PI-controller tuning optimization via PSO-based technique

Abstract. The technique of PI-controller tuning, which is based on a modification of the particle swarm optimization method, has been developed in this article. In order to take into account the most important quality indicators of plant controlling the complex criterion was developed. PI-controller tuning procedure has been reduced to the problem of criterion minimization. In the article, five benchmark transfer functions were used to estimate the technique. Comparative analysis with other well-known tuning techniques revealed the superiority of the proposed approach.

Streszczenie. W artykule przedstawiono metodę optymalizacji sterownika PI wykorzystującą algorytm rojowy. W artykule przedstawiono pięć rezultatów testów oraz porównanie tej metody z innymi powszechnie stosowanymi. Optymalizacja sterownika PI bazująca na algorytmach PSO.

Keywords: PI-controller, particle swarm optimization, tuning, criterion.

Słowa kluczowe: sterowniki PI, optymalizacja, algorytm PSO

Introduction
Proportional-integral (PI) controllers are extremely common in many fields of industrial and agricultural production. A problem of PI-controller tuning has great practical meaning since it influences the efficiency of the automated process. There are hundreds of techniques for PI-controller tuning [1], but the research in this sphere is still continuing. They are caused by new requirements for automated processes, new constraints in tuning problems statements and other reasons.

One of the approaches to the problem of PI- (or PID) controller tuning is connected with applying of optimization methods, more specific particle swarm optimization (PSO) method. Since PSO has great search abilities, it may be utilized for finding the optimal values of PI-controller coefficients. However, in already known scientific works [2-12] single-criterion optimization problems have been solved.

In these works transfer functions for heat [6], electrical [7-9], energy [10] and chemical [11] processes have been used. Note, that PSO-based PI-controller tuning may be used also for non-linear [11] or unstable [12] systems.

All of these works are related to the utilization of integral criteria. However, controlled processes are estimated with other important indicators (overshoot, settling time, etc.). Let us denote them as terminal criteria. For instance, in the article [13], mentioned criteria were used as components of the cost function to minimize.

In order to achieve better controller performance, a complex criterion should be used, which concludes both integral and terminal criteria. In the current article, such criterion has been proposed and applied to the PI-controller tuning problem (we have considered only PI-controller because of its great spreading in practical applications).

Problem statement
As it was mentioned above, the most popular industrial controller is a PI-controller. That is why, in the research, a controlled with PI-controller process (plant) is under consideration. The scheme, which corresponds to it, may be presented as shown in Fig. 1.

Fig. 1. Scheme of the closed-loop controlled process

In Fig. 1 we used followed denotations: $A_i$ – coefficients, which depend on the parameters of the plant; $n$ – the order of the plant; $\tau$ – time delay of the plant; $u$ – control function (in the following we will denote it as „control“); $K_p$ and $T_i$ – proportional and integral coefficients of PI-controller respectively, $e$ – error, which is defined as the reminder of the controlled variable $x$ and set point $r$ subtraction. Consequently, a mathematical model of PI-controller in the time domain is described with the following expression:

\[
\begin{align*}
 u &= K_p e + T_i^{-1} \int_0^t e \, dt, \\
 \end{align*}
\]

where: $t$ – time. Tuning of PI-controller is the process of finding the values $K_p$ and $T_i$ for a particular order of the plant $n$ and values $A_i$.

One of the most important demands to the PI-controller is providing the stability of the process. That demand may be expressed in the following manner:

\[
\begin{align*}
 \lim_{t \to x} x &= r; \\
 \lim_{t \to x} \frac{dx}{dt} &= 0, \quad i = (1, n).
\end{align*}
\]

In practical calculations, infinity is substituted with some moment of time:

\[
\begin{align*}
 |x(T) - r| &\leq \Delta = r_s; \\
 \frac{dx(T)}{dt} &\approx 0,
\end{align*}
\]

where: $\Delta$ – acceptable process error, which for many cases is equal to 0.05$r$ (such value has been used in the research), $r_s$ – the acceptable value of the process variable, $T$ – the moment of time when the conditions (3) are met. In the research we use the conditions (3), rather than other stability criteria, as they can be presented in the form of the following criteria to minimize:

\[
\begin{align*}
 Ter_e &= \sqrt{(x(T) - r_s)^2 + \sum_{i=1}^{n} \left(\frac{dx(T)}{dt}\right)^2} \to \min \\
 \text{or} \\
 Ter_m &= |x(T) - r_s| + \sum_{i=1}^{n} \left|\frac{dx(T)}{dt}\right| \to \min,
\end{align*}
\]

where: $Ter_e$ and $Ter_m$ – Euclidian and Manhattan norms respectively. The absolute minima of the criteria (4) and (5) are equal to zero. Indeed, reducing of (4) or (5) to zero allows to meet conditions (3). Such an approach to satisfy the stability brings the foundation for reducing the initial problem to the problem of unconstrained optimization. In the opposite case, using Hurwitz or another similar criterion involves constraints in the optimization problem statement and substantially that complicates it.
The choice of a particular criterion depends on its effectiveness. In the current investigation, better performance has revealed criterion (5) and all the further numerical data which are related to its applying.

The quantity of the numbers $K_p$ and $T_i$, which allow to minimize criterion (5), is equal to infinity. It provides the possibility of utilizing additional requirements. Such requirements may be presented as minimization of widely spread in the practice IAE (Integral Absolute Error) or ISE (Integral Square Error) criteria. The use of these, for low order transfer functions, allows finding analytical expressions for $K_p$ and $T_i$ [1]. However, IAE or ISE reflect only one aspect of control quality, which is connected with the error.

In the research, we have taken into consideration more general criterion, which includes other important indicators of the PI-controller exploitation. It can be presented as follows:

$$\text{(6)} \quad Cr = \delta_t \cdot T \cdot \text{Ter}_M \rightarrow \min_{K_p \in P, T_i \in I},$$

where $\delta_t, \ldots \delta_4$ – weight coefficients (each of these coefficients shows the impact of the particular summand), $\varepsilon_{\text{max}}$ – maximum of error, $t_i$ – settling time.

The first summand in the expression (6) corresponds to the mean integral error (it is proportional to IAE). The second summand is the similar value of control $u$. The system control. The third summand is proportional to the overshoot and the fourth one is proportional to the settling time. All of these indicators are undesirable, which causes the need of criterion (6) minimization.

Thus, we have reduced the PI-controller tuning problem to the optimization problem. It may be expressed in such a manner:

$$\text{(7)} \quad Cr + \delta_t \cdot T \cdot \text{Ter}_M \rightarrow \min_{K_p \in P, T_i \in I},$$

where $P$ and $I$ – search domains for proportional and integral coefficients of PI-controller respectively, $\delta_t$ – terminal weight coefficient, which shows the requirement of conditions (3) satisfaction. Expression (7) shows, that the minimization of the sum $Cr + \delta_t \cdot T \cdot \text{Ter}_M$ will be performed with the respect to the coefficients $K_p$ and $T_i$. Their values may be varied in domains $P$ and $I$ respectively.

**Optimization algorithm**

One of the important issues in the problem solving is the choice of an appropriate method. In the research, the modification of particle swarm optimization (PSO) was used. It is called multi-epoch PSO (ME-PSO) [14].

In the ME-PSO method, a swarm is a set of particles which move on the surface of minimized function (7). The position of a particle is described by a set of its coordinates $(K_{p,j}, T_{i,j})$ in the search domains $P$ and $I$. At the initial stage of ME-PSO algorithm, the particles’ positions are randomly initialized. During subsequent iterations, the components of position vector of a particle are updated according to the formulas:

$$\text{(8)} \quad K_{p,j}^{t+1} = K_{p,j}^{t-1} + c_1 \eta_1 (p_{K_p} - K_{p,j}^{t-1}) + c_2 \eta_2 (g_{K_p} - K_{p,j}^{t-1});$$

$$T_{i,j}^{t+1} = T_{i,j}^{t-1} + c_1 \eta_1 (p_{T_i} - T_{i,j}^{t-1}) + c_2 \eta_2 (g_{T_i} - T_{i,j}^{t-1}),$$

where $K_{p,j}$ and $T_{i,j}$ are components of the position vector of a particle on $j$-th iteration (the previous iteration is denoted with $(j-1)$ superscript); $p_{K_p}$ and $p_{T_i}$ – coordinates of the best position of a particle, that has been found on the previous iterations (personal best); $g_{K_p}$ and $g_{T_i}$ – coordinates of the best position, that has been found by the swarm on the previous iterations (global best); $c_1$ and $c_2$ – cognitive and social coefficients respectively; $\eta_1, \eta_2$ – random numbers that are generated on the interval $[0, 1]$.

An evolution of PSO algorithm includes applying the formulas (8) and updating the global and personal bests according to the rules:

$$\text{(9)} \quad \begin{cases} p_{K_p,j} = K_{p,j}, & \text{if } Cr(K_{p,j}^{t+1}, T_{i,j}^{t+1}) < Cr(p_{K_p,j}) + \delta_t \cdot T \cdot \text{Ter}_M(p_{K_p,j}); \\ p_{T_i,j} = T_{i,j}, & \text{if } Cr(T_{i,j}^{t+1}) + \delta_t \cdot T \cdot \text{Ter}_M(T_{i,j}^{t+1}) < Cr(p_{T_i,j}); \\ g_{K_p,j} = p_{K_p,j}, & \text{if } Cr(p_{K_p,j}) + \delta_t \cdot T \cdot \text{Ter}_M(p_{K_p,j}) < Cr(g_{K_p,j}) + \delta_t \cdot T \cdot \text{Ter}_M(g_{K_p,j}); \\ g_{T_i,j} = p_{T_i,j}, & \text{if } Cr(p_{T_i,j}) + \delta_t \cdot T \cdot \text{Ter}_M(p_{T_i,j}) < Cr(g_{T_i,j}) + \delta_t \cdot T \cdot \text{Ter}_M(g_{T_i,j}). \end{cases}$$

During execution of classical PSO, particles may trap to a local minimum of the function (7). In this case, the swarm tends to stagnate: its exploration features are considerably declining. Stagnant swarm is unable to find the global minimum of the criterion (7).

The novelty of the ME-PSO technique is in reintialization of the stagnant swarm. The indicator of the swarm stagnation is as follows:

$$\text{(10)} \quad AR \geq \frac{Cr(g_{K_p,j}^{t+1}, T_{i,j}^{t+1}) + \delta_t \cdot T \cdot \text{Ter}_M(g_{K_p,j}^{t+1})}{Cr(g_{K_p,j}^{t}, T_{i,j}^{t}) + \delta_t \cdot T \cdot \text{Ter}_M(g_{K_p,j}^{t})},$$

where $AR$ is an acceptable rate of the global best reduction. If condition (10) is required, then swarm should be reinitialized: positions of all particles become random. Such approach allows to continue the exploration procedure and to find the global minimum of the criterion (7).

In the conducted research we have used parameters of ME-PSO, which are set in Table 1.

### Table 1. Parameters of optimization algorithm ME-PSO

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>social coefficient $c_1$</td>
<td>2.1</td>
</tr>
<tr>
<td>cognitive coefficient $c_2$</td>
<td>0.1</td>
</tr>
<tr>
<td>swarm population</td>
<td>50</td>
</tr>
<tr>
<td>connection topology</td>
<td>full</td>
</tr>
<tr>
<td>acceptable rate $AR$</td>
<td>0.1</td>
</tr>
<tr>
<td>number of iterations</td>
<td>50</td>
</tr>
</tbody>
</table>

**Numerical experiment**

In order to investigate the impact of the values $\delta_t, \ldots \delta_4$ on the PSO-controller tuning efficiency, they have been varying through numerical experiments. Used $\delta_t, \ldots \delta_4$ values are given in Table 2.

### Table 2. Values of coefficients $\delta_t, \ldots \delta_4$

<table>
<thead>
<tr>
<th>Notation</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\delta_3$</th>
<th>$\delta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME-PSO-Error</td>
<td>1000</td>
<td>1000</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>ME-PSO-Control</td>
<td>1000</td>
<td>1000</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>ME-PSO-Duration</td>
<td>1000</td>
<td>1000</td>
<td>5</td>
<td>1000</td>
</tr>
</tbody>
</table>

In order to prove the superiority of the developed tuning technique, all the results were compared with the results of tuning PI-controller, with other well-known in the engineering practice methods: Ziegler-Nichols [15], Kappa-Tau [16], AMIGO [17], Chien-Hrones-Reswick [18], Cohen-Coon [19], Lambda Tuning [20], Skogestad [21], Tyreus-Luyben [22]. The indicators, which have been used for determination of control quality are: mean integral error $\epsilon_{\text{MIE}}$, mean integral control $t_{\text{MIC}}$, overshoot (OS) and settling time $t_s$. 
In order to prove the superiority of the developed PI-controller tuning technique, five benchmark transfer functions have been used. They are proposed by K.J. Åström and T. Hägglund in the work [23]. For each transfer function the search domains for proportional and integral coefficients were different (Table 3).

### Table 3. Conditions of the experiments

<table>
<thead>
<tr>
<th>Transfer function</th>
<th>Search domain</th>
<th>P</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>G(s)=1/(s+1)^2</td>
<td>0...10</td>
<td>0...10</td>
<td></td>
</tr>
<tr>
<td>G(s)=1/(s+1)</td>
<td>0...10</td>
<td>0...10</td>
<td></td>
</tr>
<tr>
<td>G(s)=(1-0.1s)/(s+1)^2</td>
<td>0...10</td>
<td>0...10</td>
<td></td>
</tr>
<tr>
<td>G(s)=1/(s+1)+1.01s</td>
<td>0...50</td>
<td>0...20</td>
<td></td>
</tr>
<tr>
<td>G(s)=s^2+0.5s+1</td>
<td>0...10</td>
<td>0...20</td>
<td></td>
</tr>
</tbody>
</table>

### Brief results analysis

All the obtained results are given in Table 4. The best values in Table 4 are in bold.

### Table 4. Results of numerical experiments

<table>
<thead>
<tr>
<th>Tuning method</th>
<th>Parameters</th>
<th>MIE</th>
<th>MIC</th>
<th>OS, %</th>
<th>t₀, sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ziegler-Nichols</td>
<td>2.173</td>
<td>0.899</td>
<td>0.33</td>
<td>1.39</td>
<td>9.2</td>
</tr>
<tr>
<td>Kappa-Tau</td>
<td>0.436</td>
<td>2.238</td>
<td>0.50</td>
<td>0.69</td>
<td>24.3</td>
</tr>
<tr>
<td>AMIGO</td>
<td>0.495</td>
<td>2.559</td>
<td>0.44</td>
<td>0.90</td>
<td>0.0</td>
</tr>
<tr>
<td>Chien-Hrones-Rewick</td>
<td>1.449</td>
<td>1.618</td>
<td>0.25</td>
<td>1.07</td>
<td>0.0</td>
</tr>
<tr>
<td>Cohen-Coon</td>
<td>3.001</td>
<td>0.350</td>
<td>0.22</td>
<td>1.30</td>
<td>37.2</td>
</tr>
<tr>
<td>Lambda Tuning</td>
<td>0.230</td>
<td>4.828</td>
<td>0.35</td>
<td>0.80</td>
<td>0.0</td>
</tr>
<tr>
<td>Skogestad</td>
<td>1.500</td>
<td>1.000</td>
<td>0.31</td>
<td>1.27</td>
<td>9.3</td>
</tr>
<tr>
<td>Tyreus-Luyben</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ME-PSO-Error</td>
<td>10.000</td>
<td>1.313</td>
<td>0.14</td>
<td>1.62</td>
<td>29.3</td>
</tr>
<tr>
<td>ME-PSO-Control</td>
<td>0.000</td>
<td>9.701</td>
<td>0.38</td>
<td>0.69</td>
<td>0.0</td>
</tr>
<tr>
<td>ME-PSO-Duration</td>
<td>1.257</td>
<td>1.336</td>
<td>0.51</td>
<td>1.30</td>
<td>1.9</td>
</tr>
</tbody>
</table>

**First experiment**

- Ziegler-Nichols: 2.173, 0.899, 0.33, 1.39, 9.2, 3.0
- Kappa-Tau: 0.436, 2.238, 0.50, 0.69, 24.3
- AMIGO: 0.495, 2.559, 0.44, 0.90, 0.0
- Chien-Hrones-Rewick: 1.449, 1.618, 0.25, 1.07, 0.0
- Cohen-Coon: 3.001, 0.350, 0.22, 1.30, 37.2
- Lambda Tuning: 0.230, 4.828, 0.35, 0.80, 0.0
- Skogestad: 1.500, 1.000, 0.31, 1.27, 9.3
- Tyreus-Luyben: -
- ME-PSO-Error: 10.000, 1.313, 0.14, 1.62, 29.3
- ME-PSO-Control: 0.000, 9.701, 0.38, 0.69, 0.0
- ME-PSO-Duration: 1.257, 1.336, 0.51, 1.30, 1.9

**Second experiment**

- Ziegler-Nichols: 1.229, 3.438, 0.25, 0.98, 0.0, 12.2
- Kappa-Tau: 0.245, 4.836, 0.49, 0.81, 0.6, 9.7
- AMIGO: 0.295, 5.637, 0.40, 0.81, 0.0
- Chien-Hrones-Rewick: 0.820, 6.188, 0.25, 0.88, 0.0
- Cohen-Coon: 2.057, 0.831, 0.22, 1.13, 55.7
- Lambda Tuning: 0.153, 6.464, 0.38, 0.80, 0.0
- Skogestad: 0.500, 3.000, 0.37, 1.00, 5.8
- Tyreus-Luyben: 2.500, 3.225, 0.17, 1.01, 13.6
- ME-PSO-Error: 2.450, 3.100, 0.17, 1.01, 12.8
- ME-PSO-Control: 0.000, 7.730, 0.51, 0.68, 0.0
- ME-PSO-Duration: 0.718, 2.834, 0.56, 1.06, 2.0

**Third experiment**

- Ziegler-Nichols: 1.315, 5.992, 0.23, 0.95, 0.0, 15.6
- Kappa-Tau: 0.229, 5.184, 0.49, 0.80, 0.0, 10.5
- AMIGO: 0.280, 5.957, 0.40, 0.81, 0.0
- Chien-Hrones-Rewick: 0.757, 7.245, 0.25, 0.86, 0.0
- Cohen-Coon: 1.963, 0.900, 0.20, 1.09, 55.2
- Lambda Tuning: 0.264, 6.558, 0.38, 0.80, 0.0
- Skogestad: 0.469, 3.200, 0.56, 0.96, 4.9
- Tyreus-Luyben: 1.923, 4.702, 0.17, 0.94, 1.0
- ME-PSO-Error: 3.271, 3.556, 0.15, 1.04, 30.8
- ME-PSO-Control: 0.000, 7.900, 0.51, 0.68, 0.0
- ME-PSO-Duration: 0.944, 2.558, 0.59, 1.18, 4.2

### Analysis

Analysis of the figures that are given in Table 4 shows that the used approach is effective for minimization of the undesirable indicators. For example, the optimal settling time for the first experiment is 1.20…5.28 times smaller than similar values of other PI-controller tuning methods. For the second experiment, it ranges from 1.80 to 4.58, for the third is from 1.43 to 6.38, and for the fifth one is from 1.7 to 25.0. For all results of ME-PSO-Duration approach, the overshoot is no more than 4.2% (Fig. 2, a, c, e).

Obtained results confirm the suggestion about an invariant property of the developed approach. Indeed, as the calculations of coefficients Kₚ and Tᵣ are performed numerically, more complicated transfer functions will not make significant obstacles for technique applying.

Minimization of indicator MIE allowed us to reduce mean values of error during transition mode. However, criterion MIE utilizing has a disadvantage, which is connected with quite big overshoot (Fig. 2, b). In fact, that effect to a greater or a lesser extent has been revealed almost for all experiments (except the fifth one). For instance, the minima of indicator MIE for the transfer functions G(s), Gₛ(s), Gₘ(s) vary in the range 12.8…44.0%. It means that using single indicator MIE does not lead to a good quality of tuned PI-controller performance. Indicator MIE should be used only as a part of the complex optimization criterion.

Using in the calculations criterion MIC is connected with minimization of control mean value and reducing the overshoot (Fig. 2, d). In the frame of the research, the obtained values of the overshoot were 0.0…0.6%. From this point of view, criteria MIE and MIC are opposite.

The use of the proposed approach (ME-PSO-Control) allowed us to find the smallest values of MIC for all experiments. They are less by 1.20…5.59 times than those that related to the eight engineering PI-controller tuning methods.

For the fourth numerical experiment, we have obtained zero overshoot (Fig. 2, d) while for the rest of the results that indicator varies from 9.5% to 70.2%.

Positive results have been received for the transfer function with delay Gₛ(s). These data support the previous suggestion about invariability of the technique towards the complexity of the system under PI-control.

In order to estimate the obtained results, graphics for the most popular tuning methods as well as for ME-PSO-based method have been plotted (Fig. 2). They support the previous conclusion about the superiority of the developed technique over known in engineering practice PI-controller tuning methods.

Analysis of the figures in Table 4 allows us to state that the developed technique of PI-controller tuning is effective. Indeed, almost all undesirable indicators are smaller than those that have been calculated with the known PI-controller tuning methods.
Varying the values of the weight coefficients $\delta_1, ..., \delta_4$ provides technique flexibility. That is why a user of the algorithm may obtain desirable results (in terms of minimization of criterion (6) components) by setting the values of weight coefficients.

Implementing of the proposed tuning algorithm requires a special software development. It may help engineers to tune and retune PI-controllers. Another way of using the technique is connected with its implementation in the intelligent algorithms for PI-controllers self-tuning.

The developed technique may be generalized for the systems which are described by MIMO mathematical models (including non-linear ones).

**Conclusion**

In the article, PI-controller tuning technique, which is based on a metaheuristic optimization algorithm, has been developed. It consists in the reduction of the initial problem to the problem of minimization of the devised complex criterion. Using the advanced optimization technique ME-PSO allowed us to find the coefficients of PI-controller for five benchmark transfer functions.

In the carried out research we have used as a criterion the weighted sum of mean integral error, mean integral control, overshoot and settling time. The brief analysis of the impact of weight coefficients $\delta_1, ..., \delta_4$ to the performance of the tuned PI-controller has been given. The developed PI-controller tuning technique shows its superiority over other well-known methods.

It should be noted, that the proposed approach is not limited by used in the research indicators; the optimization criterion may include other important indicators.

In addition, the problems of generalization of the developed approach to different transfer functions, MIMO systems, with taking into account constraints and control implementation via pulse width modulation will appear in future investigations.

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