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Methods of determination thermal conductivity in quasi industrial conditions

Abstract. *In the paper the classic Angstrom method of determining the value of conduction heat transfer has been successfully tested. The usefulness of the Angstrom method was investigated in terms of temperature measurement of metal samples deviating from the general assumptions of the method. A Hamming's neural network was proposed to simulate the similarity of the sample metal to previously known metals.*

Streszczenie. *W pracy sprawdzono działanie klasycznej metody Angstroma do wyznaczania wartości przewodności cieplnej właściwej λ. Sprawdzono przydatność tejże metody w warunkach cieplnych odbiegających od założeń metody. Wykorzystano sztuczną sieć neuronową Hamminga do określania stopnia podobieństwa materiału próbki testowej do znanych, typowych stali stosowanych w praktyce przemysłowej. (**Metody wyznaczania przewodności cieplnej w warunkach półprzemysłowych**)*

Keywords: Angstrom method**,** similarity, Hamming's neural network.
Słowa kluczowe: metoda Angstroma, podobieństwo, sieć neuronowa Hamming

Introduction

The knowledge of thermal diffusion or thermal conductivity is important in strength analysis of mechanical constructions operating under varying thermal conditions. For this reason, there is a need to determine these parameters with good accuracy in "in-situ" conditions which are different from research laboratory.

Properties of metals and their alloys, in particular their thermal conductivity, is very important in experimental studies due to their necessity in the numerical modelling of complex physical systems used in industry. Previous practice is rely on the data in tables of the material properties of certain metals, which are determined by making enough all measuring regimes appropriate to laboratory conditions. Based on such properties is natural common, however, researchers who model different thermal phenomena should be aware that, for example, the chemical composition of metal alloys that they model does not always correspond to those given in the tables, which may change their thermal properties.

In the paper the method of determining the diffusivity and thermal conductivity of metal samples by the method using the heat wave phenomenon has been discussed and proven.

The Angström Method - key assumptions

In the Angstrom method, one-dimensional heat flow is measured in a long, homogeneous thin rod, one end of which is placed in a heat generator of sinusoidal power P according to formula 1.

(1) $$P(t)\big|_{x=0} = P_0 \cos(\omega t + \varphi)$$

where: P_0 – amplitude of input power, ω – angular frequency, t – time, φ – initial phase of signal.

The other end of the rod is kept at constant temperature. The idea of measurement is shown in Fig. 1

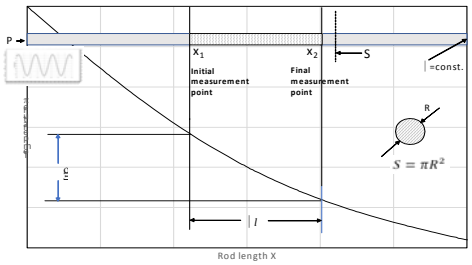

Fig. 1. The idea of determining the thermal conductivity in a metal, thermally insulated rod.

Therefore, it can be assumed that the temperature at the beginning of the rod will change according to formula 2.

(2) $$\vartheta(t)\big|_{x=0} = \vartheta_0 \cos(\omega t + \varphi)$$

where ϑ_0 – amplitude of temperature

At any point of the rod "x" the temperature dependence of time will take the form of a wave equation 3 [1,2].

(3) $$\vartheta(t,x) = \vartheta_0 e^{-a\cdot x} \cos(\omega t - \varphi - b\cdot x)$$

where: the coefficients a and b are related to the thermal diffusivity of the formula:

(4) $$a \cdot b = \frac{\omega}{2\kappa}$$

where: ω – angular frequency, $\kappa = \lambda/\rho{\cdot}c$ – thermal diffusivity.

After mathematical transformations, the formula for thermal diffusivity can be given as:

(5) $$\kappa = \frac{(\Delta l)^2}{2\Delta t \cdot \ln\left(\frac{\vartheta_1}{\vartheta_2}\right)} \left[\frac{m^2}{s}\right]$$

where: Δl – distance between points x_1 and x_2 (Fig. 1), Δt – phase shift between heat waves, ϑ_1, ϑ_2 – amplitudes of heat waves.

Hence, the thermal conductivity can be calculated from the formula:

(6) $$\lambda = \kappa \cdot \rho \cdot c_w \left[W/(m\cdot K)\right]$$

where: ρ – metal density, c_w – specific heat.

The method of measurement is shown in Fig. 2.

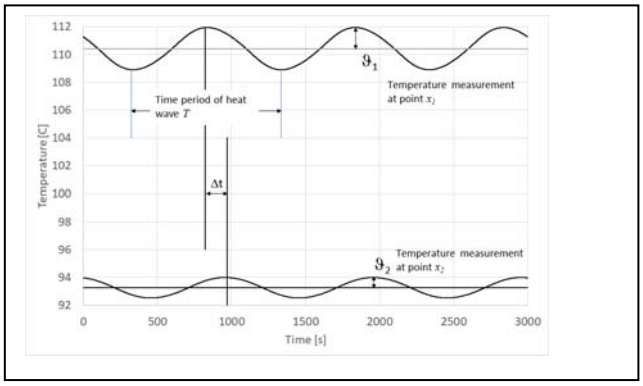

Fig. 2. The method of determining the values x_1, x_2 and Δt.

Determination thermal conductivity with Angström method

Described above method of determining the diffusivity and thermal conductivity of the metals was tested on a one-dimensional numerical model developed by the author, in which all the thermal conditions due the Angström method are met. The most important of these are: periodic force with sufficiently high power (or temperature) at the left end of the bar, no heat dissipation from its surface and maintenance of the opposite end of the rod at a constant temperature of 20ºC. The selection of measuring points x_1 and x_2 (Fig.1) and the time of the experiment due to the period of heat wave induction are the major factors influencing to the results. Using the trial and error method it has been supposed that for the length of the bar L = 0.6 m, the measurement points x_1 and x_2 are respectively 0.2 and 0.4 m (so Δl = 0.04 m).

It was presumed that the period of forcing the sinusoidal wave was To = 8000 s at the time of the experiment of 10 hours. The accuracy of the method was examined for metals with extremely different values of thermal conductivity - from 15 to 372 W / (m·K). In the calculations, the last 3 heat wave periods were analysed. Results are shown in table 1.

Table 1. Results of examined the thermal conductivity values of metals.

	Tabular values		Calculated values	
Type of material	λ [W/(m·K)]	$\rho \cdot c$ [J/(m^3·K)]	λ [W/(m·K)]	Calculation error [%]
Nickel Steel (25 Ni) (A)	15	3,80E+06	14,983	0,113
Carbon steel (B)	52	3,90E+06	52,028	0,054
Iron Armco (C)	70	3,65E+06	69,984	0,022
Brass (D)	118	3,29E+06	117,980	0,016
Aluminium (E)	230	2,42E+06	230,058	0,025
Copper (F)	372	3,22E+06	371,888	0,030

As can be seen, maintaining the above described assumptions, the Angström method gives accurate results of determine the thermal conductivity value of selected metals.

Establish measuring conditions for determining thermal conductivity in industrial environments.

In order to examine the possibility of using the Angström method under industrial conditions it is assumed that the measurements will be carried out on cuboid-shaped samples with dimensions: 0,6 x 0,6 x 0,12 m, shown in Fig. 3. The aim of the first experiment was to check two variants of temperature measurement – on the upper surface and the bottom test sample.

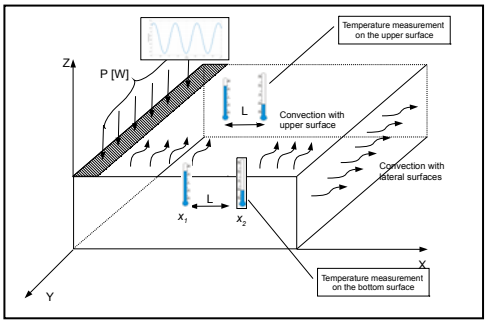

Fig.3. Simulated system of determination κ and λ under industrial conditions. Power P is given on zone 0.09 m

In the simulations it was assumed that the convection coefficient from the upper surface was 30 W/(m^2·K) and 15 W/(m^2·K) with sidewall surfaces. Results are shown in table 2. Similar experiments have been realized in the papers [3,4].

Table 2. Dependence of the value of the determination of the λ value from the temperature measurement point in the sample

Type of material	Tabular values λ	Temperature measurements on the upper surface.		Temperature measurements on the bottom surface	
	λ [W/(m·K)]	λ [W/(m·K)]	Calc. error. [%]	λ [W/(m·K)]	Calc. error. [%]
Nickel Steel (25 Ni) (A)	15	8,378	44,14	13,789	8,074
Carbon steel (B)	52	31,527	39,37	53,754	3,376
Iron Armco (C)	70	42,017	39,97	70,948	1,354
Brass (D)	118	63,441	46,23	107,134	9,208
Aluminium (E)	230	126,048	45,19	217,428	5,466
Copper (F)	372	226,267	39,17	394,962	6,172

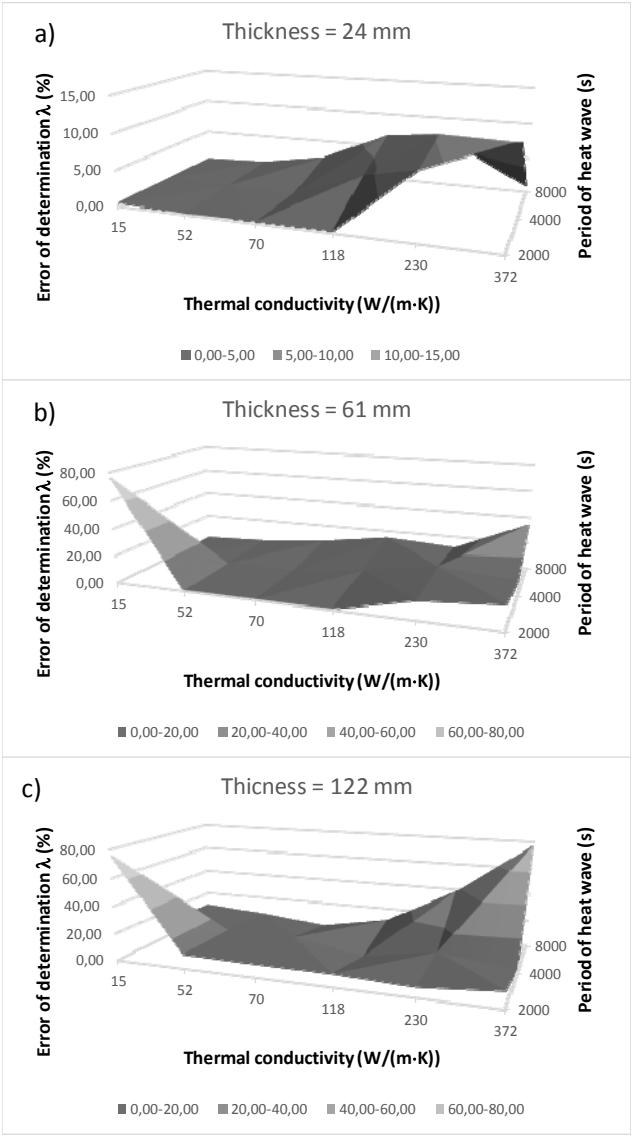

Fig. 4. Errors in determining the value of λ as a function of the thickness of test samples and the heat wave period.

Results shown in table 2 indicate that omitting phenomenon of spatial heat wave propagation (Fig 3) leads to significant errors (about 50%) of determination the λ-value. Taking this fact into account, changing the position of the sensors on the heat-insulated bottom surface of the sample gives a reduction of the error of determination of λ to a value of a few percent. In industrial practice, this may be an acceptable error.

Examination of the influence of the thickness of the test sample and the frequency of the generated heat wave on the accuracy of the determination of the value of l

In the next simulations the influence of the thickness of the test sample and the period of heat wave generation have been examined. The three values of thickness (24, 61 and 122 mm) and three periods of heat waves (2000, 4000 and 8000 s) have been checked. Results are shown in the Fig. 4.

The results indicate that the smallest errors of the λ value are found for the thinnest test plate for a heat wave period of T = 4000s. However, in the case of having a larger sample, it is possible to select the frequency of the thermal induction wave at which, for different λ values, the error of its determination will be as small as possible. It is good to initially estimate what value λ can be expected. To do this, it was proposed to use Hamming's neural network to determine the similarities of the unknown sample material to one (or more) known types of steel.

Use artificial neural network to classify temperature response of test sample into a set of target categories.

The proposed method depends on classify the temperature step input response of the sample to one or more learned categories. To solve this problem called pattern recognition, the Hamming network shown in Fig. 4 is used [5].

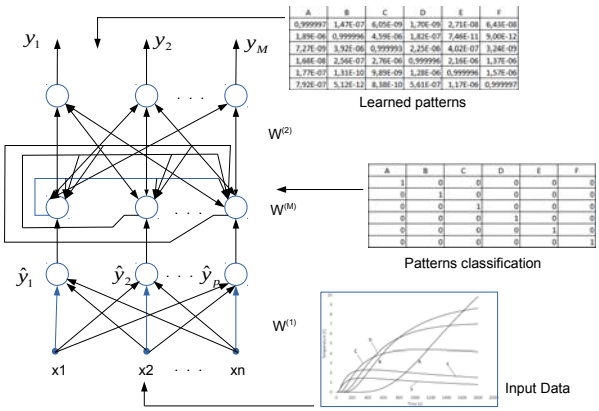

Fig.5. An example of the wide figure inserted into the text

Hamming's Neural Network consists of two layers: input layer – a layer built with neurons, all of those neurons are connected to all of the network inputs and output layer – which is called MaxNet layer. The output of each neuron of this layer is connected to input of each neuron of this layer, besides every neuron of this layer is connected to exactly one neuron of the input layer (see Fig. 5).

The operation of the Hamming network consists in the input of the master network signals that the network is about to learn. Each pattern is assigned to the classification code in a matrix m on m dimension, where m is the number of patterns to learn by a network. After learning, the network allows you to assign a previously unknown signal to one of the learned patterns.

Preparation for learning network signals.

Patterns prepared to learn the Hamming network should be easily distinguishable. For this reason, it was assumed that the standard patterns should be the difference of the temperature step response of the sample measured at points $x1$ and $x2$ due to input force P. These signals are calculated by the formula:

$$\begin{bmatrix} \vartheta_i^A \\ \vartheta_i^B \\ \vdots \\ \vartheta_i^F \end{bmatrix} = \begin{bmatrix} \vartheta_{i,x1}^A \\ \vartheta_{i,x1}^B \\ \vdots \\ \vartheta_{i,x1}^F \end{bmatrix} - \begin{bmatrix} \vartheta_{i,x2}^A \\ \vartheta_{i,x2}^B \\ \vdots \\ \vartheta_{i,x2}^F \end{bmatrix} \tag{7}$$

where: ϑ_i – i^{th} sample of temperature, A, B, …,F – kind of material (see tables).

The registration time was 0.5 hours. Figure 6 shows how, based on formula 7, to prepare the teaching pattern for steel Nickel Steel (25 Ni) (A).

The teaching patterns for each standard material are prepared in the same way. There are shown in Fig. 7.

It has been assumed that a hidden-layer neural network is consist of 20 neurons with sigmoid activation functions, while the output layer has 6 coupled by loopback neurons.. Scaled Conjugate Gradient was assumed as a method to minimize the error function. The network learning process is fast converging and lasts for about 180 epochs.

From now on, the learned network can be used to quickly determine to which element of standard set the temperature response of the sample will be most similar. This allows a quick estimate the value of the product of ρc and thermal conductivity λ. This in turn will allow to select the appropriate period of the sine heat wave and its power, so that the λ-value errors are as low as possible.

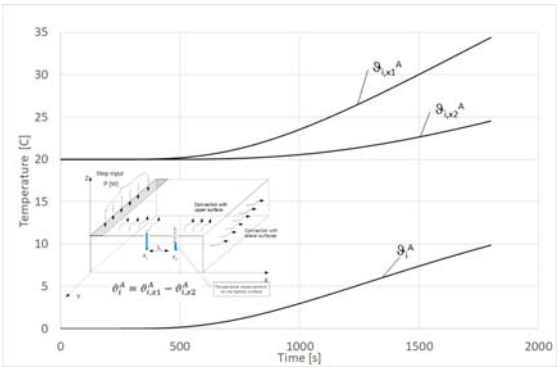

Fig.6. An example of preparing the teaching pattern.

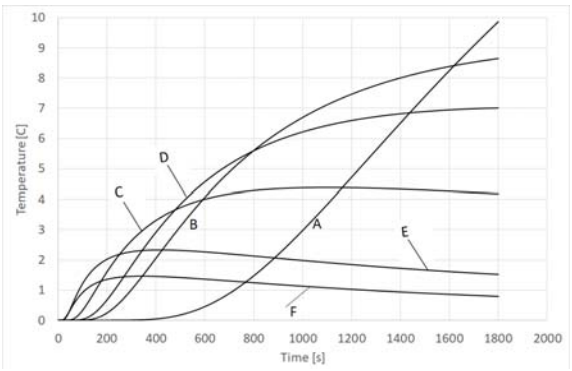

Fig.7. All patterns used to teaching Hamming's network.

The performance verification of the material class recognition system

In order to verify the operation of Hamming's network, initial identification of samples made from metals originating outside the standard set was examined. The difference in their temperature responses at points *x1* and *x2* on the heating power step, was applied to the input of the learned network. Examined network returns 6 on 1 vector classifying the sample. The tested sample is more similar to the standard sample, if the value at its position is closer to value 1. Results of samples verification are presented in the table 4.

Table 3. Results of pre-identification of material type. λ in (W/(m·K), $\rho \cdot c$ in J/(K·m^3)

Simulated properties of unknown sample	Standard samples					
	A	B	C	D	E	F
λ=400 $\rho \cdot c$=3.3e6	0	0.0031	0.0001	0.0136	0.0001	**0.983**
λ=120 $\rho \cdot c$=3.2e6	0	0	0.0754	**0.89**	0.0348	0
λ=35 $\rho \cdot c$=3.8e6	0.0004	**0.981**	0.0184	0	0	0

Table 4 shows that sample of simulated λ = 400 W/(m·K) and $\rho \cdot c$ = 3.3x10^6 J/(m^3·K) is most similar to standard sample F, that means Copper (see table 1).

Having a preliminary knowledge of the type of sample (thermal conductivity λ and the product of density ρ and specific heat c and knowing its dimensions it can be determined: the power required to achieve a measurable temperature amplitude of the forcing period (based on previously acquired knowledge) giving the smallest error of determination of λ-value and based on it time of experiment.

Conclusions

In the paper the classic Angstrom method of determining the value of conduction heat transfer has been successfully tested. The usefulness of the Angstrom method was investigated in terms of temperature measurement of metal samples deviating from the general assumptions of the method. It was found, that accuracy of λ determination depends on cuboid sample thickness and period of generating heat wave. A Hamming's neural network was proposed to simulate the similarity of the sample metal to previously known metals.

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