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Integer order vs fractional order temperature models in the forced air heating system

Abstract. *The paper presents a comparing integer order (IO) vs fractional order (FO) transfer function models of the forced air heating system applied in housing heating. The proposed FO models are simpler to identify than analogical IO models and their accuracy in the sense of fitting to experimental results is practically the same. Results are verified using experimental data from the real isothermal room, used in a pharmaceutical factory.*

Streszczenie. *W pracy zaprezentowano porównanie modeli transmitancyjnych całkowitego i niecałkowitego rzędu dla systemu nagrzewu powietrza stosowanego w ogrzewaniu budynków. Proponowane modele są tak samo dokładne jak modele całkowitego rzędu, a ich identyfikacja jest prostsza ze względu na mniejszą liczbę parametrów. Prezentowane wyniki zostały zweryfikowane z wykorzystaniem danych eksperymentalnych z pomieszczenia izotermicznego stosowanego w fabryce farmaceutycznej.* ***(Porówanie modeli całkowitego i niecałkowitego rzędu dla układu nagrzewania powietrza)***

Keywords: electric heating, temperature modeling, fractional order systems, ORA approxmation, Charef approximation, isothermal room.
Słowa kluczowe: ogrzewanie elektryczne, modelowanie temperatury, systemy ułamkowego rzędu, aproksymacja ORA, aproksymacja Charefa, pomieszczenie izotermiczne.

Introduction

The use of electric heating in building automation and air conditioning systems has many advantages. Electric heaters are easy to control and they allow to fast obtain preset temperature. The electric heating has been discussed by many Authors, e.g. the dynamics of electric floor heating systems is analyzed in the paper [6], the energy-efficient electric heating is discussed in [8]. The construction of control algorithm requires to know the model of a process. The heat processes inside buildings can be described by different models. For example the paper [7] analyzed transfer function models of electrically heated room, the paper [4] proposes transfer function, non integer order models of room heating with the use of radiator.

The fractional calculus is a very useful tool to describe many physical phenomena. Fractional Order (FO) or equivalently noninteger order models for different physical phenomena are presented by many Authors, e.g. [1], [3], [5], [13],[18], [19]. It is also well known that heat processes can be described using FO approach. This issue has been investigated e.g. by [12], [13], [15], [14]. The models can have a form of a FO transfer function, a FO partial differential equation or a FO state equation.

This paper is intented to compare integer order (IO) vs FO transfer function models of the forced electric heating system. The considered system works in a pharmaceutical factory. It is required to keep predefined, constant, relatively high temperature in insulated room with cubature about $75m^3$.

The paper is organized as follows: at the beginning, some elementary ideas from FO calculus are presented. Next, the considered heating system is described and the FO transfer function models with suitable approximations are proposed. Finally the models are verified with the use of experimental data obtained from SCADA system.

Preliminaries

0.1 Basic ideas from fractional calculus

A presentation of elementary ideas is started with a definition of a fractional order, integro-differential operator. It was given for example by [3], [9], [10] or [18]:

Definition 1 *(The elementary non integer order operator)*
The fractional order, integro-differential operator is defined as follows:

(1) $$ {}_aD_t^\alpha f(t) = \begin{cases} \frac{d^\alpha f(t)}{dt^\alpha} & \alpha > 0 \\ f(t) & \alpha = 0 \\ \int\limits_a^t f(\tau)(d\tau)^\alpha & \alpha < 0 \end{cases}, $$

where a and t denote time limits for operator calculation, $\alpha \in \mathbb{R}$ denotes the non integer order of the operation.
Next, an idea of Gamma Euler function is recalled (see for example [10]):
Definition 2 *(The Gamma function)*

(2) $$ \Gamma(x) = \int\limits_0^\infty t^{x-1}e^{-t}dt. $$

The fractional-order, integro-differential operator is described by different definitions, given by Grünvald and Letnikov (GL definition), Riemann and Liouville (RL definition) and Caputo (C definition). In the further consideration only C definition is applied:
Definition 3 *(The Caputo definition of the FO operator)*

(3) $$ {}_0^CD_t^\alpha f(t) = \frac{1}{\Gamma(N-\alpha)}\int\limits_0^\infty \frac{f^{(N)}(\tau)}{(t-\tau)^{\alpha+1-N}}d\tau, $$

where $N-1<\alpha<N$ denotes the fractional order of operation and $\Gamma(..)$ is the complete Gamma function expressed by (2).
For the C definition the Laplace transform is defined:
Definition 4 *(The Laplace transform for Caputo operator)*

(4) $$ \begin{gathered} \mathcal{L}({}_0^CD_t^\alpha f(t)) = s^\alpha F(s), \ \ \alpha < 0 \\ \mathcal{L}({}_0^CD_t^\alpha f(t)) = s^\alpha F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1}{}_0D_t^k f(0), \\ \alpha > 0, \ \ n-1 < \alpha \le n \in N \end{gathered} $$

The use of C definition (3) and (4) allows defining the fractional order transfer functions, analogical to known integer order transfer functions:

(5) $$G_1(s) = \frac{b_m s^{\beta_m} + ... + b_1 s^{\beta_1} + b_0}{a_n s^{\alpha_n} + ... + a_1 s^{\alpha_1} + a_0},$$

where $\beta_m ... \beta_1$ and $\alpha_n ... \alpha_1$ are fractional orders, $b_m ... b_0$ and $a_n, ..., a_0$ are coefficients. The transfer function (5) is often employed to the modeling of heat processes. It is presented for example by [4].
Another FO transfer function useful to the modeling of different classes of dynamic systems is as follows:

(6) $$G_2(s) = \frac{1}{(T_n s + 1)^{\alpha_n} ... (T_1 s + 1)^{\alpha_1}},$$

where $\alpha_n ... \alpha_1$ are fractional orders and $T_n, ..., T_1$ are time constants. The transfer function (6) can be also used to the modeling of heat processes as it is presented by [16].
The application of the above transfer functions requires to use an integer order approximation of the basic element s^α or $\frac{1}{(Ts+1)^\alpha}$. Such approximations, both time-continuous and time-discrete are known. In this paper the Oustaloup Recursive Approximation (ORA) and the Charef approximation are applied. They are given in the next subsections.

0.2 The Oustaloup Recursive Approximation (ORA approximation)

The method proposed by [17] allows to approximate an elementary FO transfer function s^α by the finite, integer-order transfer function, close to Pade approximation:

(7) $$s^\alpha \approx k_f \prod_{n=1}^{N} \frac{1 + \frac{s}{\mu_n}}{1 + \frac{s}{\nu_n}} = \frac{L_{ORA}(s)}{D_{ORA}(s)}.$$

In (7) N is the order of the approximation, k_f is the steady state gain, μ_n and ν_n are calculated as follows:

(8) $$\begin{aligned} \mu_1 &= \omega_l \sqrt{\eta} \\ \nu_n &= \mu_n \gamma, \ n = 1, .., N \\ \mu_{n+1} &= \nu_n \eta, \ n = 1, .., N-1 \end{aligned}$$

where:

(9) $$\gamma = \left(\frac{\omega_h}{\omega_l}\right)^{\frac{\alpha}{N}} \qquad \eta = \left(\frac{\omega_h}{\omega_l}\right)^{\frac{1-\alpha}{N}}$$

In (9) ω_l and ω_h describe the range of angular frequency, for which parameters should be calculated. The steady state gain k_f is set to assure the convergence the step response of the approximation to step response of the real plant in a steady state.

0.3 The Charef approximation

The approximation is given by [2] allows to approximate the elementary, inertial, fractional order transfer function (6):

(10) $$\frac{1}{(T_{Ch} s + 1)^\alpha} \approx \frac{\prod_{m=0}^{M-1} (1 + \frac{s}{z_m})}{\prod_{m=0}^{M} (1 + \frac{s}{p_m})} = \frac{L_{Ch}(s)}{D_{Ch}(s)},$$

where z_i and p_i denote zeros and poles of approximation, M denotes the order of the approximation. An idea of this approximation is to best fit the Bode magnitude plot of approximation to Bode magnitude plot of the plant in the given frequency band. Zeros and poles are calculated with the use of following recursive dependencies:

(11) $$p = \frac{1}{T_{Ch}}$$

(12) $$p_0 = p\sqrt{b}$$

(13) $$z_0 = ap_0$$

(14) $$\ldots$$

(15) $$p_m = p_0 (ab)^m \qquad m = 1...M$$

(16) $$z_m = ap_0 (ab)^m \qquad m = 1...M$$

where:

(17) $$\begin{aligned} a &= 10^{\frac{\Delta}{10(1-\alpha)}} \\ b &= 10^{\frac{\Delta}{10\alpha}} \\ ab &= 10^{\frac{\Delta}{10\alpha(1-\alpha)}} \end{aligned}$$

In (17) $\Delta > 0$ denotes maximal permissible error of Charef approximation, defined as the difference between Bode magnitude plot for model and plant, expressed in $[dB]$.
The order of approximation M can be estimated as follows:

(18) $$\begin{aligned} M = Int\left(\frac{log(\omega_{max} T)}{log(ab)}\right) + 1 = \\ Int\left(\frac{10\alpha(1-\alpha) log(\omega_{max} T)}{\Delta}\right) + 1 \end{aligned}$$

where ω_{max} denotes the maximal frequency band, for which the approximation is required to work properly.

The forced air heating system

The considered forced heating system is required to assure the constant, relatively high temperature in the room with cubature about $75m^3$. The system contains only heater. The cooler is not necessary due to environmental conditions and required range of preset temperatures. The simplified scheme of the system is illustrated by the SCADA screen shown in the figure 1. The cold air from the room is sucked via return duct to the heater and next, after being heated by the heater 2E1 and filtered by the HEPA filter, is supplied to the room via supply duct. The filter pollution is monitored by the pressure switch 11PS2. The electric heater 2E1 is protected from damage by monitoring air flow with the use of another pressure switch. The control signal is the power of the heater 2E1 expressed in the percent of its maximal range. The rotational speed of the blower 2M1 can be also applied as a control signal, but in this case, it is maximal and constant. The controlled temperature is measured inside the return duct by the sensor Pt 1000 (element 6TE1 in the scheme). This location of the sensor allows to avoid problems associated with the heterogeneous spatial temperature distribution in the room. Trends of temperature and control caused by the change of the preset temperature from 28^oC to 37^oC are shown in the figure 2. These trends were measured during normal work of the heating system, controlled by classic PID controller. This "passive" method was only possible to apply in the considered case. The sample time during experiments was equal: $h = 30[s]$. The outside temperature is the biggest disturbance for the system. However, in case of such a short period of time it can be assumed as constant. The short pause in control signal is caused by the standard test of the security system. It switches off for a moment heater and blower. This is illustrated by the short breakdown of control.

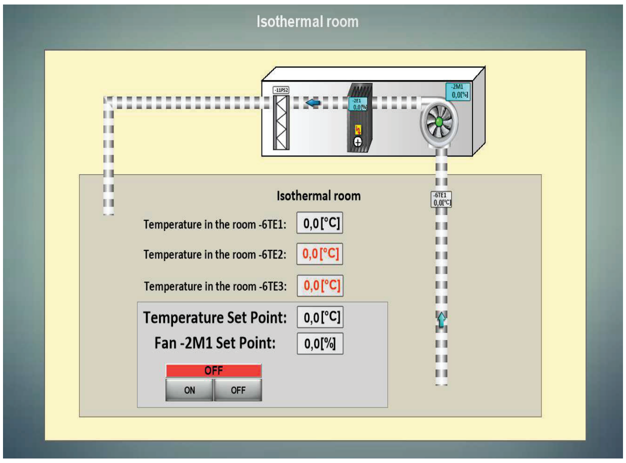

Fig. 1. The forced air heating system

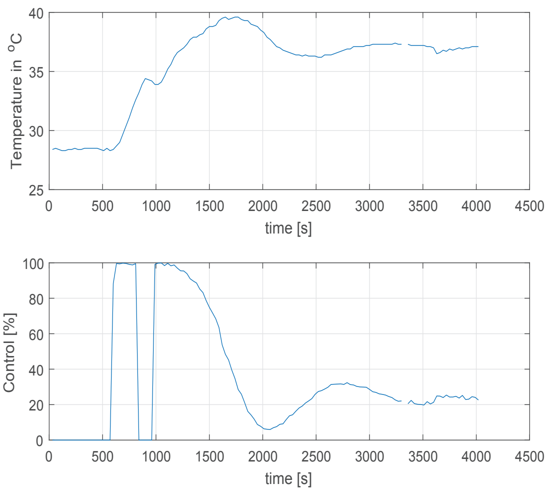

Fig. 2. The temperature measured by sensor 6TE1 and the power of heater 2E1.

The transfer function models of air temperature

0.4 Integer order models

The most obvious IO transfer function model is the model with delay:

$$G_{IO}(s) = e^{-\tau s}\frac{b_m s^m + .. + b_1 s + b_0}{a_n s^n + .. + a_1 s + a_0},$$

where $m \leq n$, τ is the dead time and a_n, b_m are transfer function coefficients. However, results of identification allow to ignore the dead time τ and the model does not contain delay:

$$G_{IO}(s) = \frac{b_m s^m + .. + b_1 s + b_0}{a_n s^n + .. + a_1 s + a_0}. \tag{19}$$

Parameters of the model (19) can be easily estimated using MATLAB function *tfest*. This model for different orders n will be compared to the FO models proposed in the next subsection.

0.5 Non integer order models

The first proposed FO model uses ORA approximation and it takes the following form:

$$G_{ORA}(s) = \frac{k_{ORA}}{T_{ORA}s^{\alpha_{ORA}} + 1}. \tag{20}$$

With respect to (7), the transfer function (20) takes the form ready to use at MATLAB:

$$G_{ORA}(s) \approx \frac{k_{ORA}D_{ORA}(s)}{T_{ORA}L_{ORA}(s) + D_{ORA}(s)}. \tag{21}$$

Next proposed elementary model has the direct form of the transfer function (10):

$$G_{Ch}(s) = \frac{k_{Ch}}{(T_{Ch}s + 1)^{\alpha_{Ch}}} \approx \frac{k_{Ch}L_{Ch}(s)}{D_{Ch}(s)}. \tag{22}$$

Results of experiments given in [14, 16] show that the accuracy of the FO model can be significantly improved by adding an IO part. Then we obtain so called hybrid models, containing FO and IO parts:

$$G_{hORA}(s) = \frac{k_{hORA}}{(T_{ORA}s^{\alpha_{ORA}} + 1)(T_1 s + 1)}. \tag{23}$$

In (23) T_1 denotes time constant of IO part. With respect to (7) the transfer function (23) takes the following form:
(24)

$$G_{hORA}(s) \approx \frac{k_{ORA}D_{ORA}(s)}{(T_{ORA}L_{ORA}(s) + D_{ORA}(s))(T_1 s + 1)}.$$

The hybrid inertial model is as follows:

$$G_{hCh}(s) = \frac{k_{hCh}}{(T_{Ch}s + 1)^{\alpha_{Ch}}(T_2 s + 1)}. \tag{25}$$

In equation (25) T_2 is the time constant of the IO part. Analogically the approximation with respect to (10) is following:

$$G_{hCh}(s) \approx \frac{k_{hCh}L_{Ch}(s)}{D_{Ch}(s)(T_2 s + 1)}. \tag{26}$$

In (20)-(26) $k_{ORA,Ch,hORA,hCh}$ denotes the steady-state gain of the model. It is used to fit the response of the model to the experimental data. This causes that it is not a free parameter during identification, because it is calculated after all other parameters. All the above transfer functions can be applied to modeling the considered temperature with the use of control signal CV and *lsim* function available at MATLAB. Results are discussed in the next section.

Identification of the models

Identification of all the proposed models was executed with the use of data shown in the figure 2. In this case only "passive" identification was possible to do. Its idea is following: the transfer function of the plant itself is the same in closed loop and open loop system. Next, the only possible to use input signal is the control signal generated by the PID controller. It is not a standard testing signal (for example the Heaviside function), but its use allows also to identify the transfer function. If the identified plant is linear in the suitable range, then results do not depend on the shape of input signal. In each case the identification consists in the best fitting a response of the model to the experimental response. To estimate this fitting the MSE and FIT functions are applied. They are given underneath.

(27)
$$MSE = \frac{1}{K_s}\sum_{k=1}^{K_s}\left(y_e(k) - y(k)\right)^2,$$

where $y_e(k)$ and $y(k)$ denote experimental and model responses to the same control signal in time moments k, K_s is the number of all collected samples.
The next considered cost function is the fitting function (28):

(28)
$$FIT = \frac{\|y - \overline{y_e}\|}{\|y - \overline{y}\|}.$$

In (28) $\overline{y_e}$ is the average value from experimental output y_e.The function (28) can be also expressed in $\%$, as it is for example in the paper [4] (equation (9) in page 8). This function is also employed by the MATLAB functions *compare* and *tfest*.

(29)
$$FIT_{100} = (1 - FIT) \cdot 100\%.$$

At the beginning the integer order model (19) for $n = 1, 2, 3$ was examined. Parameters of all models were estimated using MATLAB function *tfest*. This function employes the FIT_{100} cost function. The result transfer functions are as follows:

(30)
$$G_{IO1}(s) = \frac{0.0001886}{s + 0.0006551}$$

(31)
$$G_{IO2}(s) = \frac{0.0002493s + 2.117e-07}{s^2 + 0.002379s + 5.053e-07}$$

(32)
$$G_{IO3}(s) = \frac{6.328e-05s^2 + 3.545e-06s + 6.643e-09}{s^3 + 0.0127s^2 + 5.518e-05s + 1.896e-08}.$$

Results are given in the table 1 and figures 3-5.

Table 1. The integer order model (19).

n	MSE (27)	FIT (28)	FIT_{100} (29) [%]
1	0.4104	0.1807	81.93
2	0.1331	0.1029	89.71
3	0.0893	0.0843	91.57

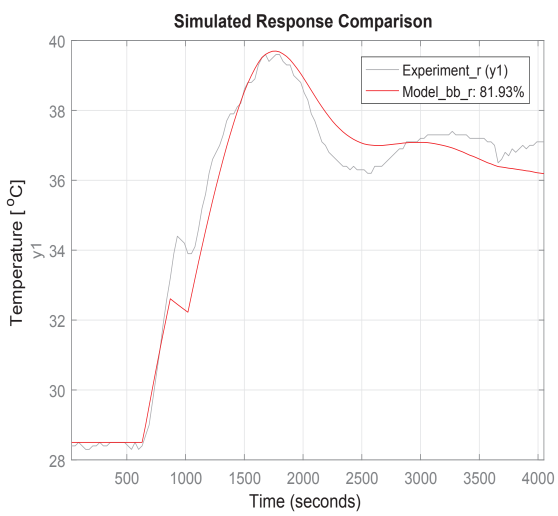

Fig. 3. The experimental data vs the 1'st order model.

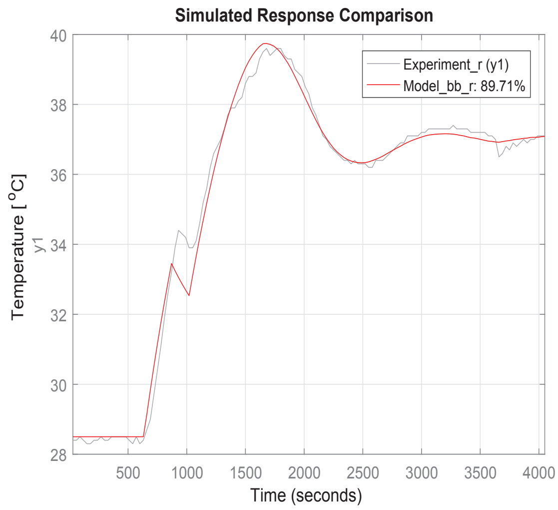

Fig. 4. The experimental data vs the 2'nd order model.

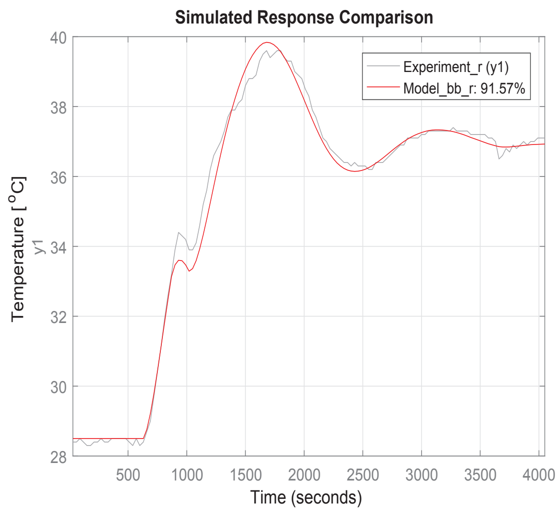

Fig. 5. The experimental data vs the 3'rd order model.

Next, the FO models were examined. The identification of all parameters was done via minimizing of the MSE cost function using the Grey Wolf Optimizer (GWO). GWO is a meta-heuristic optimization method based on the behavior of the grey wolf living in the herd proposed by [11]. It is based on the hunting phases of the grey wolf and the hierarchy of the herd. An important advantage of this algorithm is the fact that it accepts the range of initial values for the searched parameters and not the specified parameters. During use this algorithm we need to determine only the size of the herd (typically between 5 and 10)and the number of iterations.

The initial values of parameters are given in the table 2

Table 2. Inital parameters for GWO optimizer.

model	α	T_α	T_2
ORA	0-1	100-1000	-
Charef	0-1	500-1500	-
hybrid ORA	0-1	100-1000	0-500
hybrid Charef	0-1	100-5000	0-500

The parameters of ORA and Charef approximations applied in experiments are given in the tables 3 and 4.

Table 3. The ORA approximation parameters.

N	ω_l	ω_h
8	10^{-4}	10^4

Table 4. The Charef approximation parameters.

M	$\Delta[dB]$
8	0.5

The basic FO models (20) and (22) were tested firstly. The obtained models are as follows:

(33) $$G_{ORA}(s) = \frac{0.4540}{702.0088s^{0.8015}+1}$$

(34) $$G_{Ch}(s) = \frac{0.5688}{(26732s+1)^{0.4504}}.$$

The cost functions MSE and FIT are given in the table 5, and illustrated by figures 6, 7.

Table 5. The cost functions for basic FO ORA and Charef models.

model	MSE (27)	FIT	FIT_{100} (29)
ORA	0.2177	0.1316	86.84
Charef	0.224	0.1335	86.65

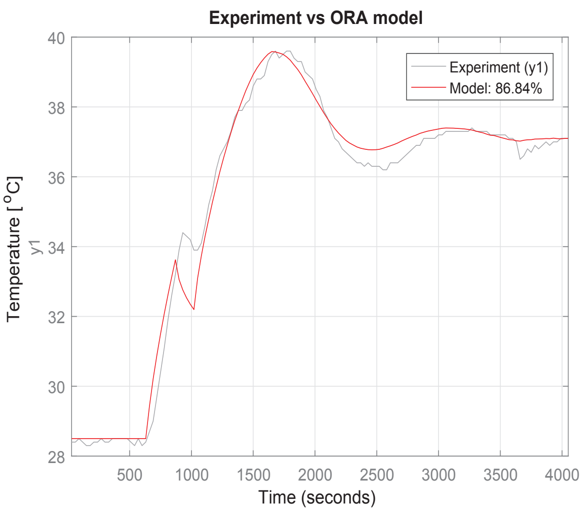

Fig. 6. The experimental data vs the FO ORA model.

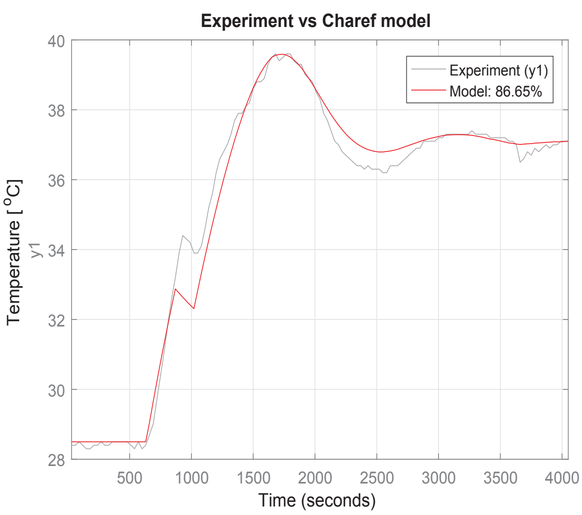

Fig. 7. The experimental data vs the FO Charef model.

Finally, the hybrid models (24) and (26) were verified. The result transfer functions are as follows:

(35) $$G_{h,ORA}(s) = \frac{1.097}{(371.9145s^{0.5692}+1)(112.566s+1)}$$

(36) $$G_{h,Ch}(s) = \frac{0.3643}{(2993.9s+1)^{0.9273}(172.5709s+1)}.$$

The accuracy of the both models is described by the table 6 and illustrated by figures 8, 9.

Table 6. The cost functions for the hybrid FO ORA and Charef models.

hybrid model	MSE (27)	FIT	FIT_{100} (29)
ORA	0.0979	0.0882	91.18
Charef	0.0939	0.0864	91.36

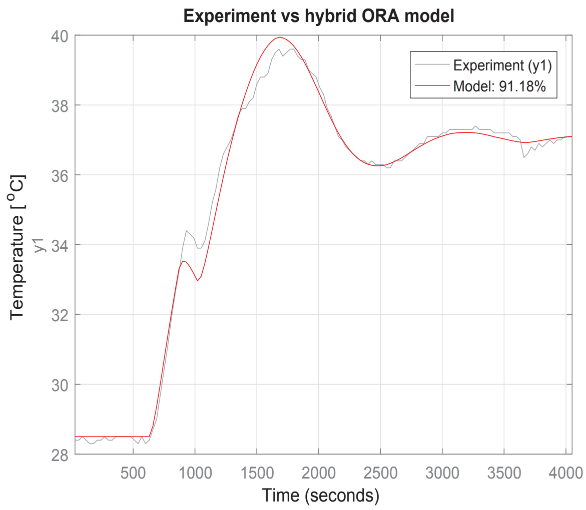

Fig. 8. The experimental data vs the FO ORA hybrid model.

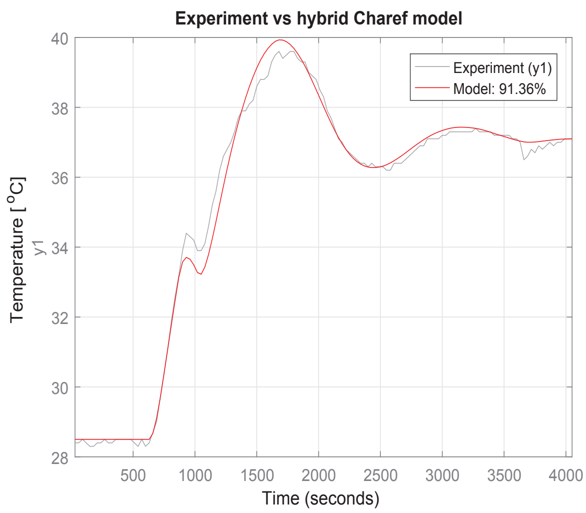

Fig. 9. The experimental data vs the FO Charef hybrid model.

The number of free parameters of all the tested models is collected in the table 7.

Table 7. Number of free parameters for all tested models.

Model	Number of parameters
1'st order	2
2'nd oder	4
3'rd order	6
ORA	2
Charef	2
hybrid ORA	3
hybrid Charef	3

Conclusions

The main conclusion of the paper is that the use of the proposed FO model allows to obtain the model comparable to integer order model in the sense of cost functions MSE and FIT, but the number of free parameters necessary to assign is smaller.

All the discussed IO and FO transfer function models are being recently applied to construct FO control algorithms for the considered forced air heating system.

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