Andriy CZABAN[1], Marek LIS[2], Marcin CHRZAN[1], Andrzej SZAFRANIEC[1] Vitaliy LEVONIUK[3]

University of Technology and Humanities in Radom, Faculty of Transport and Electrical Engineering (1), Czestochowa University of Technology, Faculty of Electrical Engineering (2), Lviv Polytechnic (3)

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Mathematical modelling of transient processes in power supply grid with distributed parameters

Abstract. *This article presents mathematical model of electric power system designed on the basis of interdisciplinary variation method, which method is based on modification of Hamilton-Ostrogradsky principle via expanding Lagrange function. The electric power system comprises two subsystems linked by power supply line with distributed parameters. The subsystems are presented in a simplified form as elements with concentrated parameters. Equations of system state are presented in normal Cauchy form.*

Streszczenie. *W pracy na podstawie interdyscyplinarnej metody wariacyjnej, która opiera się na modyfikacji zasady Hamiltona-Ostrogradskiego drogą rozszerzenia funkcji Lagrange'a, opracowano model matematyczny układu elektroenergetycznego, który składa się z dwu podukładów połączonych miedzy sobą linią zasilania o parametrach rozłożonych. Podukłady przedstawiono w formie uproszczonej, jako elementy o parametrach skupionych. Równania stanu układu przedstawione są w normalnej postaci Cauchy'ego.* (***Modelowanie matematyczne procesów nieustalonych w sieciach zasilających o parametrach rozłożonych***).

Keywords: Hamilton-Ostrogradsky's principle, electric power systems, systems with concentrated parameters, mathematical modelling.
Słowa kluczowe: zasada Hamiltona-Ostrogradskiego, układy elektroenergetyczne, układy o parametrach rozłożonych, modelowanie matematyczne.

Introduction

One of the key elements of energy systems is long power supply line with distributed parameters. All the power supply line relations are functions of time coordinate vs. spatial coordinate, this enforces use of complicated apparatus of applied mathematics while forming equations of line state, in particular, use of nonlinear differential equations with ordinary and partial derivatives. A very important issue concerning the analysed system is the fact that the power supply line is usually a component of one and only national energy system, which makes it necessary to establish very complicated boundary conditions for integration of telegraphers' equations showing relations between voltage and current in any point of the power supply line at any moment of time.

In most countries, power supply lines transmit direct and alternate current depending on the requirements. Direct current lines are used to transmit electricity to distant places at high rated voltage [3], [9].

However, electricity is transmitted most often using three-phase lines of alternate current [8]. The reason for this is simply the fact that electricity receivers requiring alternate current are most frequently accessible. Paper [4] presents design of mathematical model of two- and three-wire power supply line, wherein analysis of processes of high voltage line 500 kV were made. With use of ATP-EMTP software, high voltage lines' breakdowns were studied.

Practical approach to analysis of transient electromagnetic states occurring in power supply line are described in paper [6]. The authors present assumptions and requirements concerning modelling of separate elements of electric power systems (the energy system) in an innovative way.

This article presents mathematical model of electric power system designed on the basis of interdisciplinary approaches. The main element of the system is power supply line with distributed parameters, linking two local subsystems, which subsystems are described using simplified models of mini energetic systems, i.e. sources of EMF with R, L, C load (see figure 1). The subsystems are analysed as systems with concentrated parameters. This allows to present general mathematical model of the analysed object as sophisticated system with parameters that are both concentrated and distributed.

In order to design the mathematical model of the analysed electric power system, this work applies described in paper [1] interdisciplinary variation method, basis of which is modification of Hamilton-Ostrogradsky principle and description of electromagnetic transient states of the system. The main aim of this work is to develop a mathematical model.

Mathematical model of the system

Equation determining analytical waveform of the expanded Lagrange function [1], [3], [7]:

$$L^* = \tilde{T}^* - P^* + \Phi^* - D^* \tag{1}$$

where: L^* – modified Lagrange function, $\tilde{T}^*$ – kinetic coenergy, P^* – potential energy, Φ^* – energy of generalized dissipation forces (dispersion), D^* – energy of external and internal non-potential forces.

Since power supply lines are analysed as system of distributed parameters, then in order to determine functional of operation by Hamilton-Ostogradsky it is necessary to refer to expanded Lagrange function and its linear density [1]:

$$S = \int_{t_1}^{t_2} \left(L^* + \int_l L_l dl \right) dt, \quad I = \int_l L_l dl \tag{2}$$

where: S – functional of operation by Hamilton-Ostogradskyi, L_l – linear density of modified Lagrange function, I –functional of energy.

Modified Lagrange function elements are determined making an assumption that all parameters and functional relations for power transformer windings are described by quantity of windings that are directly connected to power supply line points (figures 1, 2):

$$T^* = \sum_{j=1}^{2} \left(\int_0^{i_{Lj,1}} \Psi_{Lj,1} di_{Lj,1} + \int_0^{i_{Lj,2}} \Psi_{Lj,2} di_{Lj,2} + \right.$$

$$(3)\quad +\int_0^{i_{Pj,1}} \Psi_{Pj,1} di_{Pj,1} + \int_0^{i_{Pj,2}} \Psi_{Pj,2} di_{Pj,2} + \frac{L_{\mathrm{L},j} i^2_{\mathrm{EL},j}}{2} + \frac{L_{\mathrm{LO},j} i^2_{\mathrm{LO},j}}{2} + $$

$$+\frac{L_{\mathrm{P},j} i^2_{\mathrm{EP},j}}{2} + \frac{L_{\mathrm{PO},j} i^2_{\mathrm{PO},j}}{2} \Bigg)$$

$$(4)\qquad P^* = \sum_{j=1}^{2} \left(\frac{Q^2_{\mathrm{LO},j}}{2C_{\mathrm{LO},j}} + \frac{Q^2_{\mathrm{PO},j}}{2C_{\mathrm{PO},j}} \right)$$

$$(5)\qquad \Phi^* = \frac{1}{2}\sum_{j=1}^{2} \left[\int_0^t \left(r_{\mathrm{L}j,1} i^2_{\mathrm{L}j,1} + r_{\mathrm{L}j,2} i^2_{\mathrm{L}j,2} \right) d\tau + \right.$$

$$\left. + \int_0^t \left(r_{\mathrm{P}j,1} i^2_{\mathrm{P}j,1} + r_{\mathrm{P}j,2} i^2_{\mathrm{P}j,2} \right) d\tau \right] + \frac{1}{2}\sum_{j=1}^{2} \int_0^t \Big(R_{\mathrm{L},j} i^2_{\mathrm{EL},j} + R_{\mathrm{LO},j} i^2_{\mathrm{LO},j} +$$

$$+ R_{\mathrm{P},j} i^2_{\mathrm{EP},j} + R_{\mathrm{PO},j} i^2_{\mathrm{PO},j} \Big) d\tau$$

$$(6)\quad D^* = \frac{1}{2}\sum_{j=1}^{2} \int_0^t \left(u_L i_{Lj,2} - \mathrm{V}_{\mathrm{L}\,j} i_{Lj,1} + e_{Lj} i_{\mathrm{EL}j} \right) d\tau +$$

$$+ \frac{1}{2}\sum_{j=1}^{2} \int_0^t \left(u_{\mathrm{P}} i_{\mathrm{P}1,j} - \mathrm{V}_{\mathrm{P},j} i_{\mathrm{P}2,j} + e_{\mathrm{P},j} i_{\mathrm{EP},j} \right) d\tau$$

$$(7)\qquad \frac{\partial T^*}{\partial x} \equiv T_l = \frac{L_0 i^2}{2},\quad \frac{\partial P^*}{\partial x} \equiv P_l = \frac{1}{2C_0} Q_x^2,$$

$$Q_x \equiv \frac{\partial Q}{\partial x},\quad Q_t \equiv \frac{\partial Q}{\partial t} = i,$$

$$(8)\qquad \frac{\partial \Phi^*}{\partial x} \equiv \Phi_l = \Phi_{l3} - \Phi_{lB} = \int_0^t \left(\frac{R_0}{2} Q_t^2 - \frac{\gamma_0}{2C_0^2} Q_x^2 \right)\Bigg|_{t=\tau} d\tau$$

where: L – applied for the left side of the line's power supply, P – applied for the right side, Ψ – linked fluxes, $i(t)$ – currents, $i(x,t)$ – power supply line current, R_0, γ_0, C_0, L_0 – dispersed line parameters, Φ_{R3} – external dissipation of energy, Φ_{RB} – internal dissipation of energy, $Q(x,t)$ – line charge, O – index, used for load circuit, E – additional EMF sources. In equations (3) and (4) left and right side currents $i_{LO} = i_{CO} = i_K$ – are determined as loop current [1].

Transformer windings on the side of energy sources (EMF) are primary.

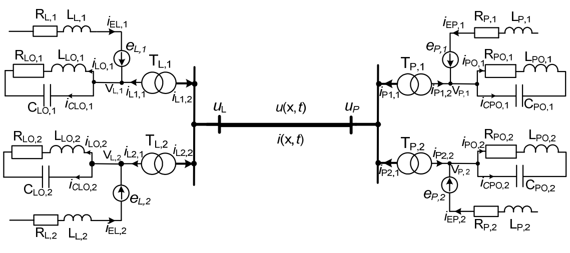

Fig.1. Basic diagram of energy system

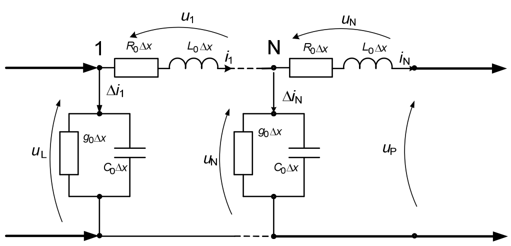

Fig.2. Calculated diagram of power supply line

When designing expanded functional of operation by Hamilton-Ostrogradsky S wherein analytical equations of elements of expanded Lagrange function (3)÷(8) are taken into account, variation of the said functional is obtained [1], [2]. By equating the variation to zero, Euler-Lagrange equation, comprising a mathematical model of the system, is received:

$$(9)\quad \frac{\partial v(x,t)}{\partial t} = \left(C_0 L_0\right)^{-1} \left(\frac{\partial^2 u}{\partial x^2} - (\gamma_0 L_0 + C_0 R_0) v - \gamma_0 R_0 u \right),\ \frac{\partial u}{\partial t} = v$$

$$(10)\qquad \frac{d\Psi_{\mathrm{L}j,1}}{dt} = V_{\mathrm{L}j} - r_{\mathrm{L}j,1} i_{\mathrm{L}j,1}, \qquad j = 1,2$$

$$(11)\qquad \frac{d\Psi_{\mathrm{L}j,2}}{dt} = u_{\mathrm{L}} - r_{\mathrm{L}j,2} i_{\mathrm{L}j,2}, \qquad j = 1,2$$

$$(12)\qquad \frac{d\Psi_{\mathrm{P}j,1}}{dt} = u_{\mathrm{P}} - r_{\mathrm{P}j,1} i_{\mathrm{P}j,1}, \qquad j = 1,2$$

$$(13)\qquad \frac{d\Psi_{\mathrm{P}j,2}}{dt} = V_{\mathrm{P}j} - r_{\mathrm{P}j,2} i_{\mathrm{P}j,2}, \qquad j = 1,2$$

$$(14)\qquad \frac{du_{C\,\mathrm{LO},j}}{dt} = \frac{i_{C\,\mathrm{LO},j}}{C_{\mathrm{LO},j}},\quad u_{C\,\mathrm{LO},j} = \mathrm{V}_{\mathrm{L},j},\quad j = 1,2$$

$$(15)\qquad \frac{du_{C\,\mathrm{PO},j}}{dt} = \frac{i_{C\,\mathrm{PO},j}}{C_{\mathrm{PO},j}},\quad u_{C\,\mathrm{PO},j} = \mathrm{V}_{\mathrm{P},j},\quad j = 1,2$$

$$(16)\qquad \frac{di_{\mathrm{LO},j}}{dt} = \frac{1}{L_{\mathrm{LO},j}} \left(u_{C\,\mathrm{LO},j} - R_{\mathrm{LO},j} i_{\mathrm{LO},j} \right),\quad j = 1,2$$

$$(17)\qquad \frac{di_{\mathrm{PO},j}}{dt} = \frac{1}{L_{\mathrm{PO},j}} \left(u_{C\,\mathrm{PO},j} - R_{\mathrm{PO},j} i_{\mathrm{PO},j} \right),\quad j = 1,2$$

$$(18)\qquad \frac{di_{\mathrm{EL},j}}{dt} = \frac{1}{L_{\mathrm{L},j}} \left(e_{\mathrm{L},j} - R_{\mathrm{L},j} i_{\mathrm{EL},j} \right),\quad j = 1,2$$

$$(19)\qquad \frac{di_{\mathrm{EP},j}}{dt} = \frac{1}{L_{\mathrm{P},j}} \left(e_{\mathrm{P},j} - R_{\mathrm{P},j} i_{\mathrm{EP},j} \right),\quad j = 1,2$$

where: all functional relations of currents and voltages are presented in figures 1 and 2.

In accordance with coordinates conversion theory, equations (10) ÷ (13) of Ψ-type are transformed to equations of A-type, which is done using Lagrange theory concerning conversion of coordinates. This results with the following:

$$(20)\qquad \frac{di_{\mathrm{L}j,1}}{dt} = A_{11\mathrm{L}j}(V_{\mathrm{L}j} - r_{\mathrm{L}j,1} i_{\mathrm{L}j,1}) +$$
$$+ A_{12\mathrm{L}j}(u_{\mathrm{L}} - r_{\mathrm{L}j,2} i_{\mathrm{L}j,2}), \qquad j = 1,2$$

$$(21)\qquad \frac{di_{\mathrm{L}j,2}}{dt} = A_{21\mathrm{L}j}(V_{\mathrm{L}j} - r_{\mathrm{L}j,1} i_{\mathrm{L}j,2}) +$$
$$+ A_{22\mathrm{L}j}(u_{\mathrm{L}} - r_{\mathrm{L}j,2} i_2), \qquad j = 1,2$$

$$(22)\qquad \frac{di_{\mathrm{P}j,1}}{dt} = A_{11\mathrm{P}j}(u_{\mathrm{P}} - r_{\mathrm{P}j,1} i_{\mathrm{P}j,1}) +$$
$$+ A_{12\mathrm{P}j}(\mathrm{V}_{\mathrm{P}j} - r_{\mathrm{P}j,2} i_{\mathrm{P}j,2}), \qquad j = 1,2$$

$$(23)\qquad \frac{di_{\mathrm{P}j,2}}{dt} = A_{21\mathrm{P}j}(u_{\mathrm{P}} - r_{\mathrm{P}j,1} i_{\mathrm{P}j,1}) +$$
$$+ A_{22\mathrm{P}j}(\mathrm{V}_{\mathrm{P}j} - r_{\mathrm{P}j,2} i_{\mathrm{P}j,2}), \qquad j = 1,2$$

where: $A_{km,\ j}$ – coefficients that depend on the inverse inductance of the power transformers [1].

Currents marked in figure 1 in V nodes are defined by the following relationships:

$$(24)\qquad i_{\mathrm{EL},j} + i_{\mathrm{L}j,1} - i_{\mathrm{LO},j} - i_{C\,\mathrm{LO},j} = 0, \quad j = 1,2$$

$$(25)\qquad i_{\mathrm{EP},j} + i_{\mathrm{P}j,1} - i_{\mathrm{PO},j} - i_{C\mathrm{PO},j} = 0, \quad j = 1,2$$

Links between line elements are determined on the basis of the second Kirchhoff law for electric circuits of distributed parameters [5] by means of the following equation:

$$(26)\qquad -\frac{\partial u(x,t)}{\partial x} = R_0 i(x,t) + L_0 \frac{\partial i(x,t)}{\partial t}$$

By discretizing equations (9) and (26) using finite difference method [1], the following is obtained:

$$(27)\qquad \frac{dv_j}{dt} = \left(C_0 L_0\right)^{-1}\left(\frac{u_{j-1} - 2u_j + u_{j+1}}{(\Delta x)^2} - (g_0 L_0 + C_0 R_0) v_j - g_0 R_0 u_j\right), \quad \frac{du_j}{dt} = v_j, \quad j = 2,...,N-1$$

$$(28)\qquad -\frac{u_{j+1} - u_{j-1}}{2\Delta x} = R_0 i_j + L_0 \frac{di_j}{dt}, \qquad j = 2,...,N-1$$

Equations (27), (28) for 1-st and N-th node of discretization take the following form:

$$(29)\qquad \frac{dv_k}{dt} = \frac{1}{C_0 L_0}\left[\frac{1}{\left(\Delta x\right)^2}\left(u_{k-1} - 2u_k + u_{k+1}\right) - \left(g_0 L_0 + C_0 R_0\right) v_k - g_0 R_0 u_k\right], \quad \frac{du_k}{dt} = v_k, \; u_1 = u_L$$

$$(30)\qquad -\frac{u_{k+1} - u_{k-1}}{2\Delta x} = R_0 i_k + L_0 \frac{di_k}{dt}, \quad k = 1, N, \quad u_N \neq u_{\mathrm{P}}$$

where: u_0, u_{N+1} – voltages of fake discretization nodes [9]. Looking for the mentioned voltages is the most difficult assignment. The said voltages must be calculated, otherwise determining boundary conditions for equation (8) is not possible [1], [2], [9].

Equations of left (k=1) and right ($k=N$) power supply line end are as follows:

$$(31)\qquad \sum_{j=1}^{2} i_{\mathrm{L}j,2} - i_1 - \Delta i_1 = 0; \qquad \sum_{j=1}^{2} i_{\mathrm{P}j,1} - i_N = 0$$

$$(32)\qquad \Delta i_1 = \Delta i_{1\mathrm{g}} + \Delta i_{1\mathrm{C}} \Rightarrow \frac{d\Delta i_1}{dt} = \frac{d\Delta i_{1\mathrm{g}}}{dt} + \frac{d\Delta i_{1\mathrm{C}}}{dt}$$

$$(33)\qquad \Delta i_{1\mathrm{g}} = \Delta x g_0 u_1, \qquad \Delta i_{1\mathrm{C}} = \Delta x C_0 \frac{du_1}{dt} = \Delta x C_0 v_1$$

Differentiating equations (31) with regard to time, wherein initial conditions are taken into account [1], lets to obtain the following:

$$(34)\qquad \frac{d}{dt}\sum_{j=1}^{2} i_{\mathrm{L}j,2} - \frac{di_1}{dt} - \frac{d\Delta i_1}{dt} = 0, \qquad \frac{d}{dt}\sum_{j=1}^{2} i_{\mathrm{P}j,2} - \frac{di_N}{dt} = 0$$

Upon transforming, equations (30) take the following forms:

$$(35)\qquad \frac{di_1}{dt} = \frac{1}{L_0}\left(\frac{u_0 - u_2}{2\Delta x} - R_0 i_1\right), \; \frac{di_N}{dt} = \frac{1}{L_0}\left(\frac{u_{N-1} - u_{N+1}}{2\Delta x} - R_0 i_N\right)$$

Upon transforming, and with reference to initial conditions, equations (32), (33) take the following forms:

$$(36)\qquad \frac{d\Delta i_{1\mathrm{g}}}{dt} = \Delta x g_0 \frac{du_1}{dt} = \Delta x g_0 v_1, \qquad \frac{d\Delta i_{1\mathrm{C}}}{dt} = \Delta x C_0 \frac{dv_1}{dt}$$

$$(37)\qquad \frac{di_N}{dt} = \frac{1}{\Delta x L_0}\left(\Delta u_N - \Delta x R_0 i_N\right) = \frac{1}{L_0}\left(\frac{u_N - u_{\mathrm{P}}}{\Delta x} - R_0 i_N\right)$$

By solving equations (30), (32) – (37), the following is obtained:

$$(38)\qquad \frac{di_1}{dt} = \frac{d}{dt}\sum_{j=1}^{2} i_{\mathrm{L}j,2} - \Delta x g_0 v_1 - \Delta x C_0 \frac{dv_1}{dt} = \frac{1}{L_0}\left(\frac{u_0 - u_2}{2\Delta x} - R_0 i_1\right)$$

$$(39)\qquad \frac{di_N}{dt} = \frac{d}{dt}\sum_{j=1}^{2} i_{\mathrm{P}j,1} = \frac{1}{L_0}\left(\frac{u_{N-1} - u_{N+1}}{2\Delta x} - R_0 i_N\right)$$

Voltage of the right rail of the energy system is:

$$(40)\qquad u_{\mathrm{P}} = \left(\frac{1}{\Delta x L_0} + \sum_{j=1}^{2} A_{11\mathrm{P}j}\right)^{-1}\left(\frac{u_N}{\Delta x L_0} - \frac{R_0}{L_0} i_N + \sum_{j=1}^{2}\left(A_{11\mathrm{P}j} r_{\mathrm{P}j,1} i_{\mathrm{P}j,1} - A_{12\mathrm{P}j}(\mathrm{V}_{\mathrm{P}j} - r_{\mathrm{P}j,2} i_{\mathrm{P}j,2})\right)\right)$$

Voltage of $\mathrm{V}_{\mathrm{L}j}$, $\mathrm{V}_{\mathrm{P}j}$ nodes are determined using equations (14) and (15) upon calculating equations (24) and (25):

$$(41)\qquad \frac{d\mathrm{V}_{\mathrm{L}j}}{dt} \equiv \frac{du_{C\,\mathrm{LO},j}}{dt} = \frac{i_{\mathrm{EL},j} + i_{\mathrm{L}j,1} - i_{\mathrm{LO},j}}{C_{\mathrm{LO},j}}, \quad j = 1,2$$

$$(42)\qquad \frac{d\mathrm{V}_{\mathrm{P}j}}{dt} = \frac{du_{C\mathrm{PO},j}}{dt} = \frac{i_{\mathrm{EP},j} + i_{\mathrm{P}j,2} - i_{\mathrm{PO},j}}{C_{\mathrm{PO},j}}, \quad j = 1,2$$

Power supply line currents are determined by means of discretization of equation (26) using method of straight lines and applying right derivative [1].

$$(43)\qquad \frac{di_j}{dt} = \frac{1}{L_0 \Delta x}(u_j - u_{j+1}) - \frac{R_0}{L_0} i_j, \quad j = 1,...,N-1$$

Computer simulation results

In order to simplify calculations the simulation is carried out with the assumptions that the left side of power supply line receives energy from two additional short lines which are treated as rails of block transformer, which transformer is powered from generators of unlimited power $T_{\mathrm{L},1}$ and $T_{\mathrm{L},2}$.

At the moment of time t = 0s the line got connected to the source of energy, wherein the above mentioned assumption were applied. Differential equations system is stable, therefore combination of Euler and simple iteration methods was used. In order to find numerical solutions of algebraic equations, method of simple iteration was applied.

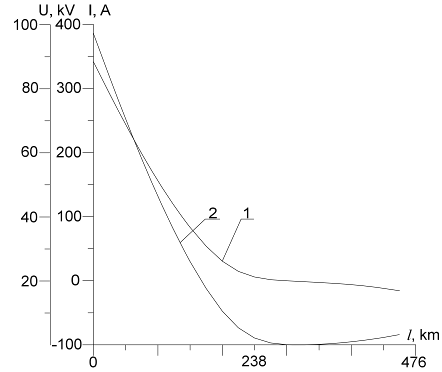

Fig.3. Spatial distribution of current (1) and voltage (2) in power supply line for time t = 0.001s

In figure 3 the current and voltage reach maximum values at the line start.

Figures 4 ÷ 5 present transient waveforms of current in different points of electric power system: load circuit, primary winding of the first power transformer. The most noticeable is the short transient process in power supply line with distributed parameters. This is due to wave processes that occur rapidly in power supply lines and partially also in power transformers.

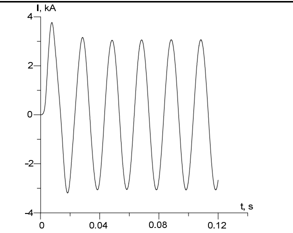

Fig.4. Transient current waveform in load circuit R-L

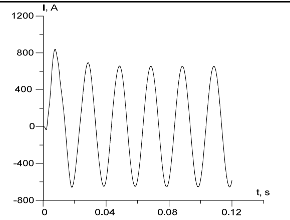

Fig.5. Transient current waveform in primary winding of right side power transformer

Figures 6 and 7 present time-spatial waveforms of current and voltage of power supply line for time interval t = (0 ÷ 0.03)s. It is possible to observe that the wave processes are rapidly absorbed. In case of increase of voltage (breakdowns), the time-spatial waveforms allow to evaluate electromagnetic waveforms of transient processes.

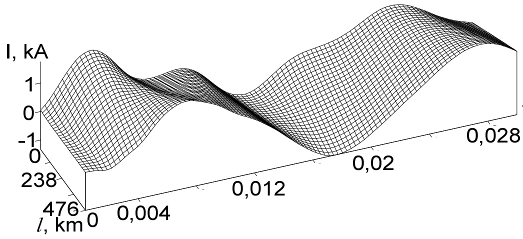

Fig.6. Time-spatial current waveform in power supply line for time interval t = (0 ÷ 0.03)s

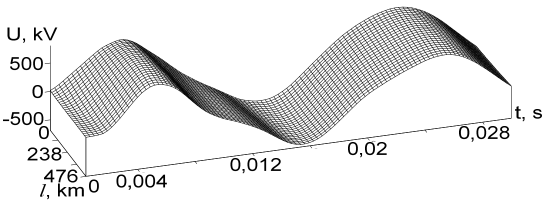

Fig.7. Time-spatial voltage waveform in power supply line for time interval t = (0 ÷ 0.03)s

Conclusions

1. Application of interdisciplinary variation method, based on modification of integral variation principle of Hamilton-Ostrogradsky via expanding Lagrange function enables designing highly sophisticated dynamic systems characterized with concentrated and distributed parameters. The main advantage of this method is the fact that while designing the mathematical model of the system there is no necessity to decompose it (like in case when using classical methods). Here model of the complex electric power system is designed and analysed as a hole.
2. Another advantage of use of interdisciplinary approaches is possibility to design a sophisticated mathematical model of a system comprised of subsystems of different science areas.
3. Mathematical modelling of transient processes in complex energy systems gets complicated due to the fact that some subsystems are analysed as systems with distributed parameters and other as systems with concentrated parameters. As a result it is necessary to integrate differential equations of the state of the system in case of both ordinary derivatives and partial derivatives.

Authors: *dr hab. inż. Andriy Czaban, prof. UTH Rad., Uniwersytet Technologiczno – Humanistyczny, Wydział Transportu i Elektrotechniki, ul. Malczewskiego 29, 26-600 Radom, E-mail: atchaban@gmail.com; dr hab. inż. Marek Lis prof. PCz., Politechnika Częstochowska, Wydział Elektryczny, al. Armii Krajowej 17, e-mail: lism@el.pcz.czest.pl; dr hab. inż. Marcin Chrzan, prof. UTH Rad., Uniwersytet Technologiczno – Humanistyczny, Wydział Transportu i Elektrotechniki, ul. Malczewskiego 29, 26-600 Radom, E-mail: m.chrzan@uthrad.pl; dr inż. Andrzej Szafraniec, Uniwersytet Technologiczno – Humanistyczny, Wydział Transportu i Elektrotechniki, ul. Malczewskiego 29, 26-600 Radom, E-mail: a.szafraniec@uthrad.pl; mgr inż. Vitaliy Levoniyk, Lwowski Narodowy Uniwersytet Rolniczy, ul. W. Wielkiego 1, Lwów, E-mail: Bacha1991@ukr.net.*

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