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doi:10.15199/48.2022.04.04

Computation of Steady-State Probabilities of the states of the SMP model in Beacon Message Dissemination

***Abstract**. With the advent of the latest technologies like AI, IoT, LiDAR systems and ToF techniques, smart cars have become a reality. Communication between machines has become possible thereby the cars on the road communicate among themselves, with the driver and with the things outside the road. Beacon messages are the most important messages, which need the highest priority and accuracy in communication. An SMP model is designed to understand and evaluate the broadcast procedure of the beacon messages. We derive the steady state probabilities of the states of the SMP according to the transmission protocol.*

Streszczenie. *Wraz z pojawieniem się najnowszych technologii, takich jak AI, IoT, systemy LiDAR i techniki ToF, inteligentne samochody stały się rzeczywistością. Komunikacja między maszynami stała się możliwa, dzięki czemu samochody na drodze komunikują się między sobą, z kierowcą iz rzeczami znajdującymi się poza drogą. Komunikaty Beacon to najważniejsze komunikaty, które wymagają najwyższego priorytetu i dokładności w komunikacji. Model SMP został zaprojektowany w celu zrozumienia i oceny procedury rozgłaszania komunikatów nawigacyjnych. Wyprowadzamy prawdopodobieństwa stanów ustalonych stanów SMP zgodnie z protokołem transmis.* (***Obliczanie prawdopodobieństw stanu ustalonego stanów modelu SMP w Beacon Message Dissemination***)

Keywords: Beacon Messages, DCF, SMP.
Słowa kluczowe: SMP, Beacon Message, stan ustalony, prawdopodoviеństwo

Introduction

The latest technologies like Artificial Intelligence (AI), Internet of Things (IoT), Light Detection and Ranging (LiDAR) systems and A Time-of-Flight (ToF) techniques, Vehicular Ad hoc Network (VANET) and smart cars from laboratory are brought to the roads where we have been so far travelling. Communication between machines has become possible thereby the cars on the road communicate among themselves, with the driver and with the things outside the road. Beacon messages are the most important messages, which need the highest priority and accuracy in communication. These messages carry details like the geographical location, the movement of the vehicle with direction and speed etc. An SMP model is designed to understand and evaluate the broadcast procedure of the beacon messages. We derive the steady state probabilities of the states of the SMP according to the transmission protocol.

On the emergence of the 5G networks one of the most sought after applications is the VANETS under ITS. Intelligent Transport Systems (ITS) is an integrated, self-reliant system to provide a simple and comfortable transportation of materials and people with cost effectiveness. The thrust areas where ITS is deployed are mainly

a. To provide a congestion free traffic flow
b. To provide an immediate detection of accidents or adverse situations and connect the apt recovery agencies.
c. Provide ambient travel time and reduce the transit time.
d. Provide adequate precautions for the unforeseen breakdowns. From the least like availability of petrol stations or cafeterias to very dangerous situations like accident prone areas.
e. Provide a clear corridor for vehicular environment only by controlling animal, pedestrians trespass, creating sufficient sub-way systems and to maintain proper lane discipline.
f. Broadcast beacon messages

The beacon messages are transmitted with the highest priority the vehicle and RSU, vice-versa and provide assistance to the driver for a safe transportation.

Model Description of the Network Architecture

a. The IEEE 802.11 DCF protocol, design and architecture.
b. The SMP design, different states in the process of capturing the channel, and the backoff behaviour of a particular station or vehicle.

Semi-Markov Process

A semi-Markov process unlike a Markov process evaluates on every state continuously and just not only on the jumps. It is a stochastic process defined by Levy (1954) and Smith (1955) in 1950s, which spans over the complete time. Since every state has to be defined for a given time, we have the state S_t at any time , defined as $S_t = X_n$, for $t \in [T_n, T_{n+1})$. Hence the process $\{S_t\}_{t\geq 0}$ is a semi Markov process, with the times $0 = T_1 < T_2 < T_3 < \cdots < T_n \cdots$ as the jump times of $\{S_t\}_{t\geq 0}$, and $\tau_n = T_n - T_{n-1}$ are the sojourn times.

IEEE 802.11 DCF

The DCF [1], the access mechanism for acquiring the channel for communication, is a process of continuous attempts made by any electronic component which attempts to transfer data from one point to the other by the IEEE 802.11 protocol [2,3,4]. We shall shortly summarise the mechanism.

Any station which is trying to access the channel to transmit any message does not transmit immediately when the message is generated. The station, immediately after the generation of the message, senses the channel whether it is busy or free. If the channel is free, the station waits for a brief moment called distributed inter-frame space (DIFS) time, and meanwhile chooses a back off counter from the contention window $(0, w-1)$ where and the counter decrements until the channel is sensed free. Meanwhile if any of the other station transmits, the decrement of the back off counter is frozen and the station persists in sensing the channel until it becomes free again. And once the channel becomes free, the decrement of the back off counter resumes. When the counter reaches zero the message is transmitted. This is the collision avoidance technique available in DCF.

Once the channel transmits the message or packet, the station waits for a Short Inter Frame Space (SIFS) and an acknowledgement ACK from the recipient terminal and continues to contend for accessing the channel again for the transmission of the next message. In case if the ACK is not received or if the channel is detected busy, it reschedules the transmission again in accordance with the back off procedure. Fig – 1 and Fig – 2.

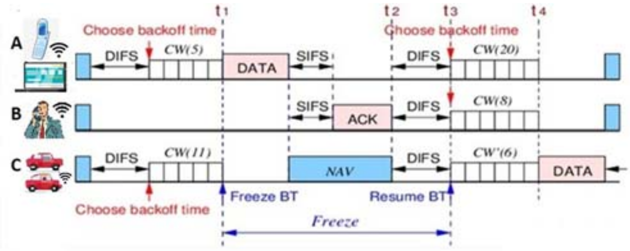

Fig – 1DCF Backoff Mechanism

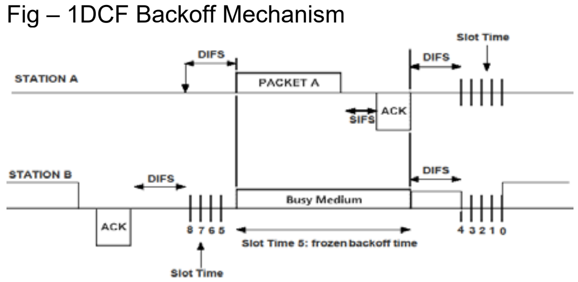

Fig – 2 DCF- Packet transmission

Semi Markov Process (SMP) Model

Definition: Markov Renewal Processes. Let S be the state space of the Markov chain. Let (X_n, T_n) be a set of random variables, where X_n are the states of the Markov chain and T_n the jump times. Let $\tau_n = T_n - T_{n-1}$ be the inter arrival time between jumps. Then (X_n, T_n) is defined as a Markov renewal processes if

$$P(\tau_{n+1} \le t, X_{n+1} = j \mid (X_0, T_0), (X_1, T_1), \cdots, (X_n = i, T_n))$$
$$= P(\tau_{n+1} \le t, X_{n+1} = j \mid (X_n = i),$$
$$\forall n \ge 1,\ t \ge 0,\ i, j \in S.$$

Equivalent to MRP, in an SMP the time that is spent on each node for every state is defined and not only just at the jumps. Hence an SMP is employed to calculate the sojourn time and its probability of the tagged vehicle or the transmitting station on study.

Model Description

The basic transmission mechanism is modelled with the states of an SMP as shown in figure 3. The transmitting station or the tagged vehicle is in different states of the SMP as described below.

Initially or at any point of time if there is no packet to be transmitted by the tagged vehicle then it is in idle state. The vehicle starts sensing the channel immediately when a message is generated and waits for a DIFS period of idle time. This state is represented by CS_1. If at this state, the channel is sensed not to be busy for a DIFS time, the message is transmitted with a probability $1-q_b$, which is denoted by the state TX, otherwise it moves to state D_{CS} with probability q_b, at this state the vehicle holds back the transmission of the packet, since some other vehicle is transmitting, and waits for DIFS period of idle time after the transmission of other vehicles. The self-loop at this stage D_{CS} represents the vehicle is sensing the transmission of other vehicles with probability r_b. Further, when the channel is sensed idle, the vehicle starts the back off procedure. It picks a back off counter from the contention window $[0, W-1]$ and decrements one by one with slot time σ with probability $1-p_b$. At any instance of the back off time, if the channel is sensed to be busy, the back off counter is frozen and resumes decrementing immediately when the channel is sensed idle again, with probability p_b. Once the counter hits zero, the message is transmitted. If there is another message to be transmitted either a new message or an updated message the vehicle goes to the state CS_2 and follows the procedure described above or else goes to the idle state. Figure 3.

The Steady-State Probabilities for the States of the SMP.

For the state D_{CS}

$$(1-r_b)\nu_{D_{CS}} + \nu_{D_{CS}} r_b = \nu_{D_{CS}} r_b + \nu_{CS_1} q_b$$

(1) $$\therefore \nu_{D_{CS}} = \frac{\nu_{CS_1}}{(1-r_b)} q_b$$

For the state CS_1

$$\nu_{CS_1} q_b + \nu_{CS_1}(1-q_b) = \nu_{idle}.1$$

(2) $$\therefore \nu_{CS_1} = \nu_{idle}$$

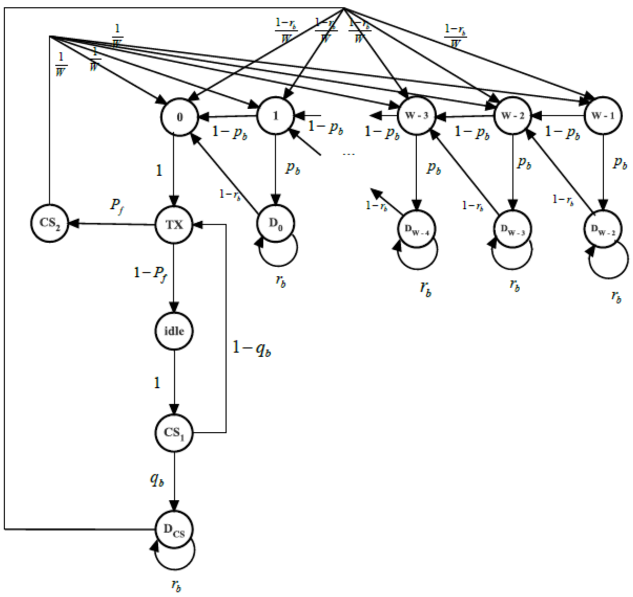

Fig – 3 SMP model of a tagged station

For the state $idle$

$$\nu_{idle}.1 = \nu_{TX}(1-p_f)$$

(3) $\nu_{idle} = \nu_{TX}(1-p_f)$

For the state TX

$$\nu_{TX}(1-p_f) + \nu_{TX}p_f = \nu_{CS_1}(1-q_b) + \nu_0.1$$

(4) $\nu_{TX} = \nu_{CS_1}(1-q_b) + \nu_0$

For the state CS_2

$$\nu_{CS_2}.1 = \nu_{TX}p_f$$

(5) $\therefore \nu_{CS_2} = \nu_{TX}p_f$

For the states 0 to $(w-1)$

$$\nu_0 = \nu_{D_{CS}}\left(\frac{1-r_b}{w}\right) + \nu_{CS_2}\frac{1}{w} + \nu_1(1-p_b) + \nu_{D_0}(1-r_b)$$

$$\nu_1 = \nu_{D_{CS}}\left(\frac{1-r_b}{w}\right) + \nu_{CS_2}\frac{1}{w} + \nu_2(1-p_b) + \nu_{D_1}(1-r_b)$$

$$\vdots$$

(6)

$$\nu_j = \nu_{D_{CS}}\left(\frac{1-r_b}{w}\right) + \nu_{CS_2}\frac{1}{w} + \nu_{j+1}(1-p_b) + \nu_{D_j}(1-r_b)$$

$$\vdots$$

(7) $\nu_{w-1} = \nu_{D_{CS}}\left(\frac{1-r_b}{w}\right) + \nu_{CS_2}\frac{1}{w}$

Using equation (7) in (6) we get

(8) $\nu_j - \nu_{j+1} = \nu_{w-1} - p_b\nu_{j+1} + (1-r_b)\nu_{D_j}$

$$j = 0,1,2,...,w-2$$

Indexing the values for j from 0 to $w-2$ in equation (8) and summing up we get

$$\nu_0 - \nu_1 = \nu_{w-1} - p_b\nu_1 + (1-r_b)\nu_{D_0}$$

$$\nu_1 - \nu_2 = \nu_{w-1} - p_b\nu_2 + (1-r_b)\nu_{D_1}$$

$$\vdots$$

$$\nu_{w-2} - \nu_{w-1} = \nu_{w-1} - p_b\nu_{w-1} + (1-r_b)\nu_{D_{w-2}}$$

(9) $\nu_0 - \nu_{w-1} = (w-1)\nu_{w-1} - p_b\left(\nu_1 + ... + \nu_{w-1}\right) + (1-r_b)\left(\nu_{D_0} + ... + \nu_{D_{w-2}}\right)$

For the states D_0 to D_{w-2}

$$(1-r_b)\nu_{D_0} + r_b\nu_{D_0} = \nu_{D_0}r_b + \nu_1 p_b$$

$$\nu_{D_0} = \nu_{D_0}r_b + \nu_1 p_b$$

$$\nu_{D_1} = \nu_{D_1}r_b + \nu_2 p_b$$

$$\vdots$$

(10) $\nu_{D_j} = \nu_{D_j}r_b + \nu_{j+1}p_b$,

i.e., $(1-r_b)\nu_{D_j} = \nu_{j+1}p_b$

$$\vdots$$

$$\nu_{D_{w-2}} = \nu_{D_{w-2}}r_b + \nu_{w-1}p_b$$

Summing up the above equations we get

$$\left(\nu_{D_0} + \nu_{D_1} + ... + \nu_{D_{w-2}}\right) = \left(\nu_{D_0} + \nu_{D_1} + ... + \nu_{D_{w-2}}\right)r_b + \left(\nu_1 + \nu_2 + ... + \nu_{w-1}\right)p_b$$

(11) $\therefore (1-r_b)\left(\nu_{D_0} + \nu_{D_1} + ... + \nu_{D_{w-2}}\right) = \left(\nu_1 + \nu_2 + ... + \nu_{w-1}\right)p_b$

Substituting (11) in (9), we get,

$$\nu_0 - \nu_{w-1} = (w-1)\nu_{w-1}$$

(12) $\therefore \nu_0 = w\nu_{w-1}$

Using eqn (10) in eqn (8) we get,

$$\nu_k - \nu_{k+1} = \nu_{w-1}, \qquad k = 0,1,2,...,w-2,$$

Indexing over k, we get,

$$\nu_0 - \nu_1 = \nu_{w-1}$$

$$\nu_1 - \nu_2 = \nu_{w-1}$$

$$\vdots$$

$$\nu_{j-1} - \nu_j = \nu_{w-1}$$

$$\nu_0 - \nu_j = j.\nu_{w-1}$$

i.e., $\nu_j = \nu_0 - j.\nu_{w-1}$ from (12)

(13) $\nu_j = (w-j)\nu_{w-1}, \quad j = 0,1,2,...,w-1$

We know that from (10),

$$(1-r_b)\nu_{D_j} = \nu_{j+1}p_b, \qquad j = 0,1,2,...,w-2,$$

and also $\nu_j = (w-j)\nu_{w-1}$,

put $j = j+1$, we get, $\nu_{j+1} = \left[w-(j+1)\right]\nu_{w-1}$

(14) $\therefore \nu_{D_j} = \dfrac{[w-j-1]p_b}{(1-r_b)}\nu_{w-1}$, $j = 0,1,2,...,w-2$

Expressing each state in terms of ν_{w-1}, and as the sum of all Probability $\sum_i p_i = 1$, we get

(15) $\therefore \nu_{D_{CS}} + \nu_{CS_1} + \nu_{idle} + \nu_{TX} + \nu_{CS_2} + \sum_{j=0}^{w-1}\nu_j + \sum_{j=0}^{w-2}\nu_{D_j} = 1$

Evaluation of $\sum_{j=0}^{w-1}\nu_j$

$$\sum_{j=0}^{w-1}\nu_j = \sum_{j=0}^{w-1}(w-j)\nu_{w-1}$$

$$= \nu_{w-1}\left[w^2 - \left(0+1+2+...+(w-1)\right)\right] \text{ from (13)}$$

(16) $\sum_{j=0}^{w-1}\nu_j = \left[\dfrac{w(w+1)}{2}\right]\nu_{w-1}$

Evaluation of $\sum_{j=0}^{w-2}\nu_{D_j}$

$$(1-r_b)\nu_{D_j} = \nu_{j+1}p_b, \qquad j = 0,1,2,...,w-2 \text{ from (10)}$$

Summing up we get,

$$\left(1-r_b\right)\sum_{j=0}^{w-2}\nu_{D_j} = p_b\sum_{j=0}^{w-2}\nu_{j+1} = p_b\left[\nu_1+\nu_2+...+\nu_{w-1}\right]$$

(17) $$\sum_{j=0}^{w-2}\nu_{D_j} = \frac{p_b}{\left(1-r_b\right)}\left[\frac{w\left(w-1\right)}{2}\right]\nu_{w-1}$$

To find ν_{TX}

$\nu_{TX} = \nu_{CS_1}(1-q_b)+\nu_0 \;\; = \nu_{idle}(1-q_b)+\nu_0$ from (4)

(18) $$\nu_{TX} = \frac{w}{\left[p_f+q_b\left(1-p_f\right)\right]}\nu_{w-1}$$

To find ν_{CS_1}

$\nu_{CS_1} = \nu_{idle} = \nu_{TX}(1-p_f)$ from (2) and (3)

(19) $$\nu_{CS_1} = \frac{w\cdot(1-p_f)\cdot\nu_{w-1}}{\left[p_f+q_b(1-p_f)\right]}$$

To find ν_{idle}

(20) $\nu_{CS_1} = \nu_{idle} = \dfrac{(1-p_f)w}{\left[p_f+q_b(1-p_f)\right]}\nu_{w-1}$, from (2)

To find ν_{CS_2}

(21) $$\nu_{CS_2} = \nu_{TX}\,p_f = \frac{w.\,p_f}{\left[p_f+q_b(1-p_f)\right]}\nu_{w-1}$$

To find $\nu_{D_{SC}}$

(22) $$\nu_{D_{SC}} = \nu_{CS_1}\frac{q_b}{\left(1-r_b\right)}$$

$$= \frac{q_b\left(1-p_f\right)w}{\left[p_f+(1-q_b)\right]\left(1-r_b\right)}\nu_{w-1}$$

Using equations (16) to (22) in equation (15), we get (23)

$$\nu_{w-1} = \frac{2\left[p_f+q_b(1-p_f)\right]\left(1-r_b\right)}{\left[\left(w+1\right)\left(1-r_b\right)+p_b\left(w-1\right)\right]\left[p_f+q_b(1-p_f)\right]w + 2\left[\left(3-p_f\right)\left(1-r_b\right)+q_b\left(1-p_f\right)\right]w}$$

For the states 0,1,2,...,n ,

$\nu_j = \Pr\left\{X=j\right\}$

(24) $$\pi_j = \frac{\nu_j\tau_j}{\sum\nu_j\tau_j}$$

Therefore, the packet transmission time

$$= \frac{PL}{Rd} = \frac{Packet\ Length}{Data\ Rate}$$

T_H: The time to transmit the packet header.

$$A_1 = \frac{PL}{Rd}+T_H = E\left[TX\right] = \tau_{TX}$$

Calculation of $\nu\tau$

(25) $\nu_j\tau_j = (w-j)\nu_{w-1}\sigma$, $j = 0,1,2,...,w-1$

(26) $$\nu_{D_j}\tau_{D_j} = \frac{(w-j-1)p_b}{\left(1-r_b\right)}\nu_{w-1}A_5$$

$$j = D_0, D_1, D_2, ..., D_{w-2}$$

(27) $$\nu_{D_{CS}}\tau_{D_{CS}} = \frac{q_b\left(1-p_f\right)w}{\left[p_f+q_b\left(1-p_f\right)\right]\left(1-r_b\right)}\nu_{w-1}A_4$$

(28) $$\nu_{CS_1}\tau_{CS_1} = \frac{\left(1-p_f\right)w}{\left[p_f+q_b\left(1-p_f\right)\right]}\nu_{w-1}A_3$$

(29) $$\nu_{idle}\tau_{idle} = \frac{\left(1-p_f\right)w}{\left[p_f+q_b\left(1-p_f\right)\right]}\nu_{w-1}A_2$$

(30) $$\nu_{TX}\tau_{TX} = \frac{w}{\left[p_f+q_b\left(1-p_f\right)\right]}\nu_{w-1}A_1$$

(31) $$\nu_{CS_2}\tau_{CS_2} = \frac{p_f w}{\left[p_f+q_b\left(1-p_f\right)\right]}\nu_{w-1}A_3$$

$$\sum_{j=0}^{w-1}\nu_j\tau_j = \sum_{j=0}^{w-1}(w-j)\nu_{w-1}\sigma$$

(32) $$\sum_{j=0}^{w-1}\nu_j\tau_j = \frac{w\left(w+1\right)}{2}\nu_{w-1}\sigma$$

(33) $$\sum_{j=0}^{w-2}\nu_{D_j}\tau_{D_j} = \frac{p_b}{\left(1-r_b\right)}\frac{w(w-1)}{2}\nu_{w-1}A_5$$

Summing up Equations from (27) to (33), we get,

$$\sum_{\alpha\in StateSpace}\nu_\alpha\tau_\alpha = \frac{q_b\left(1-p_f\right)w}{\left[p_f+q_b\left(1-p_f\right)\right]\left(1-r_b\right)}\nu_{w-1}A_4 + \frac{\left(1-p_f\right)w}{\left[p_f+q_b\left(1-p_f\right)\right]}\nu_{w-1}A_3 + \frac{\left(1-p_f\right)w}{\left[p_f+q_b\left(1-p_f\right)\right]}\nu_{w-1}A_2 + \frac{w}{\left[p_f+q_b\left(1-p_f\right)\right]}\nu_{w-1}A_1 + \frac{p_f w}{\left[p_f+q_b\left(1-p_f\right)\right]}\nu_{w-1}A_3 + \frac{w\left(w+1\right)}{2}\nu_{w-1}\sigma + \frac{p_b}{\left(1-r_b\right)}\frac{w(w-1)}{2}\nu_{w-1}A_5$$

$$\pi_{TX} = \frac{\nu_{TX}\tau_{TX}}{\sum\nu_{TX}\tau_{TX}}$$

$$\pi_{TX} = \frac{2A_1}{\left[p_f+q_b\left(1-p_f\right)\right]\left\{\left(w+1\right)\sigma+\frac{p_b}{\left(1-r_b\right)}(w-1)A_5\right\} + 2\left[A_1+A_3+\left(1-p_f\right)\left(A_2+\frac{q_b}{\left(1-r_b\right)}A_4\right)\right]}$$

Conclusion

The steady-state probability of each state of the Semi Markov Process is completely evaluated and the behaviour of the beacon message contenting for the channel resource is modelled. The above can be readily used for any throughput analysis of the beacon message transmission for different models.

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REFERENCES

[1] Bianchi G., Performance Analysis of the IEEE 802.11 Distributed Coordination Function, *IEEE Journal on Selected Areas in Communications,* 18(2000), No. 3, 535–547

[2] Xiaoyan Yin, Xiaomin Ma, Kishor S. Trivedi, MAC and application level performance evaluation of beacon message dissemination in DSRC safety communication, *Performance Evaluation,* 71(2014), 1-24

[3] Jeffrey L. Adler, Victor J. Blue, Toward the design of intelligent traveler information systems, *Transportation Research Part C* 6 (1998) 157-172

[4] Muhammad Farooq-I-Azam, , Muhammad Naeem Ayyaz, Saleem Akhtar, Connectivity based technique for localization of nodes in wireless sensor networks, *Przegląd Elektrotechniczny, ISSN 0033-2097, R. 89 NR 5/2013*, 171-175