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Mathematical modeling of processes and characteristics of wound-rotor induction motor

***Abstract**. A mathematical model and algorithm to calculating electromechanical transients and starting static characteristics of wound-rotor induction motor have been developed. The calculation is based on a mathematical model of an induction motor, which considers the saturation of the magnetic circuit and the differential method of calculating the static characteristics. The electromagnetic processes in the motor are described by a system of nonlinear electric equilibrium equations written in transformed to orthogonal coordinate axes, which are solved by the parameter continuation differential method and Newton's iterative method. Calculation of loop fluxes and differential inductances is performed using the magnetization characteristic of the main magnetic flux and the stator and rotor leakage fluxes.*

Streszczenie.** Opracowano model matematyczny i algorytm do obliczania przebiegów elektromechanicznych i rozruchowych charakterystyk statycznych silnika indukcyjnego z wirnikiem uzwojonym. Obliczenia opierają się na modelu matematycznym silnika indukcyjnego, który uwzględnia nasycenie obwodu magnetycznego oraz różnicową metodę obliczania charakterystyk statycznych. Procesy elektromagnetyczne w silniku opisane są układem nieliniowych równań równowagi elektrycznej, zapisanych w postaci przekształconej na osie ortogonalne, które są rozwiązywane metodą różniczkową z kontynuacją parametrów oraz metodą iteracyjną Newtona. Obliczanie strumieni pętli i indukcyjności różnicowych odbywa się na podstawie charakterystyki magnetyzacji głównego strumienia magnetycznego oraz strumieni rozproszenia stojana i wirnika.* (Matematyczne modelowanie procesów i charakterystyk silnika indukcyjnego z wirnikiem uzwojonym***)

Keywords: mathematical modeling, induction motor, wound-rotor, transients, static characteristics, saturation
Słowa kluczowe: silnik indukcyjny, modelowanie, wirnik uzwojony

Introduction

Most asynchronous electric drives are implemented using squirrel-cage induction motors (IM) due to their simple design and reliability in operation. Their disadvantage is a significant value of inrush current when directly connected to the nominal voltage, the value of which exceeds the nominal values by 5-8 times. In addition, they have a relatively small driving torque, which is not enough to start some mechanisms successfully, and there is also the problem of rotor speed control. The IM with a wound-rotor eliminates most of these problems, which are more complicated in their design but have better starting and regulating properties. They are used for electric drives of mechanisms with severe starting conditions. After all, rheostat control of the rotor speed is often used in regulated electric drives of relatively low power. Due to its simplicity, it finds application in drives of handling equipment. Including an additional resistor in the rotor, the circuit allows the speed to be adjusted downward from the main speed. As it is known [1-3], IM motors' type of rheostat mechanical characteristic depends on the value of rheostat resistance in the wound-rotor circuit, which means that the choice of its parameters and control laws in specific electric drives is of fundamental importance.

The stator structure of a wound-rotor induction motor and its winding do not differ from the structure of a motor with a squirrel-cage rotor. The rotor, like the stator, has a three-phase winding that is connected in a star connection. The ends of the rotor phase windings are connected to three contact rings rigidly fixed to the rotor shaft and insulated between themselves and from the rotor shaft. With the help of brushes and sliding contact, the rotor winding is connected to an external three-phase rheostat. Starting of IM is performed at the maximum value of rheostat resistances, which limit currents during starting and during reversing, braking, and, if necessary, during regulation towards reduction of rotor speed. The increase of the active resistance of the rotor circuit due to the external rheostat at start-up leads to an increase in the driving electromagnetic torque. As the rotor speed increases, the rheostat resistance decreases (usually discretely). The rheostat resistance is removed when the motor reaches the desired speed, and the rotor winding is short-circuited.

Theoretical and practical problems of rheostat control of asynchronous electric drive operation based on a wound-rotor IM are presented in fundamental [1] and many modern [2-8] works. They generally use the well-known classical IM substitution schemes or linear differential equations (DE), and the approximate Clossa formula calculates the mechanical characteristics. The problem of developing methods of researching the modes of operation of a wound-rotor IM and their characteristics, despite many publications, remains unresolved at the proper level, and known methods and calculation algorithms do not meet modern requirements for automated electric drives. Therefore, an important task is to calculate the additional resistance in the rotor circuit and the law of its regulation to obtain a mechanical characteristic of a given shape. Reliable results can be obtained only based on using a mathematical model of a wound-rotor IM with a high level of adequacy.

The essence of the problem

Since electromagnetic and electromechanical processes depend on the electromagnetic parameters of IM, which are variable in dynamic modes, the inaccuracy of their determination leads to significant deviations of the rotor speed from the present one, which is caused by inaccuracy of determination of currents and electromagnetic torque. In known control algorithms for asynchronous electric drives, parameter changes are not taken into account at all or are taken into account by appropriate coefficients, which does not guarantee the reliability of the calculation results. Substitution schemes with parameters that vary according to specific formulas do not solve the problem of adequacy of results of mathematical modeling, since due to saturation of the magnetic circuit IM, the flux linkage of loops, and therefore the electromagnetic parameters non-linearly depend on all currents.

The most accurate electromagnetic parameters of IM can be determined based on solving the problem in the field formulation [9]. Still, such programs require a significant amount of memory and calculation time, and therefore cannot satisfy the requirements for speed. An intermediate position between single-phase substitution circuits and field analysis methods is occupied by calculation methods based on the theory of electric circuits [10-12]. Thus, developing a mathematical model that allows calculating transients and static characteristics of a wound-rotor IM, taking into account changes in parameters due to saturation of the magnetic circuit, is relevant. In addition, an essential and decisive factor for effective control of the electric drive is the speed and complexity of calculation programs, which are the basis of control algorithms defined by operating conditions of technological equipment.

The paper considers an asynchronous electric drive based on an IM with a phase rotor, which has three-phase windings on the stator and rotor, with a three-phase rheostat included in the rotor circuit. The calculation algorithms outlined are based on the mathematical model of IM, developed on the basis of the theory of electric circuits and image vectors of flux linkages, currents and voltages. Processes are considered in a transformed system of orthogonal coordinates. The goal of the work is to develop a method of mathematical modeling of starting modes of a wound-rotor IM based on a mathematical model, which is based on equations of electromagnetic equilibrium of motor circuits, which make it possible to consider saturation of the magnetic core by the main magnetic flux, as well as the leakage flux leakage fluxes due to currents of stator and rotor windings.

The mathematical model

The object of the study is a three-phase IM with a wound-rotor, which is fed from a three-phase network with a symmetrical voltage system. Mathematical modeling of electric drive system operation modes requires choosing a mathematical motor model and calculation method, based on which the problem solving algorithm and computer program are developed. The mathematical model of the electric drive system must reflect the real processes with the necessary adequacy. Still, it must not be redundant in terms of the accuracy of the calculation results. An important issue is the choice of a system of coordinate axes to describe electromagnetic processes. Since symmetric modes of operation are considered for the analysis of electromagnetic processes, the optimal accuracy and complexity is the mathematical model of the motor, developed in orthogonal coordinate axes *x*, *y* [12], which allows the analysis of processes with a minimum amount of calculations, and therefore with high speed, and without reducing the accuracy of the results of mathematical modeling. To account for saturation, the magnetization characteristics of both the main magnetic flux ψ_μ and the stator $\psi_{\sigma s}$ and rotor $\psi_{\sigma r}$ winding leakage fluxes, given in the form of nonlinear dependencies, are used

$$\psi_\mu(i_\mu)\,,\quad \psi_{\sigma s}(i_s)\,,\quad \psi_{\sigma r}(i_r)\,.$$

where i_μ, i_s, i_r – modules of image vectors of currents: magnetization, stator and rotor

$$i_\mu = \sqrt{(i_{sx}+i_{rx})^2+(i_{sy}+i_{ry})^2}\ ;$$

$$i_s = \sqrt{i_{sx}^2+i_{sy}^2}\ ;\quad i_r = \sqrt{i_{rx}^2+i_{ry}^2}\ ,$$

and i_{sx}, i_{sy}, i_{rx}, i_{ry} – the currents of the corresponding transformed loops.

According to the generally accepted method, three-phase stator and rotor windings IM are converted to two-phase [1,9]. As a result, in dynamic modes, electromagnetic processes are described by the system of four DE of electric equilibrium of the stator and rotor circuits, which can be represented by a vector DE

$$(1)\qquad \frac{d\vec{\Psi}_{xy}}{dt}+\Omega_{xy}\vec{\Psi}_{xy}+R\vec{I}_{xy}=\vec{U}_{xy}\ ;$$

where

$$\vec{\Psi}_{xy}=\begin{array}{|c|}\hline \psi_{sx}\\ \hline \psi_{sy}\\ \hline \psi_{rx}\\ \hline \psi_{ry}\\ \hline\end{array}\ ;\ \vec{I}_{xy}=\begin{array}{|c|}\hline i_{sx}\\ \hline i_{sy}\\ \hline i_{rx}\\ \hline i_{ry}\\ \hline\end{array}\ ;\ \Omega_{xy}=\omega_0\begin{array}{|c|c|c|c|}\hline & -1 & & \\ \hline 1 & & & \\ \hline & & & -s\\ \hline & & s & \\ \hline\end{array}\ ;$$

$$\vec{U}_{xy}=\begin{array}{|c|}\hline u_{sx}\\ \hline u_{sy}\\ \hline 0\\ \hline 0\\ \hline\end{array}\ ;\ R_{xy}=\begin{array}{|c|c|c|c|}\hline r_s & & & \\ \hline & r_s & & \\ \hline & & r_r+r_p & \\ \hline & & & r_r+r_p\\ \hline\end{array}\ ;$$

$$\frac{d\vec{\Psi}_{xy}}{dt}=L_{xy}\frac{d\vec{I}_{xy}}{dt}=\begin{array}{|c|c|c|c|}\hline L_{sxsx} & L_{sxsy} & L_{sxrx} & L_{sxry}\\ \hline L_{sysx} & L_{sysy} & L_{syrx} & L_{syry}\\ \hline sL_{rxsx} & sL_{rxsy} & sL_{rxrx} & sL_{ryry}\\ \hline sL_{rysx} & sL_{rysy} & sL_{ryrx} & sL_{ryry}\\ \hline\end{array}\ \frac{d\vec{I}_{xy}}{dt},$$

where ψ_{sx}, ψ_{sy}, ψ_{rx}, ψ_{ry} – flux linkages of the transformed stator and rotor circuits; r_s, r_r – active resistances of these loops; ω_0 – angular frequency of the supply voltage; *s* – motor slip; r_p – the resistance of the starting rheostat phase.

The elements of the matrix *L* are the differential inductances L_{jk} of the loops (the self-inductance at *j* = *k* and mutual inductances at *j* ≠ *k*), which are determined based on magnetization curves [12]. And since the flux-circuit currents of each circuit are considered as the sum of the working and dissipation, the corresponding differential inductances L_{jj} of the matrix *L* also consist of the sum of the two resistances.

To calculate the electromechanical transient, it is necessary to supplement the DE system (2) of electric equilibrium with the rotor dynamics equation

$$(2)\qquad J\frac{d\omega_{ì}}{dt}=M_e-M_c\,,$$

where $\omega_{ì}=\omega/p$; *p* – number of pole pairs IM; *J* – moment of inertia of the drive system; ω – angular speed of the rotor in geometric rad/s; *M*e – electromagnetic torque.

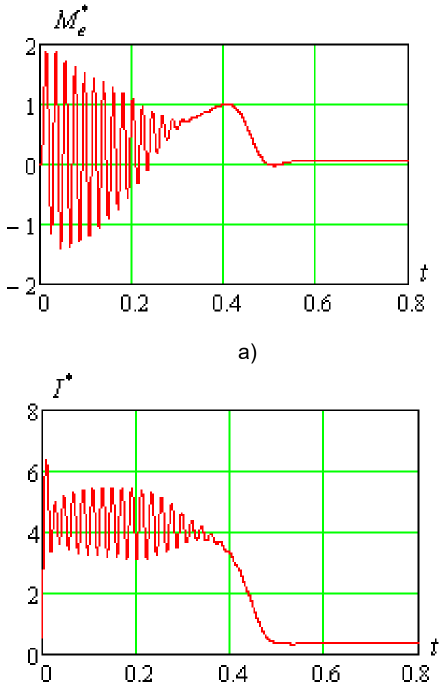
Given that

$$(3)\qquad M_e=\frac{3}{2}p\left(\psi_{sx}i_{sy}-\psi_{sy}i_{sx}\right),$$

equation (2) will have the form

$$(4)\qquad \frac{d\omega}{dt}=\frac{3}{2}\frac{p^2}{J}\left(\psi_{sx}i_{sy}-\psi_{sy}i_{sx}\right)+\frac{p}{J}M_{\hat{a}}=0\,.$$

So, the transients of a wound-rotor IM in the system of orthogonal axes x, y are described by the DE system consisting of equations (1), (4). An example of the results of calculation of the starting process of IM (*P*=250 kW, *U* = 380 V, *I* = 263 A, *p* = 3) off-load is shown in Fig. 1.



a)

b)

Fig.1. Time dependencies of relative values of electromagnetic torque (a) and current (b) during off-load starting of IM

The static characteristics

In contrast to the dynamic mode, which determines the change in coordinates over time, static characteristics allow you to research the dependence of the set of mode coordinates on a single coordinate, taken as an independent. A process is considered static when the object coordinates remain constant under constant perturbations. The basis of the static characteristic calculation algorithm is to calculate the steady-state mode of IM operation at a given slip s, and the static characteristic is calculated as a set of steady-state modes. The essence of the steady state calculation is to determine the vectors of currents and flux linkages of the motor circuits, using which you can determine the electromagnetic torque, active and reactive powers, and the like.

Under the condition of constant slip *s* of the rotor, DE systems (1) are reduced to algebraic. If we combine the image vector of the supply voltage with the x-axis, then $u_{sx} = U_m$; $u_{sy} = 0$, and the system has the form

$$\begin{aligned} &\omega_0\Psi_{sy} - r_r i_{sx} = U_m\,; \\ &-\omega_0\Psi_{sx} - r_s i_{sy} = 0\,; \\ &s\omega_0\Psi_{ry} - (r_p + r_r)i_{rx} = 0\,; \\ &-s\omega_0\Psi_{rx} - (r_p + r_r)i_{ry} = 0\,, \end{aligned} \tag{5}$$

where *U*$_m$ – peak value of the phase voltage.

Calculation of steady state mode at given values of a stator winding supply voltage, slip *s,* and rheostat resistances r_p is reduced to the solution of nonlinear system of algebraic equations (5). One way to solve system (5) for a given value of voltage is the method of differentiation by parameter [12]. For this purpose, in system (5), the vector of applied voltages $\vec{U} = \varepsilon \vec{U}_{xy}$ is presented as a product, where ε is a scalar parameter. As a result of differentiating the obtained system by ε we receive DE

$$A\frac{d\vec{I}_{xy}}{d\varepsilon} = \vec{U}_{xy}\,, \tag{6}$$

where $A = \left(\omega_0 L_{xy} - R_{xy}\right)$ – Jacobi matrix of system (5).

The algorithm for calculating the static starting characteristics as a function of slip consists of two steps. In the first one, we determine the value of coordinates at slip *s* = 1.0. In the second one, taking the voltage vector unchanged, we change the slips from one to nominal value, which allows the convergence of the iterative process, since the coordinate values obtained from the previous step are usually in the neighborhood of the convergence of the iterative process.

To calculate the multidimensional static characteristic as a function of resistance, we apply the differential method [12]. To do this, we present the system of equations (5) as a multidimensional function

$$\vec{Z}\left(\vec{\Psi}_{xy}, \vec{I}_{xy}, U_m, s, r_p\right) = 0\,, \tag{7}$$

in which the independent variable is the resistance of the rheostat r_p .

As a result of differentiating equation (7) by r_p we obtain a system of DE of the form

$$A\frac{d\vec{I}}{dr_p} = \frac{\partial \vec{Z}}{\partial r_p}\,, \tag{8}$$

Integrating DE (8) over r_p, as a parameter, in the range from r_p = 0 to a given value at slip $s = 1{,}0$ allows us to obtain the dependence of the coordinates (7), including the driving torque on the resistance of the starting rheostat. At each step of integration, we can refine the current vector $\vec{I}_{xy}$ by Newton's method, according to which the growth $\Delta\vec{I}_{xy}^{(k)}$ of the current vector at the *k*-th step of iteration is determined by the formula

$$A\Delta\vec{I}_{xy}^{(k)} = -\vec{Z}\left(\vec{I}_{xy}^{(k)}\right), \tag{9}$$

where $\vec{Z}(\vec{i}^{(k)})$ is the residuals vector of the system (5) for a given value of slip and applied voltage.

Performing step-by-step iteration allows you to integrate the DE by Euler's method and get the result in 5-10 steps.

An example of the result of calculating the dependence of the driving electromagnetic torque of the motor on the resistance r_p of the starting rheostat is shown in Fig. 2.

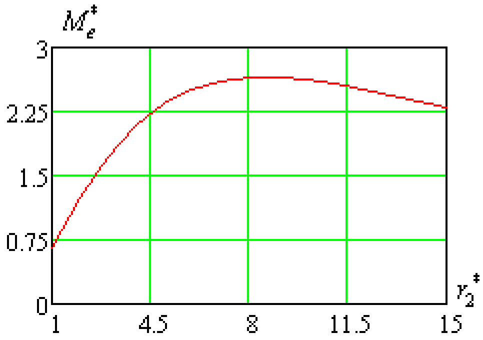


Fig.2. Dependence of the driving electromagnetic torque of the motor on the relative value of rheostat resistance

The inclusion of additional resistors in the rotor circuit is used not only to control motor current and torque during start-up but also for rheostat control of the rotor speed. By appropriate selection of additional rheostat resistance, the motor starting torque can be increased up to the critical torque M_m. This feature of the mechanical characteristic is used to start the motor at a load torque that exceeds the rated motor starting torque.

The mechanical characteristic $M = M(s)$ for each set resistance value of the starting rheostat is calculated similarly, replacing in DE (8) the variable r_p with the slip s. In this case, the resistance r_p is taken as constant, and the slip varies within the specified limits. Initial conditions for this are taken from the calculation of steady-state mode at given values of slip *s* and applied voltage $u_{sx} = U_m$. Based on the calculation of these dependencies at the value of slip *s* = 1.0, select the value of rheostat resistance at which the required starting electromagnetic torque of the motor is provided.

An example of calculating the starting mechanical characteristics of a motor for three values of the resistance of the starting rheostat in the rotor circuit is shown in Fig. 3.

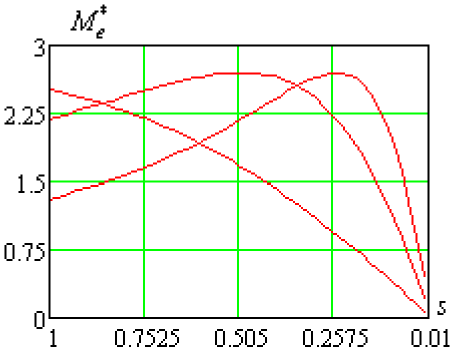


Fig.3. Starting mechanical characteristics at various values of the resistance of the rheostat in the rotor circuit

Thus, the algorithm to calculating the optimum static characteristic of a wound-rotor IM, required for starting the electric drive system, consists of two stages: the first one calculates the dependence of the starting torque on the rheostat resistance at slip *s*=1.0, and then by calculating the mechanical characteristics the selection of necessary sections and values of the starting rheostat resistance is performed. The switching moments of its sections as a function of starting current or electromagnetic torque can be determined from the static characteristic, and as a function of time - based on the calculation of the transient process. The use of the developed mathematical models makes it possible to program an algorithm, which allows maintaining the electromagnetic torque at the required level during the whole starting process.

Conclusion

1. The proposed calculation algorithms allow the methods of mathematical modeling to analyze the transients and static characteristics of induction motors with a wound-rotor with regard to saturation in order to determine the starting rheostat resistance to provide the necessary law of electromagnetic moment change.

2. The developed calculation algorithms and, on their basis, the mathematical model of IM, which uses real magnetization curves calculated on the basis of the geometry of the magnetic circuit and winding data of IM, allow adequate consideration of saturation, which ensures the accuracy of calculation results.

3. Based on the proposed calculation algorithm, mathematical models can be used to design adjustable induction motor drives and their control systems to form the necessary starting characteristics.

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