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Mathematical modeling of real time wind power density using the transformation technique

***Abstract**. Wind power density function and cumulative density function are very essential for evaluating the region's wind resource capacity. To specify the wind speed density functions, six probability density functions are considered in this study. This study suggests a transformation technique to develop a wind power density model predominantly from well-known dfs, namely, the Weibull, Gamma, Burr, Dagum, Logistic and Log-Logistic. The wind power density and cumulative density functions are derived by means of the transformation technique for all the above mentioned distributions as well as the power density and cumulative density function curves are plotted. The maximum likelihood approach is used to estimate the parameters of various distributions. The Kolmogorov-Smirnov test, Anderson-Darling test, and Chi-Squared test are used to evaluate and compare the quality of the goodness of fit. A case study including wind speed data from multiple locations demonstrates the mathematical model in action. Among the six statistical distributions shown above, the Dagum probability density function looks to be the most consistent.*

Streszczenie. Funkcja gęstości mocy wiatru i funkcja gęstości skumulowanej są bardzo istotne dla oceny potencjału zasobów wiatru w regionie. Aby określić funkcje gęstości prędkości wiatru, w niniejszym opracowaniu uwzględniono sześć funkcji gęstości prawdopodobieństwa. Niniejsze badanie sugeruje technikę transformacji służącą do opracowania modelu gęstości mocy wiatru głównie na podstawie dobrze znanych DFS, a mianowicie Weibulla, Gamma, Burra, Daguma, Logistic i Log-Logistic. Za pomocą techniki transformacji dla wszystkich w/w rozkładów wyprowadza się funkcje gęstości mocy wiatru i gęstości skumulowanej oraz wykreśla się krzywe gęstości mocy i funkcji gęstości skumulowanej. Do oszacowania parametrów różnych rozkładów stosuje się podejście największej prawdopodobieństwa. Do oceny i porównania jakości dopasowania stosuje się test Kołmogorowa-Smirnowa, test Andersona-Darlinga i test Chi-kwadrat. Studium przypadku obejmujące dane dotyczące prędkości wiatru z wielu lokalizacji pokazuje działanie modelu matematycznego. Spośród sześciu rozkładów statystycznych przedstawionych powyżej funkcja gęstości prawdopodobieństwa Daguma wydaje się być najbardziej spójna. (***Matematyczne modelowanie gęstości mocy wiatru w czasie rzeczywistym z wykorzystaniem techniki transformacji***)

Keywords: Anderson-Darling; Chi-Squared test; Dagum; Kolmogorov –Smirnov test; Weibull
Słowa kluczowe: modelowanie, turbiny wiatrowe

Introduction

The consumption of wind energy resource plays a vital role in energy supply presently throughout the world as it is cost effective and readily available. It's a fresh fuel source available in the natural form. Wind energy doesn't pollute the air like power plants that trust on burning of natural gas, for instance coal, fossil fuels which discharge particulate matter, sulphur dioxide(SO_2), and nitrogen oxides(NO) — causing human health issues and commercial damages. The harvesting of the wind resource is done by human being for several decades. Since, old Holland to the farms in the United States, windmills have been used for grinding grain and pumping water. Nowadays, the windmill's recent equivalent - a wind turbine – can be used to convert the wind's energy to produce electricity. A windmill is a mill that transfigures the wind energy into revolving energy with the practice of vanes called blades or sails. Times ago, windmills typically were used to mill grain (gristmills), pump water (wind pumps), or both. Thus, to take benefit of this spontaneously accessible energy source, an enormous quantity of investigation has remained accompanied toward the progress of a precise and consistent wind energy valuation model using various dissimilar methodologies.

The thorough understanding of wind features in a particular zone is required for an effective utilization of wind energy. The density function of wind speed is typically designated by its PDF (probability density function). Formerly, the energy density E is expressed in terms of W/m^2 for a precise wind station and a wind turbine could be attained by means of the power curve and by the wind speed PDF. To figure out the obtainability of the wind power in a particular zone, the power equation formula is given as:

(1) $$P_w(X) = \frac{1}{2} A \rho X^3$$

where A is a blade sweep area and ρ constant value of air density. Using this wind power, one can calculate the mean wind power of that area using the equation:

(2) $$\overline{P} = \int_0^\infty P_w(X) f(x) dx$$

When considering the Betz's law of power coefficient C_p, the power equation will become

(3) $$P_{C_p}(X) = \frac{1}{2} A \rho X^3 C_p(\lambda, \beta)$$

where $C_p(\lambda, \beta)$ is the power coefficient value. Thus it is necessary to implement several calculations to find the wind power using the above equation (3). Hence to avoid those calculations, we have used the transformation method to get the wind power from the perceived wind speed data.
Thus it is being established that the proper information about the PDF of wind speed is needed to evaluate the wind energy potential [1]. In this regard, many researchers have evidenced that the Weibull distribution function has a substantial part in the area of wind energy application [2].

Related Works

The PDF of Weibull distribution is used to evaluate the wind power for innumerable regions [3]. Also to verify the performance of wind power this Weibull pdf is used as an estimation model [4]. On the other hand, it is not possible to model the Weibull pdf for all the wind regimes. Alavi et al. [5] have introduced a new distribution function called Nakagami in the field of modelling the wind speed data. Out of eight different distributions they have evaluated, the Nakagami PDF provided the best fit in 2 stations though in the remaining 3 stations it ranked 3rd to 5th. Thus they have concluded that the Nakagami distribution is an effective distribution among the eight distributions they have examined. Nedaei et al.[6] have analysed over 46 existing PDF's and concluded that the Wakeby PDF has given the better performance in fitting the observed wind speed to the theoretical wind speed in the region of Shurje, Iran. They have analysed from the moment ratio diagram that the log Pearson type III, Kappa, and Generalized Gamma distributions provide the best fit in that region. Arslan et al. [7] have introduced two new distributions, Generalized Lindley (GL) and Power Lindley (PL) in the field of wind speed modelling. Also they have compared these two distributions with Weibull distribution and proved that the GL distribution offers the finest fitting to the wind speed data in the region of Turkey.

Transformation Technique To Derive Wind Power

As we discussed above the wind power can be assessed from the wind speed data using the power equations (1)-(3). Wind power evaluation using these equations, needed some engineering aspects, such as the rotational speed, the turbine blade's angle of attack, and the parameter of the pitch angle of wind turbines [8]. To overcome this Masseran [9] has adopted an alternative technique of transformation of random variables. Let P = h(X), where X is an arbitrary variable for wind speed data and $f_X(x)$ is PDF of X. Then the power PDF can be resultant by

(4) $$f_p(p) = f_X\left(h^{-1}(p)\right)\left|\frac{d\left[h^{-1}(p)\right]}{dp}\right|$$

= 0 , otherwise

Let $u = \frac{1}{2}A\rho$.

Then from the equation (1), we get $P = uX^3$

Hence $\frac{d}{dp}\left(h^{-1}(p)\right) = \frac{1}{3u^{1/3}p^{2/3}}$

Thus by substituting this in equation (4), we can derive the wind power PDF $f_p(p)$.

Statistical Analysis Methodology

The analysis of the quantity and quality of the energy is created on the ability of the PDF to designate the observed wind speed incidences distribution [10]. Though we have various PDF's, very few are suitable for the physical data. In this study, the selected six distributions which are more relevant to the purpose are Burr, Dagum, Gamma, Log-Logistic, Logistic and Weibull. The parameters[11] of the above distributions are assessed using the Maximum Likelihood Estimation (MLE). The probability density function of those six distributions and their power density and cumulative power density functions are obtained after applying the transformation method.

Results And Discussion

Description of Data

The real-time datasets were obtained from the National Renewable Energy Laboratory, which is operated by the Alliance for Sustainable Energy for the U.S. Department of Energy. The hourly data of October–December 2006 of six different wind farms have been used.

For the data from all these stations, six different distributions are evaluated and compared. The parameters of those density functions are calculated by the Maximum Likelihood Estimation. Table 1, provides the parameter values of the density functions for all the six distributions.

Performance Metrics

The Kolmogorov-Smirnov statistic (KS) test, Anderson Darling (AD) test and Chi-squared test are used to analyse the best fit among the six distributions. All these three performance metrics are computing the difference between the experimental and the theoretical data.

The Kolmogorov-Smirnov test statistic for a given theorized cumulative distribution function *Y(x)* and experimental distribution function $Y_n(x)$ is given as:

(5) $$D_n = \underset{x}{Sup}\left|Y_n(x) - Y(x)\right|$$

where $\underset{x}{Sup}$ is the supremum of the set of distances.

Table 1. Parameter estimation using the MLE

STATION	Burr	Dagum	Gamma	Logistic	Log-Logistic	Weibull
Station 1	k=719.29 α=2.6277 β=94.221	k=0.18921 α=10.27 β=9.9515	α=5.9494 β=1.1536	σ =1.5514 μ=6.8636	α=1.0989E+8 β=1.7707E+8 v=-1.7707E+8	α=2.3435 β=7.8114
Station 2	k=1129.5 α=2.9030 β=113.39	k=0.15367 α=12.775 β=13.321	α=6.9478 β=1.2924	σ =1.8781 μ=8.9792	α=8.1102E+5 β=1.6133E+6 v=-1.6133E+6	α=2.5738 β=10.183
Station 3	k=1032.1 α=2.6389 β=143.09	k=0.2041 α=9.2986 β=13.509	α=5.9584 β=1.5401	σ =2.0726 μ=9.1766	α=31.442 β=69.152 v=-60.073	α=2.4472 β=10.4
Station 4	k=645.26 α=2.7024 β=110.08	k=0.12768 α=13.83 β=13.954	α=6.1239 β=1.4596	σ =1.9914 μ=8.9384	α=416.94 β=888.62 v=-879.7	α=2.4848 β=10.119
Station 5	k=1045.7 α=2.5237 β=165.43	k=0.21268 α=8.7206 β=13.697	α=5.5717 β=1.6795	σ =2.1857 μ=9.3576	α=42.418 β=97.221 v=-87.936	α=2.2435 β=10.685'
Station 6	k=790.83 α=2.5880 β=134.16	k=0.14421 α=12.221 β=13.922	α=5.7247 β=1.5809	σ =2.0855 μ=9.0505	α=5.3900E+8 β=1.1916E+9 v=-1.1916E+9	α=2.331 β=10.283

Table 2. Results of goodness of fit for various distributions

Station	Distribution	K-S test	AD test	Chi-squared test
Station1	Burr	0.05779	13.32	132.41
	Dagum	0.02049	1.342	16.02
	Gamma	0.09168	46.109	322.61
	Logistic	0.04834	11.274	119.23
	Log-Logistic	0.03886	8.2368	95.406
	Weibull	0.04226	6.9637	78.228
Station 2	Burr	0.03073	15.909	111.17
	Dagum	0.04216	13.646	163.77
	Gamma	0.05686	78.593	355.65
	Logistic	0.04613	24.623	153.3
	Log-Logistic	0.03128	13.289	62.809
	Weibull	0.04607	24.236	206.48
Station 3	Burr	0.0270	10.063	160.61
	Dagum	0.05524	16.319	250.82
	Gamma	0.05155	51.654	325.87
	Logistic	0.04951	29.572	242.83
	Log-Logistic	0.03834	15.356	137.71
	Weibull	0.03803	11.578	174.48
Station 4	Burr	0.03535	14.017	127.47
	Dagum	0.0638	25.034	245.63
	Gamma	0.05669	62.413	319.65
	Logistic	0.05745	36.151	253.91
	Log-Logistic	0.08706	96.409	515.48
	Weibull	0.03511	12.973	137.08
Station 5	Burr	0.03899	13.013	221.99
	Dagum	0.0437	8.2613	210.40
	Gamma	0.05427	63.494	489.12
	Logistic	0.04607	18.113	251.12
	Log-Logistic	0.03952	10.323	215.12
	Weibull	0.05718	27.655	344.71
Station 6	Burr	0.04966	20.705	248.93
	Dagum	0.03774	6.0715	101.23
	Gamma	0.07254	86.777	584.43
	Logistic	0.06816	33.515	284.78
	Log-Logistic	0.05282	19.903	221.29
	Weibull	0.0464	24.13	347.85

The Anderson-Darling test statistic for a given hypothesized distribution function *Y(x)* and experimental cumulative distribution function $Y_n(x)$ is given as

(6) $$D_n = n\int_{-\infty}^{\infty}\frac{\left(Y_n(x) - Y(x)\right)^2}{Y(x)\left(1 - Y(x)\right)}dY(x)$$

The Chi-squared (χ^2) test statistic for the observed distribution function O(x) and the expected cumulative distribution function *E(x)* is given as

(7) $$\chi^2 = \sum_{i=1}^{n} \frac{\left(O_i(x) - E_i(x)\right)^2}{E_i(x)}$$

The results of goodness of fit tests are shown in Table 3. From Table 3, the distribution which is having minimum test statistic is identified as the best model for the wind speed distribution for each station.

In both the plots, X axis represents the wind power (W/m^2) data. The Y axis of power density function curve represents the power density which varies from lowest to highest possible values and of cdf curve represents the cumulative density function which is varies from zero to one as it moves from left to right on the horizontal axis.

Figure 1 and 2, represent the best fitted wind power density curves and cumulative power density curves respectively for all the stations. The real time wind data is provided by the National Renewable Energy Laboratory (NREL).

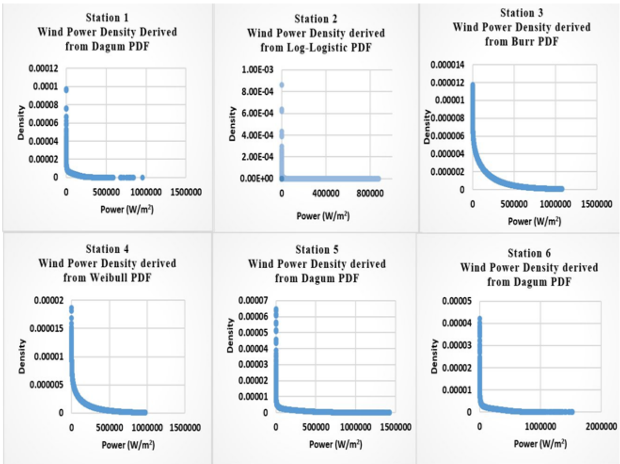

Fig.1. Power density function for each station

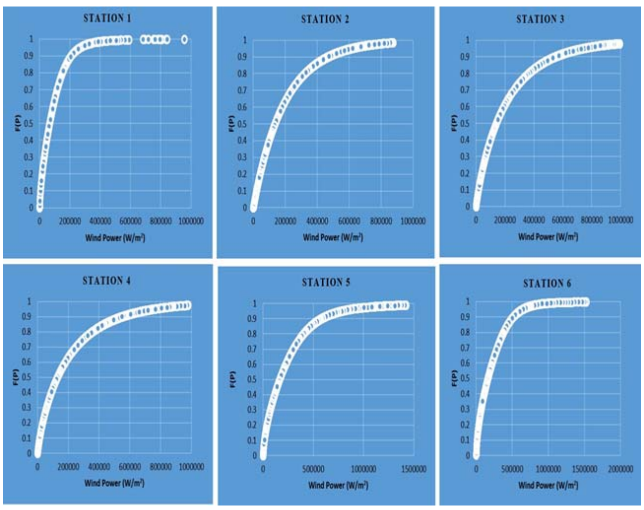

Fig.2. Cumulative power density function for each station

Conclusion

In this article, a comparison case study have been conducted to fit the best mathematical model for the wind speed data observed from the stations. To derive the theoretical wind power density function from the observed wind speed data, the transformation technique has been suggested. The speculative wind speed density fits well with the histogram of observed wind speed data. Evaluations and comparison of six different distributions for various wind speed data have been done in this study. A proficient method called MLE was proposed to evaluate the parameters of those distributions. The goodness of fit is tested using the KS statistics, AD test and Chi-squared test. The results shows that a particular distribution function cannot be recommended for all the surveyed stations. From Table 2, a comparison between the six distributions and the three goodness of fit tests (KS statistic test, AD test & Chi-squared test) reveals that the Dagum distribution function is the best modal to the wind speed data for the station 1 and station 6. Whereas for station 5, the Dagum distribution provides a finest fit according to AD test and Chi-squared test. However the KS statistic test gives the third rank to the Dagum distribution PDF for that station. The Log-logistic distribution offers the best fit to the station 2 according to AD test and Chi-squared test. Similarly the Burr and Weilbull distribution PDF's are the appropriate model for the wind speed data of the stations 3 and 4 respectively. Overall, this study showed that the Dagum distribution is more applicable to model the wind speed data since it gives the best fit in three stations and ranked in good position in other remaining stations. The scope of the statistical model using the transformation technique with other distribution functions can be carried out and performance metrics can be estimated in real time applications.

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REFERENCES

[1] Safari, B., Modeling wind speed and wind power distributions in Rwanda. Renewable and Sustainable Energy Reviews, Vol.15, No.2, pp.925-935, 2011.
[2] Celik, A., "A statistical analysis of wind power density based on the Weibull and Rayleigh models at the southern region of Turkey", Renewable Energy, vol.29, No.4, pp.593-604, 2004.
[3] Islam, M., Saidur, R. and Rahim, N., "Assessment of wind energy potentiality at Kudat and Labua, Malaysia using Weibull distribution function", Energy, Vol. 36, No.2, pp.985-992, 2011.
[4] Lydia, M., Kumar, S., Selvakumar, A. and Prem Kumar, G., "A comprehensive review on wind turbine power curve modeling techniques", Renewable and Sustainable Energy Reviews, Vol.30, pp.452-460, 2014.
[5] Alavi, O., Mohammadi, K. and Mostafaeipour,A."Evaluating the suitability of wind speed probability distribution models: a case study of east and southeast parts of Iran", Energy conversion and Management, Vol. 119, pp.101-108, 2016.
[6] Nedaei, M., Assareh, E. and Walsh, P., "A comprehensive evaluation of the wind resource characteristics to investigate the short-term penetration of regional wind power based on different probability statistical methods", Renewable Energy, Vol. 128, pp.362-374, 2018.
[7] Arslan, T., Acitas, S. and Senoglu, B., "Generalized Lindley and Power Lindley distributions for modeling the wind speed data", Energy Conversion and Management, Vol.152, pp.300-311, 2017.
[8] Jaramillo, O. and Borja, M., "Bimodal versus Weibull Wind Speed Distributions: An Analysis of Wind Energy Potential in La Venta, Mexico", Wind Engineering, Vol.28, No.2, pp.225-234, 2014.
[9] Masseran, N., "Evaluating wind power density models and their statistical properties", Energy, Vol.84, pp.533-541, 2015.
[10] Chang, T., "Estimation of wind energy potential using different probability density functions", Applied Energy, Vol.88, No.5, pp.1848-1856, 2011.
[11] Savino MM, Manzini R, Selva VD, Accorsi R (2017) A new model of environmental and economic evaluation of renewable energy systems: the case of wind turbines. Appl Energy 189:739–752