_{n\in N} ||\boldsymbol{i}_{nu}||^2} \,. \tag{28}$$

Mutual orthogonality of the active, reactive and unbalanced currents in circuits with sinusoidal voltages and currents was proven in [19]. This applies, of course, to individual harmonic of any order n. Therefore, the active, reactive and unbalanced components of the n^{th} order harmonic of the load current $\boldsymbol{i}_n$ in decomposition (24) are mutually orthogonal, i.e.,

$$(\boldsymbol{i}_{na},\, \boldsymbol{i}_{nr}) = (\boldsymbol{i}_{na},\, \boldsymbol{i}_{nu}) = (\boldsymbol{i}_{nr},\, \boldsymbol{i}_{nu}) = 0 \,. \tag{29}$$

The CPC in decomposition (24) are sums of harmonics. Because harmonics of different order r and s, are mutually orthogonal, then the scalar products of two currents, which are sums of harmonics, can be expressed generally as

$$(\boldsymbol{i}_x, \boldsymbol{i}_y) = (\sum_{r\in N} \boldsymbol{i}_{rx}, \sum_{s\in N} \boldsymbol{i}_{sy}) = \sum_{n\in N} (\boldsymbol{i}_{nx}, \boldsymbol{i}_{ny}) \,. \tag{30}$$

Thus, if harmonics of the same order n of two currents are mutually orthogonal, i.e., $(\boldsymbol{i}_{nx}, \boldsymbol{i}_{ny}) = 0$, then such currents are orthogonal as well. Consequently, all terms on the right side of (24) are mutually orthogonal and hence

$$||\boldsymbol{i}||^2 = ||\sum_{n\in N} \boldsymbol{i}_{na}||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2 \,. \tag{31}$$

Since

$$\sum_{n\in N} \boldsymbol{i}_{na} = \boldsymbol{i}_a + \boldsymbol{i}_s \tag{32}$$

the question arises, are the active and scattered currents mutually orthogonal or not?

The scalar product defined by (25) in time-domain, can be calculated in the frequency-domain as follows

$$(\boldsymbol{i}_x, \boldsymbol{i}_y) = \frac{1}{T}\int_0^T \boldsymbol{i}_x^{\mathrm{T}}(t)\, \boldsymbol{i}_y(t)\, dt = \mathrm{Re} \sum_{n\in N} \boldsymbol{I}_{xn}^{\mathrm{T}} \boldsymbol{I}_{yn}^{*} \,. \tag{33}$$

Therefore, the scalar product of the active and the scattered currents is equal to

$$\begin{aligned} (\boldsymbol{i}_a, \boldsymbol{i}_s) &= \mathrm{Re} \sum_{n\in N} \boldsymbol{I}_{an}^{\mathrm{T}} \boldsymbol{I}_{sn}^{*} = \mathrm{Re} \sum_{n\in N} G_b\, \boldsymbol{U}_n^{\mathrm{T}} (G_{nb} - G_b)\, \boldsymbol{U}_n^{*} = \\ &= G_b \sum_{n\in N} (G_{nb} - G_b) ||\boldsymbol{u}_n||^2 = \\ &= G_b (\sum_{n\in N} G_{nb}\, ||\boldsymbol{u}_n||^2 - G_b \sum_{n\in N} ||\boldsymbol{u}_n||^2) = G_b (P - P) = 0. \end{aligned} \tag{34}$$

Thus these two currents are also mutually orthogonal, so that the three-phase rms values of the load CPC satisfy the relationship

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2 \,. \tag{35}$$

This relationship between the load CPC three-phase rms values provides clear information on how specific phenomena in the load contribute to the load current three-phase rms value increase. It increases above the minimum three-phase rms value $||\boldsymbol{i}_a||$, needed for permanent energy delivery with the averaged rate equal to the active power P; because of change of the load conductance G_{nb} with harmonic order; because of a phase-shift between the voltage and current harmonics and because of the load current asymmetry. The scattered, reactive and unbalanced currents are associated with these three phenomena.

It is important to observe that decomposition of the load current into the Currents' Physical Components does not require any knowledge on the load structure and its parameters. It can be done based on measurements performed at the load terminals. Such measurement has to provide only the crms values of the load voltage and current harmonics, i.e., the values $\boldsymbol{U}_{Rn}$, $\boldsymbol{U}_{Sn}$, $\boldsymbol{U}_{Tn}$, $\boldsymbol{I}_{Rn}$, $\boldsymbol{I}_{Sn}$, and $\boldsymbol{I}_{Tn}$.

Numerical illustration. The presented above current decomposition into CPC is illustrated numerically with the circuit shown in Fig. 6. This decomposition into CPC, as presented in the previous chapter, was obtained without any restrictions as to the level of the supply voltage asymmetry, its distortion and the load imbalance. Therefore, to demonstrate that this decomposition is valid independently of the supply voltage asymmetry and distortion, and independently of the load imbalance, very high level of them was assumed in this illustration. They are much higher than could be observed in distribution systems. These, rather unrealistic assumptions, can enhance credibility of the developed current decomposition, however.

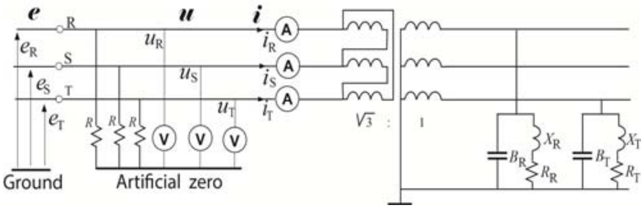

Fig. 6. Circuit used for numerical illustration

It is assumed in this illustration that the internal voltage e_R of the distribution system is

$$e_R(t) = \sqrt{2}\ \mathrm{Re}\{100e^{j\omega_1 t} + 30e^{j3\omega_1 t} + 20e^{j5\omega_1 t} + 10e^{j7\omega_1 t}\}\ \mathrm{V}$$

i.e., it is strongly distorted by harmonics of the 3rd; 5th and the 7th order, thus the set N is equal to

$$N = \{1, 3, 5, 7\}.$$

It is assumed that the voltage at terminals S and T are:

$$e_S(t) = e_R(t - T/3), \qquad e_T(t) = 0.5\, e_R(t + T/3)$$

thus the supply voltage is strongly asymmetrical. The load parameters for the fundamental harmonic are assumed to be equal to

$$R_R = X_R = R_T = X_T = 1.0\ \Omega, \quad B_R = B_T = 0.5\ \mathrm{S}.$$

The load is supplied from an ideal transformer in Δ/Y configuration with the turn ratio: $\sqrt{3} : 1$. The line S on the transformer secondary side is not loaded.

The crms values of the zero sequence component of the supply voltage harmonics are equal to

$$\boldsymbol{U}_1^z = 16.67e^{-j60^o}\ \mathrm{V}, \quad \boldsymbol{U}_3^z = 25.0\ \mathrm{V},$$
$$\boldsymbol{U}_5^z = 3.33\, e^{j60^o}\ \mathrm{V}, \quad \boldsymbol{U}_1^z = 1.67e^{-j60^o}\ \mathrm{V}.$$

The crms values of harmonics of lines R, S and T voltages referenced to the artificial zero are

$$U_1 = \begin{bmatrix}\boldsymbol{U}_{R1}\\ \boldsymbol{U}_{S1}\\ \boldsymbol{U}_{T1}\end{bmatrix} = \begin{bmatrix}92.8e^{j8.9^o}\\ 92.8e^{-j128.9^o}\\ 66.7e^{j120.0^o}\end{bmatrix}\mathrm{V}, \quad U_3 = \begin{bmatrix}\boldsymbol{U}_{R3}\\ \boldsymbol{U}_{S3}\\ \boldsymbol{U}_{T3}\end{bmatrix} = \begin{bmatrix}5.0e^{j0.0^o}\\ 5.0e^{j0.0^o}\\ 10.0e^{j180.0^o}\end{bmatrix}\mathrm{V}$$

$$U_5 = \begin{bmatrix}\boldsymbol{U}_{R5}\\ \boldsymbol{U}_{S5}\\ \boldsymbol{U}_{T5}\end{bmatrix} = \begin{bmatrix}18.6e^{-j8.9^o}\\ 18.6e^{j128.9^o}\\ 13.3e^{-j120.0^o}\end{bmatrix}\mathrm{V}, \quad U_7 = \begin{bmatrix}\boldsymbol{U}_{R7}\\ \boldsymbol{U}_{S7}\\ \boldsymbol{U}_{T7}\end{bmatrix} = \begin{bmatrix}9.3e^{j8.9^o}\\ 9.3e^{-j128.9^o}\\ 6.7e^{j120.0^o}\end{bmatrix}\mathrm{V}$$

while the crms values of the load current harmonics:

$$I_1 = \begin{bmatrix}\boldsymbol{I}_{R1}\\ \boldsymbol{I}_{S1}\\ \boldsymbol{I}_{T1}\end{bmatrix} = \begin{bmatrix}139.2e^{j8.9^o}\\ 86.6e^{-j150.0^o}\\ 66.1e^{j160.9^o}\end{bmatrix}\mathrm{A}, \quad I_3 = \begin{bmatrix}\boldsymbol{I}_{R3}\\ \boldsymbol{I}_{S3}\\ \boldsymbol{I}_{T3}\end{bmatrix} = \begin{bmatrix}18.1e^{j85.2^o}\\ 0\\ 18.1e^{-j94.8^o}\end{bmatrix}\mathrm{A}$$

$$I_5 = \begin{bmatrix}\boldsymbol{I}_{R5}\\ \boldsymbol{I}_{S5}\\ \boldsymbol{I}_{T5}\end{bmatrix} = \begin{bmatrix}128.5e^{j80.1^o}\\ 79.9e^{-j120.1^o}\\ 61.1e^{-j71.8^o}\end{bmatrix}\mathrm{A}, \quad I_7 = \begin{bmatrix}\boldsymbol{I}_{R7}\\ \boldsymbol{I}_{S7}\\ \boldsymbol{I}_{T7}\end{bmatrix} = \begin{bmatrix}93.5e^{j98.6^o}\\ 58.2e^{-j60.3^o}\\ 44.4e^{-j109.4^o}\end{bmatrix}\mathrm{A}.$$

The rms values of the line voltages and currents are shown in Fig. 7.

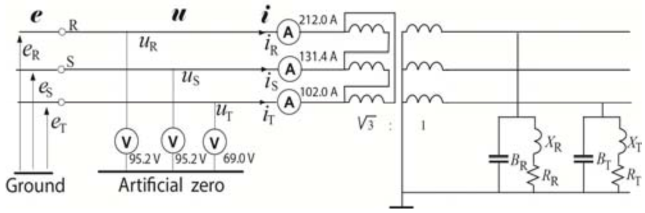

Fig. 7. Rms values of the line-to-artificial zero voltages and line currents in the circuit shown in Fig. 6

Results of the circuit analysis with respect to the load equivalent parameters for harmonics and harmonic active and reactive powers are compiled in Table.1.

Table 1. Results of the circuit analysis

	n	1	3	5	7
P_n	W	23750	23	73	9
Q_n	VAr	0	2700	43846	15960
$\|\|\boldsymbol{u}_n\|\|$	V	147.20	12.25	29.44	14.72
G_{nb}	S	1.0962	0.1500	0.0843	0.0438
B_{nb}	S	0	1.800	5.059	7.366

The active power of the load is P = 23855 W and the three-phase rms value of the supply voltage as referenced to the artificial zero is $||\boldsymbol{u}||$ = 151.33 V, so that, the equivalent balanced conductance of the load is

$$G_b = \frac{P}{||\boldsymbol{u}||^2} = 1.0417\ \mathrm{S}.$$

Having values of equivalent parameters of the load as compiled in Table 1, three-phase rms values of all Currents' Physical Components of the load current can be calculated, namely

$$||\boldsymbol{i}_a|| = G_b||\boldsymbol{u}|| = 157.6\ \mathrm{A}$$

$$||\boldsymbol{i}_r|| = \sqrt{\sum_{n\in N} B_{nb}^2\, ||\boldsymbol{u}_n||^2} = 185.5\ \mathrm{A}$$

$$||\boldsymbol{i}_s|| = \sqrt{\sum_{n\in N} (G_{nb} - G_b)^2\, ||\boldsymbol{u}_n||^2} = 34.5\ \mathrm{A}$$

$$||\boldsymbol{i}_u|| = \sqrt{||\boldsymbol{i}||^2 - (||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2)} = 110.4\ \mathrm{A}.$$

The power factor of the load is equal to

$$\lambda = \frac{P}{S} = \frac{||\boldsymbol{i}_a||}{||\boldsymbol{i}||} = \frac{||\boldsymbol{i}_a||}{\sqrt{||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2}} = 0.58.$$

This last result provides us with information on how specific properties of the load contribute to the power factor reduction, i.e., to an increase in the load current three-phase rms value $||\boldsymbol{i}||$.

Power equation

Although decomposition of the load current into the Currents' Physical Components and calculation or measurement of all three-phase rms values of these components provide full information on the power properties of the load, these properties are commonly specified in the electrical engineering community in terms of powers. To meet such expectations, let us multiply (35) by the square of the three-phase rms value of the supply voltage referenced to the artificial zero $||\boldsymbol{u}||$, namely

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2 \ \Big|\ \mathrm{x}\ ||\boldsymbol{u}||^2.$$

and the power equation

(36) $$S^2 = P^2 + D_s^2 + Q^2 + D_u^2$$

is obtained. In this power equation

(37) $$S = ||\boldsymbol{u}||\,||\boldsymbol{i}||$$

is the apparent power,

(38) $$D_s = ||\boldsymbol{u}||\,||\boldsymbol{i}_s||$$

is the scattered power,

(39) $$Q = ||\boldsymbol{u}||\,||\boldsymbol{i}_r||$$

is the reactive power and

(40) $$D_u = ||\boldsymbol{u}||\,||\boldsymbol{i}_u||$$

is the unbalanced power of the load.

Powers in equation (36), apart from the active power P, which has also a clear physical interpretation, are only formal products of three-phase rms values of the supply voltage and the load Currents' Physical Components. Nonetheless, apart from the apparent power S, all these powers are associated with distinctive physical phenomena in the load.

Conclusions

This paper shows that power properties of unbalanced linear time-invariant loads supplied by a three-wire line from a source of asymmetrical and nonsinusoidal voltage can be described with the Currents Physical Components – based power theory. The supply current of such loads consists of four mutually orthogonal components, namely the active, scattered, reactive and unbalanced currents, each of which is associated with a distinctive physical phenomenon.

Presented in this paper results conclude development of the CPC-based power theory of three-phase circuits with LTI loads supplied by three-wire lines from voltage sources with periodic voltages.

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