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Mathematical models of the electric arc of variable geometrical parameters and various heat dissipation methods

***Abstract**. The paper deals with three mathematical models of the electric arc of variable geometrical parameters, in which the dissipated power depends respectively on the arc lateral area (the Voronin model), on its volume or on these two parameters simultaneously (two variants proposed in this study). Simulations were carried out to verify the applicability of these models. At the first stage of the simulation it was assumed that variation in arc length was significantly slower than variation in current and an arc variant was tested with a constant cross-section area. At the second stage a constant length arc with the cross-section area varying in a quasi-steep manner was investigated. The effectiveness of these mathematical models is presented in the form of dynamic voltage-current characteristics, for various prescribed values of parameters.*

***Streszczenie.** Rozpatrzono trzy przypadki modeli matematycznych łuku elektrycznego o zmiennych rozmiarach geometrycznych, w których rozpraszana moc cieplna zależy od pola powierzchni bocznej kolumny, od jej objętości, lub jednocześnie od tych dwóch parametrów. Na pierwszym etapie badań symulacyjnych potraktowano zmiany długości kolumny jako znacznie wolniejsze w porównaniu ze zmianami wymuszenia prądowego. Wybrano wariant łuku, w którego modelach pole przekroju poprzecznego kolumny jest stałe. Na drugim etapie badań rozpatrzono łuk o stałej długości i o quasi skokowej zmianie pola przekroju poprzecznego kolumny. Efektywność stosowania opisanych modeli matematycznych łuku zaprezentowano w postaci charakterystyk napięciowo-prądowych dynamicznych, odpowiadających różnym wartościom zadanych parametrów. (**Modele matematyczne łuku elektrycznego o zmiennych rozmiarach geometrycznych kolumny i różnych metodach rozpraszania ciepła**)*

Keywords: electric arc, Voronin model, parameter distortions.
Słowa kluczowe: łuk elektryczny, model Woronina, zaburzenia parametrów.

Introduction

There are many possible factors causing variation in length or in cross-section area of an electric arc in electric or power devices. They can be divided into distortion factors and control factors. Among the external causes of arc length variations, the following are the most common: changes of the electrode positions, inflection of the plasma column due to a transverse gas flow, inflection of the plasma column due to a transverse magnetic field, shifts if electrode spots due to mechanical or magnetic factors. External factors causing variation in the cross-section area of the arc column include: variation in the pressure of the gas atmosphere, variation in the composition of the plasma gas, variation in the speed of gas flowing around the arc column, constricting the plasma by the walls of the channel or of the plasma generator.

Varying the arc length is the most common and important method used to influence the arc stability in power switches and thermal power in electrothermal devices. The arc column size in power switches should change as fast as possible. When electrodes or plasma jets are immersed in the furnace chamber with sufficiently high speed, this enables fluent changes of the voltage and consequently of the contribution of the thermal power components in the arc energy balance [1, 2].

There are many difficulties that have to be overcome in the modelling of an electric arc of variable geometrical parameters. These difficulties arise due to nonlinear dependences among the electrical, thermal and geometrical characteristics of the plasma column [1, 3].

The Voronin model of the arc column with variable geometrical parameters and with heat dissipation processes dependent on the arc lateral area

In order to construct the Voronin model [4, 5] it is necessary to adopt the following simplifying assumptions:

1. The arc column is cylindrical with a constant cross-section area at any point along the axis;
2. Plasma is homogeneous along the transverse and axial direction;
3. Heat is dissipated only from the arc lateral surface;
4. Arc length and cross-section area can vary in time.

The foundation of the model is the simplified equation of the arc thermal balance

$$\frac{dQ}{dt} = P_{el} - P_{dis} = u_{col} i - P_{dis} \quad (1)$$

where: Q – enthalpy of the plasma, J; P_{el} – electric power supplied to the arc column, W; P_{dis} – thermal power dissipated from the arc column, W; u_{col} – voltage on the arc column, V. The following notations will be now introduced:

$$Q = q_V V = q_V l S \quad (2)$$

$$g = \frac{\sigma \cdot S}{l} \quad (3)$$

$$P_{dis} = P_S(l,S) = p_S S_s = p_S l \sqrt{4\pi S} \quad (4)$$

where: q_V – volumetric density of enthalpy, J/m^3; σ – specific conductivity of the arc column, S/m; l – arc column length, m; p_S – area density of the power dissipated through the arc lateral surface, W/m^2; S – cross-section area of the arc, m^2; S_s – lateral area of the arc, m^2; $S_s = l\sqrt{4\pi S}$.

The arc conductance is a function of the arc enthalpy $g = F(Q)$. If one of the Mayr model assumptions is adopted

$$\sigma = \sigma_0 \cdot exp\left(\frac{q_V}{q_0}\right) \quad (5)$$

where: σ_0 and q_0 are the approximation coefficients of the plasma conductivity function, then the dynamic arc model with variable geometrical parameters $S(t)$ and $l(t)$ becomes [4]

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{q_0 S l}\left(u_{col} i - P_{dis}\right) + -\frac{1}{l}\frac{dl}{dt}\left(1 + ln\frac{g\,l}{\sigma_0 S}\right) + \frac{1}{S}\frac{dS}{dt}\left(1 - ln\frac{g\,l}{\sigma_0 S}\right) \quad (6)$$

After Eq. (4) is taken into account and transformations are applied

$$\frac{1}{g}\frac{dg}{dt}=\frac{p_S}{q_0}\sqrt{\frac{4\pi}{S}}\left(\frac{u_{col}i}{p_S l\sqrt{4\pi S}}-1\right)+ -\frac{1}{l}\frac{dl}{dt}\left(1+ln\frac{g\,l}{\sigma_0 S}\right)+\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right) \tag{7}$$

The ultimate form of the Voronin formula is

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_S(S)}\left(\frac{u_{col}i}{P_S(l,S)}-1\right)+ -\frac{1}{l}\frac{dl}{dt}\left(1+ln\frac{g\,l}{\sigma_0 S}\right)+\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right) \tag{8}$$

where: $\theta_S(S)=\frac{q_0}{p_S}\sqrt{\frac{S}{4\pi}}$, $P_S(l,S)=p_S l\sqrt{4\pi S}$.

In the special case with the constant parameters (dl/dt = 0, dS/dt = 0), the equation can be brought to the Mayr model (P_S = const) or to the Cassie model ($P_S = U_C^2 g$) with the time constant θ_S = const [4]. In the Mayr model the arc cross-section area is constant, and changes in current cause changes in temperature distribution and related physical parameters of plasma. In the Cassie model, the temperature distribution and the related plasma parameters are constant and the arc cross-section area varies. In the case of an unconstricted arc, increase in current causes increase in the arc diameter and consequently increase in energy dissipation. In a constricted arc, decreasing the arc column diameter leads to increase in the plasma temperature, arc voltage and heat dissipation.

When the changes in the above-mentioned parameters or distortions occur slowly, the static arc characteristic is represented by the formula

$$U_{col}=\frac{P_S(l,S)}{I} \tag{9}$$

and the resultant arc voltage is

$$U=U_{AK}+U_{col}(I,l,S)=U_{AK}+\frac{P_S(l,S)}{I} \tag{10}$$

All the three parameters p_S, q_0, σ_0 being components of the model (8) are obtained experimentally and assumed to be constant. Special cases of this model with the dimensions $l(t)$ and $S(t)$ being constant or varying gradually are discussed in [6-10], where it is demonstrated that such special cases lead either to the Mayr model, to the Cassie model or to their modifications. These modified models, however, are capable of representing electrotechnological processes only to a limited extent.

A model of the arc column with variable geometrical parameters and with heat dissipation processes dependant on its volume

On the basis of the previously presented model it can be assumed that the thermal power dissipated from the arc column is, in an analogous case, proportional to the plasma volume

$$P_{dis}=P_V(l,S)=p_V V=p_V lS \tag{11}$$

where: p_V – density of power dissipated from the arc column volume, W/m^3, V – volume of the arc column, m^3. Applying the assumptions (1)-(3) and (5), the following equation can be obtained

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{q_0 Sl}\left(u_{col}i-P_{dis}\right)-\frac{1}{l}\frac{dl}{dt}\left(1+ln\frac{g\,l}{\sigma_0 S}\right)+ +\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right) \tag{12}$$

Taking into account Eq. (10) and after applying transformations

$$\frac{1}{g}\frac{dg}{dt}=\frac{p_V}{q_0}\left(\frac{u_{col}i}{p_V lS}-1\right)-\frac{1}{l}\frac{dl}{dt}\left(1+ln\frac{g\,l}{\sigma_0 S}\right)+ +\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right) \tag{13}$$

The ultimate form of the equation becomes [6, 8]

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_V}\left(\frac{u_{col}i}{P_V(l,S)}-1\right)-\frac{1}{l}\frac{dl}{dt}\left(1+ln\frac{g\,l}{\sigma_0 S}\right)+ +\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right) \tag{14}$$

where: $\theta_V=\frac{q_0}{p_V}$; $P_V(l,S)=p_V lS$.

When the changes in the relevant parameters or distortions occur slowly, the static arc characteristic is represented by the formula

$$U=\frac{P_V(l,S)}{I} \tag{15}$$

and the resultant arc voltage is

$$U=U_{AK}+U_{col}(I,l,S)=U_{AK}+\frac{P_V(l,S)}{I} \tag{16}$$

A model of the arc column with variable geometrical parameters and with heat dissipation processes dependent on its lateral area and volume

As analyses of heat transfer processes in the arc column indicate, there are several channels of dissipation, dependent on kind of gas and its pressure, the material and shape of the electrodes, as well as current and frequency [1]. A more general assumption can be introduced, in which heat dissipation depends simultaneously on the lateral area S_s and on the arc column volume V

$$P_{dis}=P_{SV}=P_S+P_V=p_S S_s+p_V V= =p_S l\sqrt{4\pi S}+p_V lS \tag{17}$$

Applying the assumptions (1)-(3), (5) and (17), a new equation of power balance can be obtained

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{q_0 Sl}\left[u_{col}i-\left(p_S l\sqrt{4\pi S}+p_V lS\right)\right]+ -\frac{1}{l}\frac{dl}{dt}\left(1+ln\frac{g\,l}{\sigma_0 S}\right)+\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right) \tag{18}$$

after transformations it becomes

(19) $$\frac{1}{g}\frac{dg}{dt}=\frac{p_S\sqrt{\frac{4\pi}{S}}+p_V}{q_0}\left(\frac{u_{col}i}{p_S l\sqrt{4\pi S}+p_V lS}-1\right)+ -\frac{1}{l}\frac{dl}{dt}\left(1+ln\frac{g\,l}{\sigma_0 S}\right)+\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right)$$

and the ultimate form of the mathematical model is

(20) $$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{SV}(S)}\left(\frac{u_{col}i}{P_{SV}(l,S)}-1\right)+ -\frac{1}{l}\frac{dl}{dt}\left(1+ln\frac{g\,l}{\sigma_0 S}\right)+\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right)$$

where the following notation is introduced:

$$\theta_{SV}(S)=\frac{q_0}{p_S\sqrt{\frac{4\pi}{S}}+p_V},\ P_{SV}(l,S)=p_S l\sqrt{4\pi S}+p_V lS .$$

When the changes in parameters or external distortions occur gradually, the arc static characteristic is

(21) $$U_{col}=\frac{P_{SV}(l,S)}{I}=\frac{p_S l\sqrt{4\pi S}+p_V lS}{I}$$

and the resultant voltage is

(22) $$U=U_{AK}+\frac{P_{SV}(l,S)}{I}=U_{AK}+\frac{p_S l\sqrt{4\pi S}+p_V lS}{I}$$

Properties of the arc models in which fluctuations of the arc column size are taken into account

In the mathematical models of the arc presented above, the value of the damping function depends on the cross-section area of the arc column ($\theta_S(S)\propto\sqrt{S}$, $\theta_{SV}(S)\propto\sqrt{S}$), but only if heat dissipation is dependent on the lateral area S_s. The time constant of the model (14) θ_V = const. The Voronin model disregards the causes of the fluctuations of the arc column size. Such causes can be external or internal, the latter including changes in the range of high current of a free arc, which are responsible for variation in the arc cross-section area $S(g(i))=var$. Changes in the range of low current of a free arc cause changes in the temperature distribution in the arc cross-section and of other physical parameters.

The ties between the arc damping functions in the arc models are as follows:

(23) $$\frac{\theta_S(S)}{\theta_V}=\frac{p_V}{p_S}\sqrt{\frac{S}{4\pi}}$$

(24) $$\frac{\theta_{SV}}{\theta_V}=\frac{p_V}{p_V+p_S\sqrt{\frac{4\pi}{S}}}$$

(25) $$\frac{\theta_{SV}(S)}{\theta_S}=\frac{p_S}{p_S+p_V\sqrt{\frac{S}{4\pi}}}$$

As it is evident from the previously presented formulas, the value of dissipated power depends linearly on the arc length in every case ($P_S\propto l$, $P_V\propto l$, $P_{SV}\propto l$). The cross-section area S, however, affects the power in various ways in various models ($P_S\propto\sqrt{S}$, $P_V\propto S$). Experimental studies carried out on a constricted arc indicate that $U_{stat}\propto l$ and $U_{stat}\propto d^{-1}\propto S^{-1/2}$, which proves that heat is dissipated from the column in various ways [11, 12].

Rapid changes in the arc column length are typically caused by variable magnetic field. Let variation of the arc length be relatively slow ($dl/dt\approx 0$) as compared to variation in current. Then the models can be simplified in the following way:

- Eq. (8) becomes

(26) $$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_S(S)}\left(\frac{u_{col}i}{P_S(l,S)}-1\right)+\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right)$$

- Eq. (13) becomes

(27) $$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_V}\left(\frac{u_{col}i}{P_V(l,S)}-1\right)+\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right)$$

- Eq. (19) becomes

(28) $$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{SV}(S)}\left(\frac{u_{col}i}{P_{SV}(l,S)}-1\right)+\frac{1}{S}\frac{dS}{dt}\left(1-ln\frac{g\,l}{\sigma_0 S}\right)$$

The effectiveness of the mathematical models was verified by means of simulations. In the arc circuit there was a source of sinusoidal current with the amplitude of 120 A and frequency 50 Hz. In the first case it was assumed that the cross-section area of the arc column is constant S = const and that the arc column increases its length relatively slowly (with the speed v) and linearly l = variab starting from the value l_0. The sum of voltage drops near the cathode was U_{AK} = 18 V.

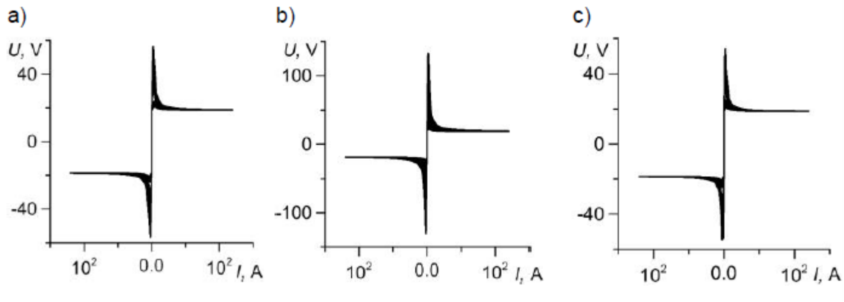

Fig. 1. Dynamic voltage-current characteristics of an expanded arc: a) according to model (8) with the parameters (p_S = 150 000 W/m^2, q_0 = 3000 J/m, l_0 = 5·10^{-3} m, v = 1·10^{-2} m/s, S = 5·10^{-5} m^2); b) according to model (14) with the parameters (p_V = 6·10^6 W/m^3, q_0 = 200 J/m, l_0 = 8·10^{-3} m, v = 5·10^{-2} m/s, S = 5·10^{-4} m^2); c) according to model (20) with the parameters (p_V = 6·10^6 W/m^3, p_S = 3000 W/m^2, q_0 = 200 J/m, l_0 = 5·10^{-3} m, v = 1·10^{-2} m/s, S = 5·10^{-4} m^2)

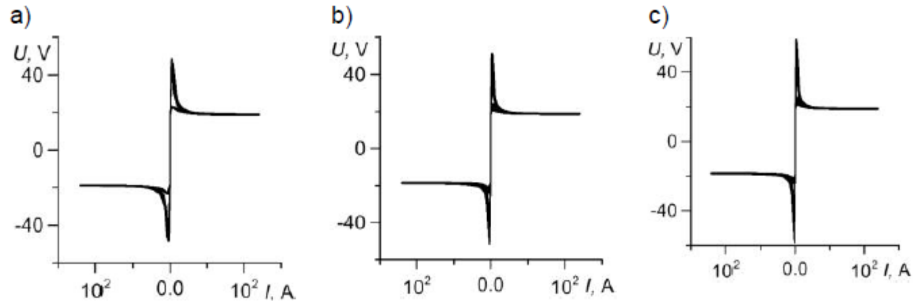

Fig. 2. Dynamic voltage-current characteristics of an arc with a quasi-step decrease of the cross-section area: a) according to model (26) with the selected parameters (p_S = 150 000 W/m^2, q_0 = 3000 J/m, σ_0 = 8000 S/m, l_0 = 10·10^{-3} m, S_{min} = 5·10^{-5} m^2, S_{max} = 10·10^{-5} m^2); b) according to model (27) with the selected parameters (p_V = 6·10^6 W/m^3, q_0 = 200 J/m, σ_0 = 8000 S/m, l_0 = 1·10^{-3} m, S_{min} = 3·10^{-4} m^2, S_{max} = 6·10^{-4} m^2); c) according to model (28) with the selected parameters (p_V = 6·10^6 W/m^3, p_S = 4000 W/m^2, q_0 = 200 J/m, σ_0 = 8000 S/m, l_0 = 10·10^{-3} m, S_{min} = 3·10^{-4} m^2, S_{max} = 6·10^{-4} m^2)

The dynamic voltage-current characteristics $u(i)$ corresponding to the three arc models are presented in Fig. 1. As can be seen, the three curves are very similar, which indicates that the approximations assumed in the models may have a limited potential for representing experimental data.

The simulation experiment was repeated, this time with the assumed constant arc length l_0 = const and quasi-steep decrease in the arc column cross-section area S = variab from S_{max} to S_{min}. The dynamic voltage-current characteristics $u(i)$ corresponding to the three arc models are presented in Fig 2.

Conclusions:

1. The mathematical model of the arc known as the Voronin model takes into account only one kind of heat dissipation through the arc lateral surface, mainly by radiation. This may limit its applicability to represent electrical processes occurring during heating in electrotechnological devices and in power switches.
2. The modified arc model proposed in this paper (14) takes into account the condition of heat dissipation from the arc column volume both by radiation and by convection. It is then a viable alternative for representing the electrical processes occurring during heating in devices and power switches.
3. Another modified arc model offered in this paper (20) takes into account the condition of heat dissipation from the lateral surface and from the arc column volume by radiation and by convection, thereby providing a more general variant than the previous ones, suitable for representing electrical processes occurring during heating in devices and power switches.
4. In the mathematical models proposed herein it is assumed that plasma conductivity depends exponentially in its specific enthalpy (5), which is analogous to the Mayr model with a constant column diameter and constant dissipated power. At the same time, it is held that the dissipated power depends on the arc cross-section area (4), (11) or (17), which is analogous to the Cassie model with a proportional dependence of conductance on plasma enthalpy. This entails that the model can be applied to a wide range of current variation. It should also be noted that the static curves (9), (15) and (21) are hyperbolas, as in the Mayr low current arc model.
5. The simulations of processes occurring in an electric circuit carried out with the use of the three mathematical arc models indicate that further research is needed to improve the applicability of arc models for representing real processes.

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