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Characterization of defects in conductive materials by the intrusive stochastic method using the random variable Lognormal

***Abstract**. The aim of this study is to evaluate the reliability of an electromagnetic system due to the presence of a rectangular defect in non-destructive testing by eddy currents. The distribution of the hazard which corresponds to the electrical conductivity of the conductive material is represented by the random variable of Lognormal type which is associated with the spectral stochastic finite element model (SSFEM). The results obtained are compared with those of the Monte Carlo simulation and the experimental measurements. an assessment of the probability of failure is made.*

Streszczenie.** Celem pracy jest ocena niezawodności systemu elektromagnetycznego ze względu na obecność wady prostokątnej w badaniach nieniszczących prądami wirowymi. Rozkład zagrożenia, który odpowiada przewodności elektrycznej materiału przewodzącego, jest reprezentowany przez zmienną losową typu Lognormal, która jest powiązana ze spektralnym stochastycznym modelem elementów skończonych (SSFEM). Otrzymane wyniki porównuje się z wynikami symulacji Monte Carlo i pomiarami eksperymentalnymi. dokonuje się oceny prawdopodobieństwa niepowodzenia.* (Charakteryzacja defektów materiałów przewodzących intruzyjną metodą stochastyczną z wykorzystaniem zmiennej losowej Lognormal***)

Keywords: Lognormal random variable , Intrusive stochastic finite element method, Reliability and probability of failure
Słowakluczowe: Lognormalna zmienna losowa stochastyczna metoda elementów skończonych, Zmiana impedancji .

Introduction

The study of electromagnetic systems requires knowing the physical input parameters to obtain the output information such as their performance. The difficulty encountered in solving a practical problem is the ability to identify physical properties like electrical conductivity, magnetic permeability, which must be known before the study is tackled. Most of cases, the input data are often sought with the aid of an experiment or are used with some uncertainty [1,2].

Today, researchers are focusing more and more on modeling in uncertain environments such as stochastic calculations. The deterministic calculation requires going towards an inverse calculation. Often, for post-processing, such as assessing the reliability of the electromagnetic system, uncertainty about the physical properties of the system is not taken in to account. Knowing the forms of defects is also necessary in the inspection process. In this situation, the input data is often used with some uncertainty [1- 4].

The spectral stochastic finite element intrusive method (SSFEM) considers the input variables as being random variables, they can be of Gaussian, Lognormal or other type. The developed method (SSFEM) also has a fairly significant advantage, in particular; the study and the analysis of the sensitivity of the system in a single step [1] .

This work deals with defect inspection on copper represented by a plate with a rectangular defect. An eddy current non-destructive testing technique is used to assess the reliability of the system studied by determining the impedance variation in the area suspected by the presence of a defect. This type of defect can be present in the copper windings of squirrel-cage asynchronous electric motors during brazing, and can also be encountered in the rotor bars during conductor injection. The inspection of this type of defect is carried out during the manufacturing process by measuring the electrical conductivity at several points of the part to be controlled using a sigma meter which is calibrated before any measurement. The values gathered will be compared to the reliability data in terms of electrical conductivity.

This type of defect can be inspected by a non-destructive testing device through a simulation that uses a spectral stochastic finite element code which consists of considering the electrical conductivity as random variable to integrate the uncertainty on the physical property of the copper. The study is performed under 2D Axi-symmetric assumptions.

The shape of the defect is rectangular, the random variable is of Lognormal type, characterized by an average value and a standard deviation fixed for this study at 0.9, they are developed in series of Hermit polynomials.

The post-processing makes it possible to evaluate the impedance variation in the defect, the impedance data for a fault of rectangular shape are compared to those derived by the simulation of Monte Carlo and the experimental measurements [4].

The experimental part is carried out at the Research Center for Real Time NDT Chosun University Gwangju, Korea, as part of an internship to prepare a doctoral thesis. Nortek 500 portable eddy current flaw detectors are used to inspect and characterize Inconel 600 and copper faults with random physical properties. We have exploited the results of the copper material as part of our study [5-7].

The reliability study is represented by the evaluation of the probability of failure of the system.

1. Deterministic Electromagnetic Model

The electromagnetic equation is obtained from Maxwell's equations associated with those of the middle relations and the law of ohms. In case of harmonic hypothesis, the electromagnetic formulation for the deterministic problem is given as follows [1,2].

$$\nabla \wedge \frac{1}{\mu}\left(\nabla \wedge \vec{A}\right) + j\sigma_s \omega \vec{A} = \vec{J}_{sz} \tag{1}$$

A: Magnetic vector potential[T. m] ; μ : Magnetic permeability [H.m^{-1}] ; f - Frequency [Hz]; σ_s electrical conductivity [S.m^{-1}]; J_{sz} : Source current density next z [A / m^2]

The finite element formulation leads, considering homogeneous Dirichlet boundary conditions, to the matrix system, and whose elements are as

(2) $$[M_{stoc}][A]=[F]$$

(3) $$M_{stoc_{ij}}=\iint_{\Omega}\nu\left(\frac{\partial\alpha i}{\partial x}\frac{\partial\alpha j}{\partial x}+\frac{\partial\alpha i}{\partial y}\frac{\partial\alpha j}{\partial y}\right)dxdy$$

(4) $$F_i=\iint J_{sz}\alpha_i dxdy$$

(5) $$[A]=[A_1,A_2,...,A_n]^T$$

2. Formulation of the intrusive method of spectral stochastic finite elements SSFEM

we assume X to be a random variable, whose input parameters are characterized by the vector M with independent stochastic variables, under these conditions, if X has a finite variance, X can be written as a linear combination of multivariate polynomials on the basis of Hermit polynomials.

(6) $$X=\sum_{i=1}^{p-1}\alpha_i H_i(\xi_1,...,\xi_n)$$

p , represents the degree of polynomial chaos.

$\{H_i(X), i=0,\ldots,\infty\}$ are Hermits polynomials,

$\{\alpha_i, i=0,\ldots,\infty\}$ are coefficients of Hermits polynomials where ξ is a reduced centred Gaussian random variable (r.c.g.r.v) with the following density of probability [2,5].

(7) $$\Phi(G)=\frac{1}{\sqrt{2\pi}}e^{\frac{-\sigma^2}{2}}$$

In (6) α_i are unknown coefficients to be determined.

We notice that all Hermit polynomials are orthogonal With regard to the Gaussian measure according to the orthogonally property.

(8) $$\mathrm{E}[\mathrm{H_n}(\xi(\theta))\,\mathrm{H_m}(\xi(\theta))]=0 \quad \text{if } \mathrm{n}\neq\mathrm{m}$$

E [.] denotes the mathematical expectation.

For the computation of the coefficients of Hermits polynomials, the work develops the method of projection. Thus, the coefficients of Hermits polynomials are given by the following expression [1].

(9) $$\alpha_i=\frac{E[GH_i(\xi)]}{i!}$$

The identification of the Hermit polynomial coefficients in the case of Iso-probabilistic transformation is performed using the next formula:

(10) $$\alpha_i=\int_R F_G^{-1}(\Phi(t))H_i(t)\phi(t)dt$$

2.1. Random Electrical Conductivity

In the context of our study, the electrical conductivity is distributed in the Hermit polynomial by the random variable of type Lognormal.

When the variable follows a lognormal distribution of parameters (λ, ζ), the coefficients a_i can be calculated analytically [7,8], as follows.

(11) $$a_i=\frac{\zeta\, e^{\lambda+\frac{1}{2}\zeta^2}}{i!} \qquad i\geq 0$$

Table.1. Stochastic distribution of electrical conductivity coefficients

Lognormal random variable	**Average value**	**standard deviation**	**Stochastic distribution coefficients**		
			a_{00}	a_{11}	a_{22}
(λ ,ζ)	ζ=76MSm[1]	λ=0.9	3.20	2.8	1.3

2.2. Stochastic algebraic matrix [1]

The expressions of electric conductivity and the unknown magnetic vector potential as random variables in the base of Hermit polynomials permit to write [9-13]

(12) $$\sigma=\sum_{i=1}^{p-1}\sigma_i H_i(\xi_1,...,\xi_M)$$

(13) $$A=\sum_{j=0}^{n_A}A_j\Psi j(\xi_1,\cdots,\xi_M)$$

$\{\xi_1,..,\xi_2\}$ centered reduced Gaussian variables

p: The order of development of polynomial chaos

$n_A=p-1$

σ: random electrical conductivity

The final form of the stochastic algebraic system is as follow [1, 2]

(14) $$\sum_{j=0}^{p-1}(M^{s\,\prime}_{jk}+j\omega N^s_{jk})A_j=F^s_k$$

(15) $$M^{s\,\prime}_{jk}=d_{0jk}M_0$$

(16) $$N^s_{jk}=\sum_{i=0}^{p-1}d_{ijk}N^s_i$$

(17) $$D_{ijk}=\begin{cases}\dfrac{i!j!k!}{\left(\frac{i+j-k}{2}\right)!\left(\frac{j+k-i}{2}\right)!\left(\frac{k+i-j}{2}\right)!} & if \begin{cases}(i+j+k)\ even\\ k\in[|i-j|,i+j]\end{cases}\\ 0 \quad \text{otherwise}\end{cases}$$

D_{ojk} and D_{ijk} are constants to be determined as follows [1].

By fixing p=3, In this case:
j=0, 1, 2 and k=0, 1, 2.

We obtain the following matrix system:

M^s, N^S, F^s are the random linear matrixes and source vector respectively related to solving problem. The stochastic system obtained from (14), (15), (16) and (17) is:

(18) $$\begin{bmatrix}M^s_{stoc00} & M^s_{stoc10} & M^s_{stoc20}\\ M^s_{stoc01} & M^s_{stoc11} & M^s_{stoc21}\\ M^s_{sfoc02} & M^s_{stoc12} & M^s_{stoc22}\end{bmatrix}\begin{bmatrix}A_0\\A_1\\A_2\end{bmatrix}=\begin{bmatrix}F_0\\F_1\\F_2\end{bmatrix}$$

where A_0 , A_1 , A_2 are the solutions of the stochastic complex algebraic system and F_0 , F_1, F_2 are the source vector components.

2.3. Post Treatment [1-3]

After solving the matrix system, we obtain three random solutions of the potential vector are A_0, A_1, A_2, the number of solution is equal to p, which represents the degree of polynomial chaos.
The post treatment, in the first step, consists of calculating the impedance Z, in the suspected defect zone, the expression of the impedance is as follows:

(19) $$\mathrm{Re}(Z) = -\frac{N^2}{J.S^2}\,\omega.\iint_S 2.\pi.r.\mathrm{Im}(A).ds$$

(20) $$\mathrm{Im}(Z) = \frac{N^2}{J.S^2}\,\omega.\iint_S 2.\pi.r.\mathrm{Re}(A).ds$$

J:Current density [A/m^2]; S: Conductors Section [m^2] ; N: Number of turns

The limit state function G_{SSFEM} is obtained with the threshold value of the impedance and the stochastic solutions of the impedance .

(21) $$G_{SFEM} = (Z_s)_{\max} - \sum_{j=0}^{p-1} Z_j^{i0}\Psi_j(\xi_1,\xi_2)$$

$(Z_s)_{max}$: reference value of the impedance [Ω]

Z^{i0}: Stochastic impedances [Ω].

The reliability index β is calculated from the limit state function G_{SSFEM}.

(22) $$\beta = Min\sqrt{|G_{SSFEM}|}$$

From the values of the reliability index a numerical calculation is carried out on the basis of the Lagrange polynomial, this one allows us to obtain the formulation of the probability of failure as well as its evolution according to the index of reliability [1,9 ,10].

(23) $$P_D = \phi(-\beta)$$

3. Description of the application

The chosen application is a non-destructive testing device, which represents a squirrel cage of an asynchronous motor subjected to eddy current inspection.
The test is carried out at the short circuit ring of the asynchronous motor.
The NDT device consists of a 2D approximation of a conductive cylinder with a conductivity of 58 MS/m. The defect is characterized by a length of 10 mm and a depth of 1 mm.
The inductor, representing the sensor, has 200 turns, it is supplied by an alternating current of amplitude 0.008A and a frequency of 150 kHz [1,2,13].

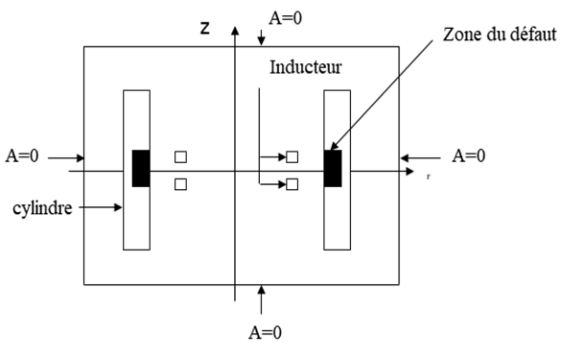

Fig.1. Geometry of the study device in (r,z) plan

3.1. Simulation and results

The obtained results are presented by figures 3, 4, 5, 8 and 9. Figure.3 shows the mesh of the defect study domain with a rectangular geometry. This mesh is used to solve the 2D axi-symmetric electromagnetic system.

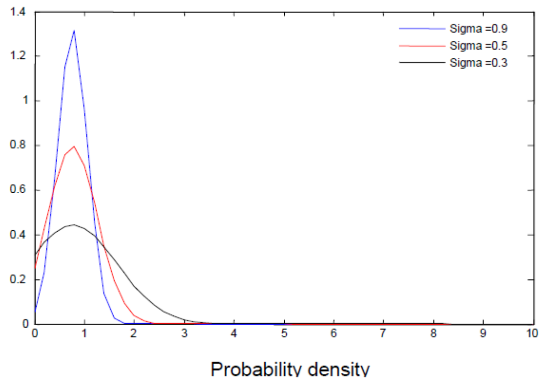

Fig. 2. Probability density of the Lognormal random variable for different standard deviations.

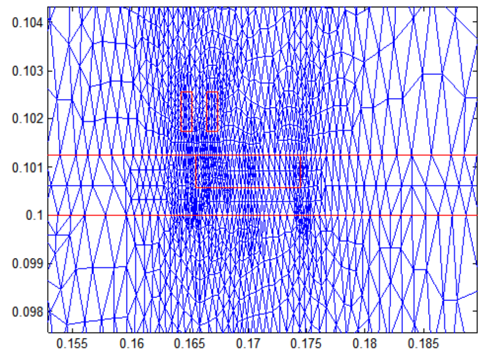

Fig.3. Mesh of the studied domain for rectangular geometry

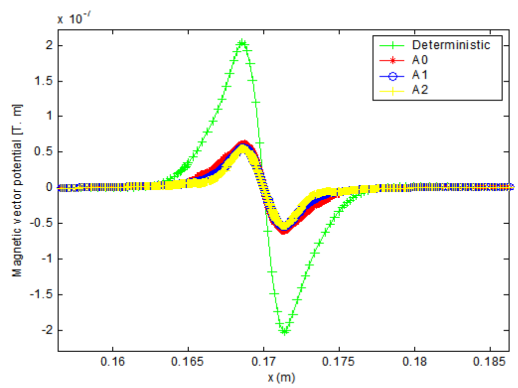

Fig.4. Comparison between deterministic and stochastic Solution

Table 2. Parameters of the test experiment

Coil	Physical and geometric characteristics	Copper
Inner diameter10 mm	Electric conductivity	58 MS/m
Outer diameter12 mm	Relative permeability	1
Number of turns 200	Thickness	2 mm
	Crack length	10 mm
Lift-off	Crack width	2 mm
1 mm	Crack depth	1mm

The figure 4 shows the comparison between the evolution of the vector potential A in a deterministic and stochastic model, in this case we see the fluctuation of the stochastic solution in the default zone. the three solutions practically match each other.

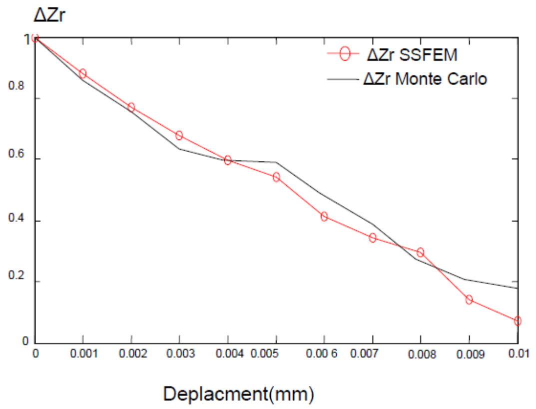

Fig.5. Confrontation of the spectral stochastic finite element method with the Monte Carlo simulation

Figure 5, shows us the evolution of the variation of the impedance in reduced value, obtained by the specral stochastic finite element model and the Monté Carlo draw. We find an important agreement for the two models which allows us to assert a significant advance for the spectral stochastic finite element model (intrusive

3.2. Experimental setup

The system used consists of Nortek 500 eddy current flaw detectors placed parallel to the specimen being inspected, which moves on the x-y plane of the fault zone.
The copper plates contain cracks of various shapes and sizes. We scan the rectangular shaped one with the length and width and different depth varying from 100% air (correspond to 1.15mm) to 20%.

The experiment is characterized by the parameters given in table 2.

The impedance signal obtained from the ECT probe is recovered and represented as a function of the sensor's position and impedance plane. [3-5]

Fig.6.Nortek 500 flaw detectors

A comparison of the impedance variation in the rectangular shaped defect is illustrated in Figure 8, which shows a good agreement between the experimental data obtained from the conditions mentioned in Table 1, and those obtained by the simulation of the spectral stochastic finite element method SSFEM.

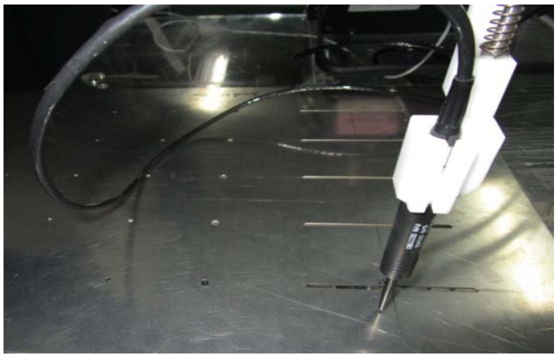

Fig.7. Inspected specemen

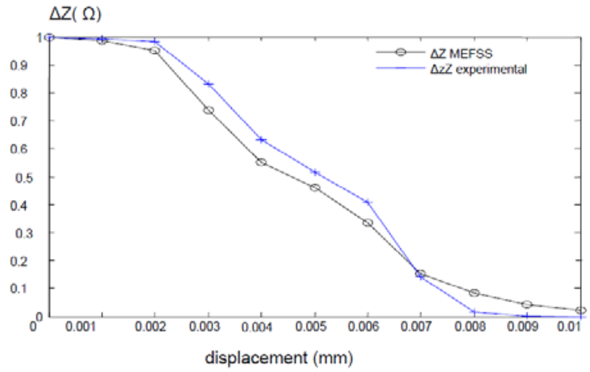

Fig.8. Comparison of impedance variation between SSFEM and experimental measurements

The Figure.9 shows the shape of the correlation of the probability of failure as a function of the reliability index. This correlation is derived from the expansion of the Lagrange polynomial using the reference data [10] and the values obtained by the stochastic calculation of the reliability index.

The two curves practically overlap,whichs allow us to argue that the exploitation of the Lagrange polynomial is a rather appreciable numerical technique in the evaluation of the correlation in terms of probability of failure.

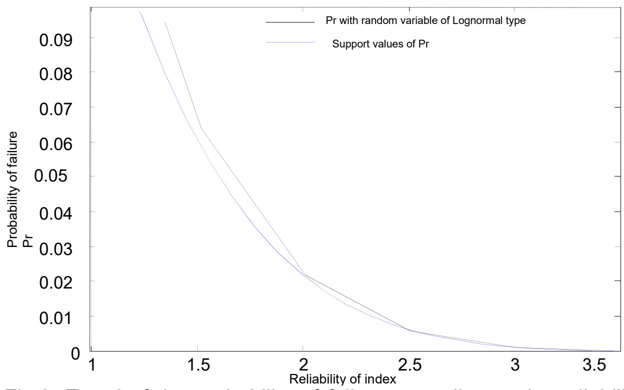

Fig.9. Trend of the probability of failure according to the reliability index

Conclusion

This work presents the development of the intrusive spectral stochastic finite element method, on one hand, generate the geometric random shape of a defect and, on the other, solve the 2D electromagnetic equation under an Axisymmetric assumption by considering the electrical conductivity as a lognormal random variable with a standard deviation of 0.9. The Gaussian type random variable has already been studied in previous works with very satisfactory results [1- 4].

Our objective is to assess the relevance of using the lognormal random variable in the SSFEM model. Eddy current non-destructive testing is used in our application to detect and evaluate the presence of a defect in the form of a rectangular crack on a copper plate in order to simulate the occurrence of this type of fault on a copper bar rotor in a squirrel cage asynchronous motor. The use of the SSFEM model, allowed us to calculate the impedance variations in the defect zone, a comparison of the variation of the latter according to the position carried out, on the basis of a separation between the model SSFEM and the simulation of Monte Carlo [1],[14-16], and on the other hand with values resulting from the experience performed in the NDT research center of the Chosun university.

The first observation appeared in the importance of the variation of the impedance in the default zone compared to the healthy zone where this variation is null [3,4].

Then comes the comparison of the two simulation methods the SSFEM model and the Monte Carlo simulation [1,5]. When comparing the SSFEM model and the experimental one, we can affirm that the approach of the random variable of lognormal type provides information on the performance of the SSFEM model. It shows a good agreement with the experimental data. Attention is drawn when the relevance of using a particular method in this type of testing. Experiments are not often available and require standardized and accredited test platforms, which is not very accessible, Monte Carlo requires large series and remains very expensive in an industrial environment Nevertheless, its place remains privileged despite the need to go to the opposite problem for exploitation .

Compared to the spectral stochastic finite element method, it offers the advantage of studying the system and evaluating it in a single step, which is an advantage in terms of computation time and control of the simulations compared to the Monte Carlo simulation, and above all, it also makes it possible to consider a hazard on the physical properties or the source from the design stage, which is not negligible with regard to the possibilities of evaluating the reliability and the probability of failure, which remains a fundamental theme in the industry.

On figure 4, we notice that the deterministic solution gives a variation of the vector potential A, hence of the impedance, more important that of the stochastic solution, this is due to the fact that in the defect it is supposes that it there is a lack of matter, so the electrical conductivity is null in this area for the deterministic calculation, on the other hand the stochastic calculation distributes the electrical conductivity according to the coefficients of the random variable, which are not all null; represented by a so-called stochastic matrix.

This study is completed to consolidate the SSFEM model by evaluating the probability of failure in the presence of a defect compared to the reference values, the results obtained show the important contribution of the stochastic model in terms of evaluation and inspection of defects in conductive materials, Therefore we are optimistic about its use in composite materials, particularly in electrical networks.

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REFERENCES

[1] Zehor OUDNI, Azouaou BERKACHE, Hamid MEHADDENE, Hassane MOHELLEBI, Jinyi LEE, Comparative study to assess reliability in the presence of two geometric defect shapes for non-destructive testing,Przegląd Elektrotechniczny, ISSN 00332097,R.95NR12/2019. (2019).
[2] Z.Oudni, M.Feliachi, H.Mohellebi, Assessment of the Probability of Failure for EC Nondestructive Testing Based on Intrusive Spectral Stochastic Finite Element Method, *Eur. Phys. J. Appl. Phys*, 66 (2014), No.5, 133-137.
[3] M.Bervellier, Elément finis stochastiques: Approche intrusive et non intrusive pour des analyses de fiabilité, PhD Thesis, (2005),Blaise Pascal-Clermont II University.
[4] P.Burascano, E.Cardelli, Faba Antonio, Fiori Simone, Massinelli Andrea, "Numerical analysis of eddy current non destructive testing JSAEM benchmark problem # 6 –cracks with different shapes", Proc. ENDE-V, pp. 333-340, Jun. Budapest,(2000).
[5] A.Berkache, Z.Oudni, H.Mehaddene, H.Mohellebi, J. Lee, Inspection and characterization of random physical property defects by stochastic finite element method *PRZEGLĄD ELEKTROTECHNICZNY*, R. 95 NR 8/2019, ISSN 0033-2097, Doi:10.15199/48.2019.08.23.(2019).
[6] K. Kosmas and E. Hristoforou, "The effect of magnetic anomaly detection technique in eddy current non-destructive testing," *Int. J. Appl. Electromagn. Mech.*, v. 25, no. 1–4, 319–324, (2018).
[7] J.Lee, J.Hwang, J.Jun, S.Choi, Nondestructive Testing and Crack Evaluation of Ferromagnetic Material by Using Integrated Hall Sensor Array, *J. Mech. Sci. Technol*, 22 No.12, 2310-2317,(2008).
[8] Mac H., Clénet S., Mipo J. C., O.Moreau, Solution of Static Field Problems with Random Domains,IEEE Trans.Magn.,Vol 46, N°8, pp 3385-3388,(2010).
[9] Loeve M., Probability Theory I, forth edition, Springer-Verlag Inc, 45, DOI 10.1007/978-1-4684-9464-8,(1977).
[10] JRC. R. A.Da Silva, A. T.Beck, E.Da Rosa, of the stochastic beam bending problem by Galerkin Method and the Askey-Wiener scheme, Latin American Journal of Solids and Structures, 6, pp 51 – 72, (2009) .
[11] R.Gaignaire, F.Guyomarch, O.Moreau,S.Clénet,and B.Sudret, *IEEE Trans. Magn.* 45-1432,(2009).
[12] K. Kosmas and E. Hristoforou, "The effect of magnetic anomaly detection technique in eddy current non-destructive testing," *Int. J. Appl. Electromagn. Mech.*, vol. 25, no. 1–4, pp. 319–324, (2018).
[13] J.Lee, J.Hwang, J.Jun, S.Choi, Nondestructive Testing and Crack Evaluation of Ferromagnetic Material by Using Integrated Hall Sensor Array, *J. Mech. Sci. Technol*, 22 No.12, 2310-2317,(2008).
[14] Mac H., Clénet S., Mipo J. C., O.Moreau, Solution of Static Field Problems with Random Domains,IEEE Trans.Magn.,Vol 46, N°8, pp 3385-3388,(2010).
[15] Loeve M., Probability Theory I, forth edition, Springer-Verlag Inc, 45, DOI 10.1007/978-1-4684-9464-8,(1977).
[16] JRC. R. A.Da Silva, A. T.Beck, E.Da Rosa, of the stochastic beam bending problem by Galerkin Method and the Askey-Wiener scheme, Latin American Journal of Solids and Structures, 6, pp 51 – 72, (2009) .
[17] R.Gaignaire, F.Guyomarch, O.Moreau,S.Clénet,and B.Sudret, *IEEE Trans. Magn.* 45-1432,(2009).
[18] M.Lemaire**,** in Proceedings of the 15th European Conference on Numerical Methods in Electromagnetism, Lille, pp. 3,(2006).
[19] K.Beddek, S.Clénet, O.Moreau, Y.LeMenach, Solution of Large Stochastic Finite Element Problems—Application to ECT-NDT, *IEEE Trans. Magn*., 49 ,2013, No.5, 1605-1608 Pike G.E., Preparation of Papers for Transactions, *IEEE Trans. Magn*., 50, No. 5, 133-137,(2002).
[20] J-L.Thomas, Simplified Modeling of Eddy Current Control of Steam Generator Tubes Report of internship ESA IGELEC, University of Nantes, France,(1998).
[21] Azouaou Berkache, Jinyi Le, and Eunho Choe, Evaluation of Cracks on the Welding of Austenitic Stainless Steel Using Experimental and Numerical Techniques, Applied Sciences ,
[22] DOI: 10.3390/app11052182, (2021).
[23] Srikara Pranesh, Debraj Ghosh, A FETI-DP based parallel hybrid stochastic finite element method for large stochastic systems , Computers and StructuresVol. 195, No. CA,(2018)