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## Testing and modeling of selected heat dissipation systems for high power LED light sources

Abstract. All important parameters of LED light sources depend strongly on temperature of semiconducting materials. Durability and efficiency of mentioned devices decrease dramatically in the function of temperature. According to thermal generation of current carriers, also electrical parameters (like dynamic resistance of the diode) varies with junction temperature. In some cases, such phenomena can provide a very fast degradation of LED by means of overheating or by exceeding maximal current of the diode. Basing on such reasons, heat dissipation systems, seems to be a very important equipment of semiconducting light sources. All solutions presented in the paper are connected with automotive solutions, where limited space of heat dissipation system is a very important factor. Due to this feature, selected types of heat sinks equipped with heat pipes, were analyzed and described in the paper. Methods for analysis of such devices were presented and some Author's procedures for effective determination of temperature distribution were described. Accuracy of proposed solutions were compared to test results of physical models. Efficiency of heat pipes heat sinks was compared to classical heat dissipation systems in natural convection conditions.

Streszczenie. Istotne parametry eksploatacyjne diod elektroluminescencyjnych istotnie zależą od temperatury. Zarówno trwałość, sprawność, jak i parametry elektryczne (jak rezystancja dynamiczna diody) zmieniają się w funkcji temperatury złącza, co prowadzić może do szybkiej degradacji LED'ów wynikającej z przegrzania lub przekroczenia granicznej wartości prądu (w przypadku stosowania źródeł napięciowych). Z uwagi na opisane czynniki, poprawna konstrukcja elementów rozpraszających ciepło jest szczególnie istotna w źródłach o znaczących mocach. Rozwiązania prezentowane w niniejszym artykule dotyczą źródeł światła stosowanych w pojazdach, gdzie ograniczone rozmiary wymuszają poszukiwania najbardziej efektywnych sposobów rozpraszania ciepła. W pracy scharakteryzowano radiatory wyposażone w rurki cieplne. Podano metody analizy tego typu układów oraz autorskie rozwiązania pozwalające na efektywne obliczenia rozkładu temperatury. Sprawność radiatorów wyposażonych w rurki cieplne porównano z klasycznymi w warunkach charakterystycznych dla eksploatacji w maszynach roboczych. (Badania i modelowanie wybranych układów rozpraszania ciepła dedykowanych dla źródeł LED wysokich mocy).

Keywords: LED light source, modelling, heat pipes Słowa kluczowe: źródła światła LED, modelowanie, rurki cieplne

### Introduction

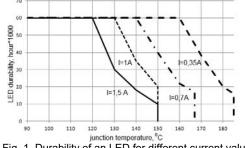
LED light sources, similarly to other semiconducting devices, are characterized by strong temperature impact on operating characteristics and parameters [1 ÷ 3]. Such effect is observed in reference to static and dynamic characteristics and results mainly from thermal generation of current carriers in semiconducting materials. In most lighting applications of LEDs, issues of connection frequency of the light source are not important as they work in low frequency range. In such group of utilities, large temperature of the junction can provide a reduction of luminous efficacy, changes in the color of the emitted light or limitation of diodes durability. Basic diode equation (1) enables to determine ideal current vs. voltage characteristics and to show temperature dependency of mentioned characteristic [4].

(1) 
$$I = Aqn_i^2 \left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D}\right) e^{\frac{Uq}{kT} - 1}$$

where: A - cross section area of the junction; q - charge; D diffusion coefficient; L - depth of penetration of electrons (n) and holes (p); N - dopands concentration; k - Boltzmann constant; T - temperature; U - voltage

Temperature impact on the output characteristics of diodes results from different concentration of the charge carriers  $(n_i^2)$ , mainly in high doped semiconductors [5]. Additionally, in presented formula (1), there is a factor called the thermal force (kT/q), which describes the phenomena of thermal generation of current carriers in p-n junction. Basing on above formula, one can conclude that limitation and stabilization of LED junction temperature is a very required task. Exploitation of LED with maximal junction temperatures provides a significant (about 20%) reduction of luminous flux [4]. In the figure 1 exemplary characteristics of LED durability for different diode forward

current were shown [6]. Diodes can operate with maximal junction temperatures only, when efficient heat dissipation systems are in use.





In the paper popular types of heat sinks were compared to heat sinks equipped with heat pipes. Heat pipes are a very interesting alternative for typical heat dissipation sinks. Due to extremely low thermal resistances of heat pipes, they allow to the homogenization of temperature distribution improvement within all volume of the device. This feature have a significant role for intensification of heat transfer to the environment as the heat transfer coefficient depends on temperature value. Such feature, in combination with simple construction and reliability of heat pipes, results, that they are widely used in heat dissipation systems, especially in electronic devices [7, 8]. In the paper some aspects of heat pipes utility in LED light sources was presented. In general, designing process of such systems requires a coupled thermal and fluid flow analysis to take into account all physical phenomena that occur in heat pipes systems. Such approach is uncomfortable in engineers point of view. Author's procedure for designing of heat sinks equipped with heat pipes was presented. Accuracy of proposed numerical models was confirmed by tests of physical devices. Additionally, some numerical simulations were performed to compare the efficiency of heat pipes in comparison to typical aluminum heat sinks. Basing on calculations results, it was possible to determine conditions and range of applications of analyzed heat sinks in solutions connected with LED light sources, especially used in automotive solutions [9, 10].

### Modeling of heat sinks equipped with heat pipes

In the chapter more important rules of numerical modeling of the heat pipes were described. Typical coupled field analysis of thermal and fluid flow fields were used. Such solution is required to present all physical phenomena of the heat pipes operation. Basing on Authors' tests and simulations, the simplified model of heat pipes was proposed.

All benefits of heat pipes result from heat and mass transfer in closed space that is partly filled with liquid and vapor of low pressure. Effective heat transfer coefficient in the heat pipes results from evaporation, boiling and liquefaction of the operating agent inside the pipes (irrespectively of the pipe type) [11, 12]. There is possibility to mark out four basic regions of heat pipes, according to the figure 2.



Fig. 2. Typical regions of heat pipes. 1 - boiling and evaporation region; 2 - liquefaction region; 3 - gas phase; 4 - return of liquid phase

In the boiling area (1 in the fig. 2), that is located in vicinity of cooled device (for example LED), there is a liquid phase. Type of the liquid and pressure determine the boiling temperature. Due to the phase change in boiling area, convection heat transfer coefficient can reach a extremely large values [12, 13]. The largest part of heat pipe is filled with liquid vapor (3 in fig. 2), that is carried by convection to the colder part of the pipe (2 in fig. 2). In this area the liquefaction process occurs. Similarly to the boiling, phase change provides a very large heat transfer coefficient value. Circulation of the active agent is closed by side walls or wick of the pipe (4 in fig. 2), where liquid phase returns to the hot side by the capillary effect or (in some cases) gravity [14].

Due to given description, one can conclude that basic equations of heat transfer in heat pipes should be extended by mass transport equations and, in general, fluid dynamics computations. The computations of boiling process require to use two equations for the mass flow. Formula (2) represents amount of weight received from boiled liquid. Amount of mass transferred to the gas can be described by similar equation, but one have to consider opposite direction of mass transfer [15].

(2) 
$$M_c = 0.1 \rho_c \alpha_c \frac{t_{mix} - t_{pc}}{t_{pc}}$$

Similarly, liquefaction process require to determine the mass flow from gas to liquid phase (3) [15].

$$M_c = -0.1 \rho_{gas} \alpha_{gas} \frac{t_{pc} - t_{mix}}{t_{pc}}$$

where:  $t_{pc}$  - temperature of the phase change;  $t_{mix}$  - mixture temperature;  $\rho$  - density;  $\alpha$  - heat transfer coefficient

Apart from the mass of phase changing substance, a very important factor is the heat flux transferred between gas and liquid phase (4). In this equation  $L_H$  denotes the latent heat.

(4) 
$$Q_c = -0.1 \rho_{gas} \alpha_{gas} \frac{t_{pc} - t_{mix}}{t_{przem}} \cdot L_H$$

Equations (2) to (4) describe the phase change phenomena. In the area filled with the vapor, basic equations of substance (5), momentum (6) and energy (7) balances apply [15].

(5) 
$$\frac{\partial \rho}{\partial \tau} + \frac{\partial (\rho w_x)}{\partial x} + \frac{\partial (\rho w_y)}{\partial y} + \frac{\partial (\rho w_z)}{\partial z} = 0$$
  
(6) 
$$\rho \frac{dw_x}{d\tau} = \rho g_x - \frac{\partial \rho}{\partial x} + \mu \left( \frac{\partial^2 w_x}{\partial x^2} + \frac{\partial^2 w_x}{\partial y^2} + \frac{\partial^2 w_z}{\partial z} \right)$$

(7) 
$$\frac{+\frac{1}{3}\mu\frac{\partial}{\partial x}\left(\frac{\partial n_{x}}{\partial x} + \frac{y}{\partial y} + \frac{\partial n_{z}}{\partial z}\right)}{\frac{dt}{d\tau} = a\nabla^{2}t$$

where: *t* - temperature; *p* - pressure; *w* - gas velocity;  $\mu$  - kinematic viscosity; *a* - thermal diffusivity;  $\tau$  - time

Direct analytical solution of above equations is not possible in most cases. Due to this fact some specialized numerical methods of computational fluid dynamics are in use. Mathematical description of presented phenomena is generally inconvenient for engineering analysis. Especially, equations for mass flow in boundary surfaces are characterized by high level of complexity and their accuracy is not confirmed [16]. To avoid such complicated calculations, simplified models of heat pipes, presented in figure 2 were used. Most important simplification results from omission of the capillary action of the liquid flowing in the direction of boiling area. The effect is not significant in the heat exchange phenomena at all, but, it can limit the value of thermal energy that can be transferred inside the heat pipes. Due to mentioned disadvantages of the utility of equations (2) ÷ (7), calculations were performed by using criterion equations for thermal convection and conduction. In the boiling area, third type of boundary condition was used. Heat transfer coefficient was determined basing on equation (8). Nonlinear characteristic of heat transfer coefficient was included.

(8) 
$$\alpha = 0.145 p^{0.5} (T_w - T_n)^{2.33}$$

where: p - pressure;  $T_n$  - saturation temperature

Similar method was used to describe the liquefaction effects that occur in opposite end of the heat pipe (9).

(9) 
$$\alpha = 0.72B \frac{\sqrt[4]{r}}{\sqrt[4]{l(T_n - T_w)}}$$

Values of *B* and *r* depend on the temperature [15]. In the space of vapor inherence (area of convection heat transfer), typical models of fluid flow analysis (CFD) were used. Full analytical model included all important conditions used during physical tests :

- heat source of constant temperature (60°C) placed in front surface of the heat pipe;

- thermal convection heat transfer from external surfaces of the pipe to the environment of constant temperature.

All parameters of numerical model were selected to compare results of numerical and physical tests in steady state. In figure 3, the gas velocity distribution inside numerical model of the heat pipe have been presented.

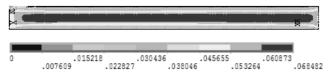
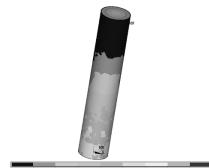


Fig. 3. Gas velocity (in m/s) in the cross section of heat pipe model

Basing on simulations results, it can be mentioned that in the heat pipes a typical convection cells do not occur. Maximal velocities are noticed outside the thin boundary layers. Such distribution have a positive impact on homogenization of temperature field distribution.

In thermal analysis point of view, most important factor is the temperature field distribution, shown in figure 4. In analyzed case, temperature value changes from 49,5°C to 60°C. Heterogeneity of temperature distribution in analyzed pipe was larger than 17 %.



49,46 50,63 51,80 52,97 54,15 55,31 56,48 57,65 58,82 6

Fig. 4. Temperature field in the heat pipe numerical model in steady state

Physical model of the heat pipe (of geometry similar to numerical model) equipped with 4 thermocouples was used to compare results of numerical calculations and physical measurements. In the figure 5 temperature distribution on the external surface of both models (physical and numerical) were shown in the function of pipe height (h). Results of physical experiment were interpolated between temperature measuring points.

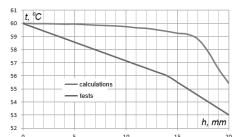


Fig. 5. Temperature distributions on external surfaces of physical and numerical models

Results of physical experiment and numerical calculations are characterized by qualitative and quantitative differences. However, according to complexity of analyzed physical systems, total quality of numerical models is sufficient. Maximal differences were not higher than 8 %.

All models of heat pipes are characterized by high complexity and they are difficult to implementation in systems characterized by complex geometry. Additionally, calculations of coupled fields are very time consuming. Therefore, it was assumed that better solution is to use simplified models of heat pipes, where all volume of the pipe will be analyzed as a solid materials of thermal conductivity determined basing on numerical models and physical tests. Such solution results directly from the definition of thermal resistance (10). After modifications, there is possibility to determine actual value of thermal conductivity.

(10) 
$$W = \frac{\delta}{\lambda F}$$

where:  $\delta$  - length of the thermal flux; *F* - cross section of the heat flux inside heat pipe;  $\lambda$  - thermal conductivity of the heat pipe

Basing on numerical simulations and tests of heat pipes of 8 mm diameter and 200 mm length, values of average thermal conductivities were determined.

# Influence of heat sink type on heat dissipation conditions

According to theoretical description presented in previous parts of the paper, thermal power dissipated to the environment from solid bodies, is proportional to the temperature of all parts that have physical contact with liquids and heat transfer coefficient [6]. Utility of heat sinks equipped with heat pipes require to maximize areas of their external surfaces. In most cases, heat sinks designed for cooperation with heat pipes are characterized by small distances between their fins [14]. Such solution have some important advantages, like reduction of thermal time constants (11). Due to this effect, heat dissipation systems are very desirable for cooling object characterized by dynamic changes of power values (12). Basic disadvantage of low fins distance heat sinks is reduction of their utility in natural convection conditions. To determine mentioned effect, analysis of heat sinks presented in figure 6 was performed for constant power of LED source of 6 W.

(11) 
$$N_c = \frac{m \cdot c_p}{\alpha \cdot F_z}$$

$$(12) t = (t_p - t_U)e^{-it_Q}$$

where: m - mass;  $c_p$  - specific heat;  $\tau$  - time;  $F_z$  - external surface

 $\tau_t$ 

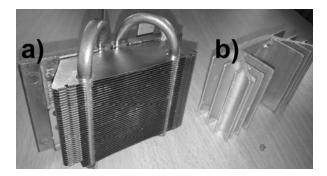


Fig 6. Types of heat sinks used for tests

Tests of physical model was performed in steady state, when temperatures weren't change more than 0.5 K/min. Measured temperatures in specific locations in heat sinks are presented in table 1.

Table 1. Steady	state tempera	tures for heat	sinks shown ii	n fig. 6

Heat sink type	Temperature of LED source, °C	Fins temperature, °C
fig. 6a	56.1	47
fig. 6b	47	42.7

Basing on performed tests one can conclude, that for classic aluminum heat sink (without heat pipe) it was obtained:

- lower value of LED light source;

- lower temperature drop between heat source (LED) and fins.

Such effects are strongly connected with heat sinks design. For the heat sink equipped with heat pipe, small air gap between fins reduces the velocity of gas flowing in this region by natural convection. Through high value of air viscosity, air inside the air gaps can be considered as motionless. This effect has been presented in figure 7, where velocity vectors in air gaps of 1 mm and 5 mm thickness were presented. Figure 7 applies to natural convection, where temperatures of lower part of air gap was characterized by temperature of 150°C and source located above gap was at temperature 20°C.

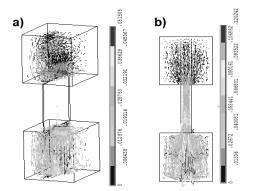


Fig. 7. Vectors of air velocity in air gaps of 1 mm (a) and 5 mm (b) thickness

Presented results of physical tests were used to develop authors numerical procedure for heat sinks modeling in natural convection conditions. For heat sinks, where distance between fins is lower than 10 mm, heat transfer can be calculated using equation (7) for thermal conduction. Basic difference from typical conduction equation lays from the utility of relative thermal conductivity except of "normal" conductivity ( $\lambda$ ). Such relative conductivity can be determined basing on criterion equation (13) that enable to consider velocity of the air. In proposed procedure it was assumed that from all external surfaces of heat sink (including external surfaces of the air gaps), thermal energy can be transferred by convection and radiation to the environment of given temperature  $t_0$  (14). Heat transfer coefficient cam be determined by simplified formula (2).

(13) 
$$\lambda_z = \lambda_{\sigma} C (Gr \cdot \Pr)^n$$

where: Gr - Grashoff number; Pr - Prandtl number; C, n - coefficient for flow type characterization;  $\lambda_g$  - thermal conductivity of air with zero velocity

For all heat sinks, where the distance between fins is higher than 10 mm, third type of boundary condition (14) can be used in all internal and external surfaces of the heat sink. Thermal radiation heat transfer between fins can be omitted according to similar temperature values of the fins.

(14) 
$$\lambda \frac{\partial t}{\partial n} = \alpha_k (t_F - t_0)$$

where:  $t_F$  - temperature of external surface of radiator;  $t_0$  - ambient temperature

Proposed procedure is characterized by the ease of implementation in most FEM systems. Performed simulations obtained by using authors procedure and computational fluid dynamics (CFD) procedure show a good accuracy of results (fig. 8).

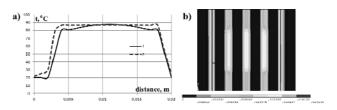


Fig. 8. Temperature distributions (a) on external surface of the heat sinks and air velocity in air gaps (b). 1 - CFD model, 2 - authors procedure

The procedure was used to determine the efficiency of two heat sinks shown in figure 9. For both cases external dimensions (cross sections of the basis and fins height of heat sinks) were the same. Heat source of constant power (6 W) was used. Basic calculations were performed for natural convection. Heat sink, shown in fig. 9a, was characterized by a large distance between fins. According to authors procedure, in this case third type of boundary conditions were used in all surfaces of the heat sink. For the second heat sink, procedure of relative thermal conductivity was used to determine heat exchange between the fins.

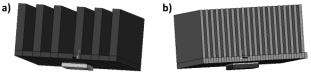


Fig. 9. Geometry of analyzed heat sinks. a) typical heat sink; b) heat sink designed for utility with heat pipes

Additional analysis were performed for the heat sink shown in fig. 9b to determine its efficiency for forced convection. Constant velocity of the air was assumed at 1 m/s in all air gaps between fins. Results has been shown in figure 10.

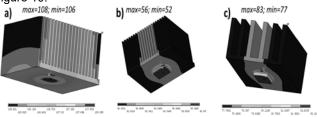


Fig. 10. Temperature fields in heat sinks shown in fug. 16. a), c) - natural convection; b) forced convection

In the figure 11 temperature distribution on internal part of heat sinks pads (from the side of LED) has been shown.

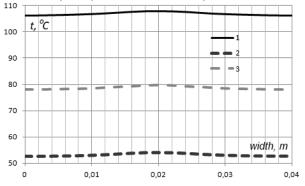


Fig. 11. Steady state temperature distribution in internal surface of analyze heat sinks. 1 - heat sink from fig. 16b, natural convection; 2 - heat sink from fig. 16a natural convection; 3 - heat sink from fig. 16b - forced convection

Basing on performed analysis, it was proven, that heat sinks characterized by small air gaps between fins (designed for cooperation with heat pipes) should operate in forced convection conditions. For natural convection, according to viscosity, air can't flow between fins and potential benefits of heat pipes cannot be reached.

#### Summation

In the paper different types of heat sinks, especially designed for cooperation with heat pipes were analyzed. Basic purpose was to determine possibility and expediency of utility of such solutions for heat dissipation systems in LED headlights for low-speed vehicles. Basing on performed tests and simulations, some conclusions can be listed:

- performed physical tests of heat pipes and Perkin's tubes enable to determine real possibilities of heat dissipation intensity in function of used pipes and their spatial orientation,

- proposed analytical and numerical model of the thermal energy propagation within heat pipes was characterized by high accuracy, confirmed by physical tests. Model was used for multivariate analysis;

basing on performed simulations, it was proven that utility of heat pipes in classic heat sinks does not lead to intense performance increment of such systems (in terms of significantly higher heat flux dissipated to the environment);
basing on modeling results, it was proven, that special heat sinks designed for cooperation with heat pipes, can reach a high efficiencies only for forced convection;

- all calculations and numerical models accuracy was confirmed by physical tests.

5 pages. The whole paper is written using type size 9 and Arial font with the indentation set at 5 mm. Directly after section headings (type size 9 using Arial bold), place the text in justified style. For figure captions, names of the tables and footnotes use Arial font with type size 8. When symbols need to be inserted into the text, use option "insert/symbol" (for example  $\Delta \alpha$  or  $\vartheta \in \infty$ ). Please do not use the equation editor for this purpose. The equation editor should only be used if a symbol does not exists in the symbol list – for example  $\hat{H}$ . Although the whole text is written using Arial font, for symbols please use the Times New Roman italic – for example *J* and not *J* (i.e. the same symbols as in the equations).

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