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doi:10.15199/48.2022.09.03

Mathematical modeling and creation of algorithms for analyzing the ranges of the amplitude-frequency response of a vibrating rotary crusher in the software Mathcad

Abstract. *The article is devoted to the study of motion laws for rotary vibration crusher. Kinematic and dynamic analysis was performed. Differential equations of rotor motion are solved and analyzed, frequency response and energy consumption graphs in MathCad 15.0 software environment are presented. Verification of the mathematical model was carried out by comparing the results of experimental research with theoretical research. It was proved that the proposed mathematical models are adequate (discrepancy are 7.2 to 12.1%).*

Streszczenie. *Artykuł poświęcony jest badaniu praw ruchu obrotowego kruszarki wibracyjnej. Przeprowadzono analizę kinematyczną i dynamiczną. Równania różniczkowe ruchu wirnika są rozwiązywane i analizowane, prezentowane są wykresy odpowiedzi częstotliwościowej i zużycia energii w środowisku oprogramowania MathCad 15.0. Weryfikację modelu matematycznego przeprowadzono poprzez porównanie wyników badań eksperymentalnych z badaniami teoretycznymi. Wykazano, że zaproponowane modele matematyczne są adekwatne (rozbieżność wynosi od 7,2 do 12,1%).* (***Modelowanie matematyczne i tworzenie algorytmów analizy zakresów odpowiedzi amplitudowo-częstotliwościowej wibracyjnej kruszarki obrotowej w programie Mathcad***)

Keywords: Lagrange's equations II kind's, differential equations, parameters of oscillations, analytical mechanics, vibrating machine
Słowa kluczowe: równanie różnicowe, odpowiedź częstotliwościowa, kruszarka wibracyjna

Introduction

For feeding livestock very often use cheap fodder grain, which is not suitable for food purposes. Usually such grain has high humidity [1, 2]. Dry food grain is rarely used. But the specific energy consumption, which is specified in the technical characteristics for hammer crushers are reliable only for quality dry grain (humidity about 14-16%) [2, 3]. During the grinding of wet (fodder) grain, productivity is significantly reduced and specific energy consumption increases [4]. Therefore, there is a need to develop an equally energy-efficient crusher for both dry and wet grain.

Analysis of literary sources and problem statement

During the destruction process of materials with plasticity properties, energy is used to overcome molecular bonds and irreversible plastic deformations [5, 6]. The energy spent on deformation is converted into heat [7, 8, 9].

With increasing moisture content, fragility and ultimate strength decrease, ductility and ultimate deformation before fracture increase [5, 10].

In addition, during the grinding process of forage grains can often be adhesion to the sieve of the wet shredded product [10, 11, 12].

The development of a vibratory crusher consists of many complex stages [13, 14]. The first steps in the development process of a machine are theoretical studies [15, 16]:

1) development of kinematic scheme (the new design should solve the problem of effective grinding (dry and wet grain) and prevent blockage of the sieve with crushed material);

2) kinematics research (these results are the basis for dynamic analysis) [17];

3) dynamics research (these results are the basis for calculating the crusher drive and creating a design drawing) [18].

Using the infrastructure of the laboratory of the process and processing of equipment and food industry Vinnytsia National Agrarian University, a design of vibratory crusher was developed [6, 19, 20]. Sharp disks will be installed in this crusher instead of rectangular plates (hammers). That is the methods of impact and cutting to destroy the material will be combined [19]. Thus, a local overvoltage of surface micro volumes at the places of application of loads will be created [13, 21]. When the disc hits the grain quickly, it causes the sharp blade to sink into the body and create the pressure needed to break the material [8, 22]. In addition, for more intensive sieving of the product that already crushed oscillations of the sieve will be provided [19, 23].

Purpose and tasks of research

The purpose of the research is to development of a mathematical model for the crusher being designed and determination of ranges of amplitude-frequency characteristics in which energy consumption will be rational.

To achieve this goal, it is necessary: a kinematics study performed and to obtain the laws of motion; develop differential equations describing the motion of the disk rotor and energy consumption by the drive; find the solutions of the differential equations and determine the most optimal ranges of values for amplitude-frequency characteristics; check whether mathematical models are adequate by comparing the results of theoretical and experimental studies using a laboratory model of a vibrating crusher.

Materials and methods

Scientific articles position based on the classical theory of mechanical oscillations of the laws of theoretical mechanics and physics, kinematics analysis was done analytical method, the principle of superposition [12, 15].

The machine is represented mathematical model with 6 degrees of freedom, namely the shifting of the centre of mass of the container along with the axis OX (Fig. 1), shifting the centre of mass of the rotor along with the axis OX, shifting the centre of mass of the container along the axis OZ, shifting the centre of mass of the rotor along the axis OZ, angular shifting of the rotor relative to the axis O1Y1, angular shifting of the disc relative to axis O2Y2 [24].

To determine the crusher's laws of motion along each of the independent coordinates (x, y, z, φ_1, φ_2, φ_3), Lagrange's equations of the second kind will be used [17, 24] (1). In this vibrational system (Fig. 1), four characteristic moving masses can be distinguished in the total mass *m* (2).

To determine the equations of the generalized linear velocity of the centres of mass of the structural elements of the vibratory disk crusher, the mechanism will be divided into elementary components (Fig. 2), which will be studied separately [25]. To solve and analyze the obtained equations of motion of the executive body of the vibratory disk crusher, the mathematical environment MathCad 15.0 was used [6]. The use of this program allowed us to determine the patterns of change of oscillation parameters depending on the angular velocity of the drive shaft.

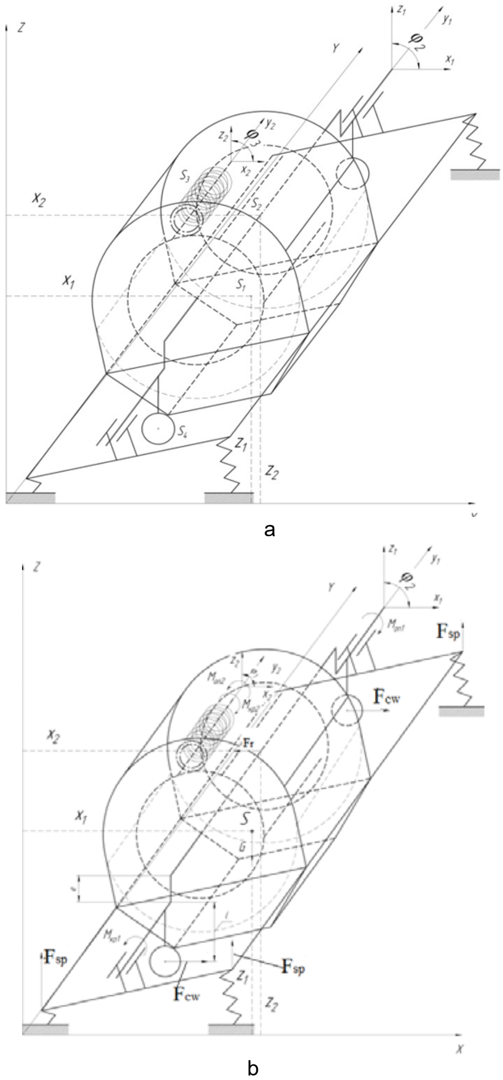

Fig. 1. Calculation scheme for studying: a) kinematics; b) forces and moments

$$\begin{cases} \frac{d}{dt}(\frac{\partial T}{\partial \dot{x}}) - \frac{\partial T}{\partial x} = Qx \\ \frac{d}{dt}(\frac{\partial T}{\partial \dot{z}}) - \frac{\partial T}{\partial z} = Qz \\ \frac{d}{dt}(\frac{\partial T}{\partial \dot{y}}) - \frac{\partial T}{\partial y} = Qy \\ \frac{d}{dt}(\frac{\partial T}{\partial \dot{\phi}_1}) - \frac{\partial T}{\partial \phi_1} = Q\phi_1 \\ \frac{d}{dt}(\frac{\partial T}{\partial \dot{\phi}_2}) - \frac{\partial T}{\partial \phi_2} = Q\phi_2 \\ \frac{d}{dt}(\frac{\partial T}{\partial \dot{\phi}_2}) - \frac{\partial T}{\partial \phi_3} = Q\phi_3 \end{cases} \quad (1)$$

where *T* – kinetic energy of the system; Qx, Qz, Qy,$Q\phi_1$,$Q\phi_2$,$Q\phi_3$ – generalized resistance forces.

$$\begin{cases} m = m_1 + m_2 + m_3 + m_4; \\ m_1 = m_\text{к} + m_m + m_{sf} + m_b; \\ m_2 = m_r + m_c; \\ m_3 = m_d; \\ m_4 = m_{cw}; \\ m_r = m_{esh} + m_{cd} + m_{var} + m_{sup} + m_{axles}; \end{cases} \quad (2)$$

where m_κ– mass of frame, kg; m_m– mass of material, kg; m_{sf}– mass of support frame, kg; m_b– mass of bearing units, kg; m_r– rotor weight, kg; m_c– mass of couplings, kg; m_d– mass of impact discs, kg; m_{cw}– weight of counterweight, kg; m_{esh}– mass of eccentric shaft, kg; m_{cd}– mass of intermediate discs, kg; m_{var}– a mass of eccentric variation mechanisms, kg; m_{sup}– mass of support discs, kg; m_{axles}– mass of disc axles, kg.

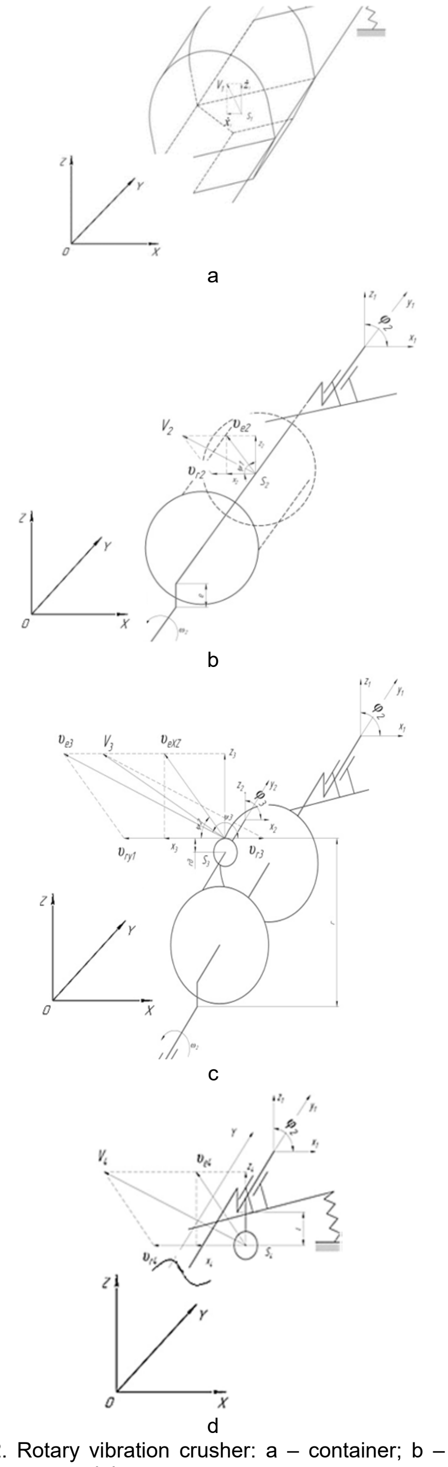

Fig. 2. Rotary vibration crusher: a – container; b – rotor; c – disc; d – counterweight.

As a result of research studies performed earlier, the approximate ranges of oscillation amplitude, vibration velocity, vibration acceleration and oscillation intensity were determined. However, to assess energy efficiency, it is necessary to experimentally investigate the effect of amplitude-frequency characteristics on energy consumption for the crusher drive.

Alsow, based on previous study [8, 20], a database was adopted, which included the values: the range of angular velocity of the drive shaft ω_2 =0…150 rad s^{-1}, and the time factor interval t = 0…60 s, as well as the values of the accepted constants [19] of the studied system (Table 1).

Table 1. Numerical values of the main constants adopted for the studied system

Constants	Value
Total moving weight, kg	83.2
- m_1, kg	32.7
- m_2, kg	31.9
- m_3, kg	13.1
- m_4, kg	5.5
The distance from the axis of rotation to the center of mass of the rotor *e*, m	0.005
Working disk radius r_d, m	0.045
The radius of the support disk r_{sup}, m	0.14
The distance from the top of the working disk to the axis of rotation *r*, m	0.19
The distance from the axis of rotation to the center of mass of the counterweight *l*, m	0.044
Stiffness of elastic elements C, N/m	
- along the axis *OX* : C_x	3900
- along the axis *OZ* : C_z	3900

The experimental study of ranges of amplitude-frequency characteristics at which energy consumption will be rational was carried out at Vinnytsia National Agrarian University using the laboratory model of a rotary vibration crusher [6].

To record the angular velocity values of the drive shaft, the UNI-T UT372 wireless tachometer (Uni-Trend Technology Limited, Dongguan, China) was used. To manage and change rotation frequencies of the motor shaft (up 0 to 150 s^{-1} with step 5 s^{-1}), the AOSN-20-220-75 (PJSC «Megommeter», Uman, Ukraine) autotransformer was used [26]. To determine the energy consumption to drive the crusher, the EMF-1 (ELTIS Electric, Lviv, Ukraine) electronic wattmeter was used [20]. The operating principle and operating rules for thos devices are described in the technical documentation. To record the amplitude-frequency characteristics of the vibratory disk crusher, a sensor based on the ST Microelectronics LIS3DH accelerometer was developed [8, 27].

Taking into account the permissible errors of the measuring equipment [28], a critical value of the discrepancy between the experimental and theoretical research was taken by 15% [20]. Exceeding this boundary indicates the unreliability of the mathematical model and the inability to use it when designing a crusher of this type. Processing and analysis of the research results were carried out in the Microsoft Excel 2019 software environment.

Research results

To determine the linear velocity of the centres of mass of structural elements of oscillatory system we will divide the mechanism into elementary components are links and we will study them separately [15, 29]. Velocity (V_1, V_2, V_3, V_4) for the generalized mass S_1 (container),), S_2 (rotor), S_3 (disc), S_4 (counterweight) (Fig. 2), m·s^{-1}:

(3) $$V_1 = \sqrt{\dot{x}_1^2 + \dot{z}_1^2};$$

(4) $$V_2 = \sqrt{v_{r2}^2 + v_{e2}^2 + 2v_{r2}v_{e2}\cos\psi_1};$$

(5) $$V_3 = \sqrt{v_{r3}^2 + v_{e3}^2 + 2v_{r3}v_{e3}\cos\psi_3};$$

(6) $$V_4 = \sqrt{v_{r4}^2 + v_{e4}^2 + 2v_{r4}v_{e4}\cos\psi_4};$$

where $\dot{x}_1, \dot{z}_1$ – the velocity S_1 along with the axis *OX* and *OZ*, m·s^{-1}; v_{r2}, v_{r3}, v_{r4} – respectively the relative velocity of S_2 (the moving coordinate system – $x_1y_1z_1$), S_3 (the moving coordinate system – $x_2y_2z_2$) and S_4 (the moving coordinate system – $x_1y_1z_1$), m·s^{-1}; v_{e2}, v_{e3}, v_{e4},– respectively the frame velocity of S_2 and coordinate system – $x_1y_1z_1$ (the fixed coordinate system – XYZ), S_3 and coordinate system – $x_2y_2z_2$ (the fixed coordinate system – $x_1y_1z_1$), S_4 and coordinate system – $x_1y_1z_1$ (the fixed coordinate system – XYZ), m·s^{-1}; ψ_1, ψ_3, ψ_4 – respectively the angle between the vectors $\vec{v}_{r2}$ and $\vec{v}_{e2}$, $\vec{v}_{r3}$ and $\vec{v}_{e3}$, $\vec{v}_{r4}$ and $\vec{v}_{e4}$, rad.

(7) $$v_{r2} = e\cdot\dot{\varphi}_2,\ v_{e2} = \sqrt{x_2^2 + z_2^2}\cdot\dot{\varphi}_2 \to \sqrt{x_1^2 + z_1^2}\cdot\dot{\varphi}_2;$$

(8) $$v_{r3} = r_o\cdot\dot{\varphi}_3\cdot ku,\ v_{e3} = \sqrt{v_{ry1}^2 + v_{eXZ}^2 + 2v_{ry1}v_{eXZ}\cos\psi_2}$$

(9) $$v_{r4} = l\cdot\dot{\varphi}_2,\ v_{e4} = \sqrt{x_4^2 + z_4^2}\cdot\dot{\varphi}_2 \to \sqrt{x_1^2 + z_1^2}\cdot\dot{\varphi}_2,$$

where: *e* – distance from S_2 to the axis 0_1y_1, m; $\dot{\varphi}_2$ – the angular velocity of the rotor, rad·s^{-1}; x_2, z_2 – displacement of the S_2 relative to fixed axes OX and OZ, m; r_d– the radius crusher disc, m; $\dot{\varphi}_3$ – the angular velocity crusher disc, rad·s^{-1}; v_{ry1} – the relative velocity of cutting edge (the moving coordinate system – $x_1y_1z_1$), m·s^{-1}; v_{eXZ} – the frame velocity of the cutting edge and coordinate system – $x_2y_2z_2$ (the fixed coordinate system – $x_1y_1z_1$), m·s^{-1}; ψ_2 – the angle between the vectors $\vec{v}_{eXZ}$ and $\vec{v}_{ry1}$, rad; *l* – distance from S_4 to the axis 0_1y_1, m; x_4, z_4 – displacement of the S_4 relative to fixed axes OX and OZ, m; ku – the transfer coefficient of torque ($ku = 0 \ldots 1$).

When rotating the working rotor equipment between the edge crusher disc and material, the friction forces F_{fm}, the rotation is working disk is only possible where $F_{fm} > F_{fa}$. F_{fa} is the friction forces in conjunction with friction «crusher disk – disk axis» [12]. If there is $ku \to 1$ an increase F_{fm}, whereas at $ku = 0$, $F_{fm} = 0$, and as a result $\dot{\varphi}_3$=0 [12, 24]. If $ku > 0$, when

(10) $$\dot{\varphi}_3 = \frac{\dot{\varphi}_2\cdot r_{sup}}{r_d\cdot ku};$$

(11) $$v_{ry1} = r\cdot\dot{\varphi}_2;$$

(12) $$v_{eXZ} = \sqrt{x_3^2 + x_3^2}\cdot\dot{\varphi}_2 \to \sqrt{x_1^2 + z_1^2}\cdot\dot{\varphi}_2.$$

where: r_{sup} – the radius of the support disk, m; *r* – the distance from the edge of the crusher disc to the axis 0_1y_1, m; x_3, z_3 – displacement of the S_3 relative to fixed axes OX and OZ, m.

Equation (3-6) for velocites $V_{1\text{-}4}$, takes the form:

(13) $$V_1 = \sqrt{\dot{x}_1^2 + \dot{z}_1^2};$$

(14) $$V_2 = \sqrt{(e\cdot\dot{\varphi}_2)^2 + (x_1^2 + z_1^2)\dot{\varphi}_2 + 2\cdot e\cdot\dot{\varphi}_2\cdot x_1};$$

(15) $$V_3 = \sqrt{\begin{aligned}&(r_d\cdot\dot{\varphi}_3\cdot ku)^2 + (r\cdot\dot{\varphi}_2)^2 + (x_1^2 + z_1^2)\cdot\dot{\varphi}_2 + \\ &+2(r\cdot\dot{\varphi}_2)x_1 - 2(r_d\cdot\dot{\varphi}_3\cdot ku)(2x_1 + r\cdot\dot{\varphi}_2)\end{aligned}}$$

(16) $$V_4 = \sqrt{(l\cdot\dot{\varphi}_2)^2 + (x_1^2 + z_1^2)\cdot\dot{\varphi}_2 + 2\cdot l\cdot\dot{\varphi}_2\cdot x_1}.$$

Total kinetic energy of the system:

(17) $$\begin{aligned}\mathrm{T} = &\frac{1}{2}m_1[\dot{x}_1^2 + \dot{z}_1^2] + \frac{1}{2}m_2[(e\cdot\dot{\varphi}_2)^2 + (x_1^2 + z_1^2)\cdot\dot{\varphi}_2 \\ &+2\cdot e\cdot\dot{\varphi}_2\cdot x_1] + +\frac{1}{2}m_3[(r_d\cdot\dot{\varphi}_3\cdot ku)^2 + (r\cdot\dot{\varphi}_2)^2 + \\ &(x_1^2 + z_1^2)\cdot\dot{\varphi}_2 + 2(r\cdot\dot{\varphi}_2)x_1 - 2(r_d\cdot\dot{\varphi}_3\cdot ku)\times\end{aligned}$$

$$\times (2x_1 + r \cdot \dot{\varphi}_2)] + \frac{1}{2} m_4 [(l \cdot \dot{\varphi}_2)^2 + (x_1^2 + z_1^2) \cdot \dot{\varphi}_2 + 2 \cdot l \cdot \dot{\varphi}_2 \cdot x_1] + \frac{1}{2}[(I_2 \dot{\varphi}_2^2) + (I_3 \dot{\varphi}_3^2 ku) + (I_3 \dot{\varphi}_2^2) + (I_4 \dot{\varphi}_2^2)].$$

The generalized force can be interpreted as a coefficient before the variation of the generalized coordinate in the expression for the sum of the elementary works of all active forces.

Using the calculation scheme in Fig. 1b generalized forces can be identified:

$$\begin{cases} Qx = (m_2 + m_3)\omega_2{}^2 e \cos(\omega_2 \cdot t) - m_4 \omega_2{}^2 l \times \cos(\omega_2 \cdot t) - c_x x \\ Qz = \begin{bmatrix} (m_2 + m_3)\omega_2{}^2 e \cdot \sin(\omega_2 \cdot t) - m_4 \omega_2{}^2 l \times \\ \times \sin(\omega_2 \cdot t) - (m_1 + m_2 + m_3 + m_4) g - c_z z \end{bmatrix} \\ Q\dot{\varphi}_2 = \begin{bmatrix} \mathrm{M}_{\text{кр}1} + m_2 \omega_2{}^2 e^2 \cdot \sin(\omega_2 \cdot t) - m_4 \omega_2{}^2 l^2 \times \\ \times \sin(\omega_2 \cdot t) - \mathrm{M}_{\text{оп}1} \end{bmatrix} \\ Q\dot{\varphi}_3 = (\mathrm{M}_{\text{кр}2} - \mathrm{M}_{\text{оп}2}) \cdot ku \end{cases} \quad (18)$$

where c_x,c_z – the stiffness of the elastic elements along the respective axes.

Using the MathCad 15.0 software environment, first and second parts for expressions of equation (1) were solved analytically. Therefore, expressions of equation (1) will take the following form:

$$\ddot{x} + \alpha_x \cdot \dot{x} - x \left[\frac{\dot{\varphi}_2 (m_2 + m_3 + m_4) + c_x}{m_1} \right] = \cos(\omega_2 \cdot t) \times \left(\frac{(m_2 + m_3)\omega_2{}^2 e - m_4 \omega_2{}^2 l}{m_1} \right) + \frac{m_2 \cdot \dot{\varphi}_2^2 \cdot e}{m_1} + \frac{m_3 \cdot r \cdot \dot{\varphi}_2^2 - m_3 \cdot 2ku \cdot r_d \cdot \dot{\varphi}_3^2 + m_4 \cdot \dot{\varphi}_2^2 \cdot l}{m_1}; \quad (19)$$

$$\ddot{z} + \alpha_z \cdot \dot{z} - z \cdot \left[\frac{\dot{\varphi}_2 (m_2 + m_3 + m_3) + c_z}{m_1} \right] = \frac{(m_2 + m_3)\omega_2{}^2 e - m_4 \omega_2{}^2 l}{m_1} \times \sin(\omega_2 \cdot t) - \frac{(m_1 + m_2 + m_3 + m_4) g}{m_1}; \quad (20)$$

$$\frac{m_3 (2rx - 2\ddot{\varphi}_2 r^2 - 2ku \cdot r_d \dot{\varphi}_3 \cdot r + x^2 + z^2)}{2} + \frac{m_2 (x^2 + 2ex + z^2 + 2\ddot{\varphi}_2 e^2)}{2} + \frac{m_4 (2\ddot{\varphi}_2 l^2 + 2lx + x^2 + z^2)}{2} + \ddot{\varphi}_2 (I_2 + I_3 + I_4) = \mathrm{M}_{\text{кр}1} + m_2 \omega_2{}^2 e^2 \cdot \sin(\omega_2 \cdot t) - m_4 \omega_2{}^2 l^2 \sin(\omega_2 \cdot t) - \mathrm{M}_{\text{оп}1}; \quad (21)$$

$$ku \cdot r_d \cdot m_3 \big(ku \cdot r_d \cdot \ddot{\varphi}_3 - (2x + r \cdot \dot{\varphi}_2) \big) + I_3 \cdot ku \cdot \ddot{\varphi}_3 = (\mathrm{M}_{\text{кр}2} - \mathrm{M}_{\text{оп}2}) \cdot ku. \quad (22)$$

To establish the laws of motion for the axes OX and OZ, it is sufficient to use only the first two equations (19, 20), which after mathematical transformations will take the form:

$$\alpha_x \cdot \dot{x} - x \left[\frac{\dot{\varphi}_2 (m_2 + m_3 + m_4) + c_x}{m_1} \right] = \cos(\omega_2 \cdot t) \times \left[\left(\frac{(m_2 + m_3)\dot{\varphi}_2{}^2 e - m_4 \dot{\varphi}_2{}^2 l}{m_1} \right) + \frac{m_2 \cdot \dot{\varphi}_2^2 \cdot e + m_3 \cdot r \cdot \dot{\varphi}_2^2 - m_3 \cdot 2ku \cdot r_d \cdot \dot{\varphi}_3^2 + m_4 \cdot \dot{\varphi}_2^2 \cdot l}{\cos(\omega_2 \cdot t) m_1} \right]; \quad (23)$$

$$\ddot{z} + \alpha_z \cdot \dot{z} - z \cdot \left[\frac{\dot{\varphi}_2 (m_2 + m_3 + m_4) + c_z}{m_1} \right] = \sin(\omega_2 \cdot t) \times \left[\frac{(m_2 + m_3)\omega_2{}^2 e - m_4 \omega_2{}^2 l}{m_1} - \frac{(m_1 + m_2 + m_3 + m_4) g}{\sin(\omega_2 \cdot t) \cdot m_1} \right]. \quad (24)$$

The solutions of equations (23) and (24) will be found for both second-order linear differential equations with constant coefficients, assuming that $\dot{\varphi}_2 = \omega_2$. The dissipation coefficients of this system can be represented as [17]:

$$\alpha_x = 2\sqrt{k_x^2 - \omega_2^2}; \; \alpha_z = 2\sqrt{k_z^2 - \omega_2^2}. \quad (25)$$

Specific modulus of forcing force (Lanets *et al.*, 2019):

$$F_{mx} = \left(\frac{(m_2 + m_3)\omega_2{}^2 e - m_4 \omega_2{}^2 l}{m_1} \right) + \frac{m_2 \cdot \omega_2^2 \cdot e + m_3 \cdot r \cdot \omega_2^2 - m_3 \cdot 2ku \cdot r_d \cdot \omega_3^2 + m_4 \cdot \omega_2^2 \cdot l}{\cos(\omega_2 \cdot t) m_1}; \quad (26)$$

$$F_{mz} = \frac{(m_2 + m_3)\omega_2{}^2 e - m_4 \omega_2{}^2 l}{m_1} - \frac{(m_1 + m_2 + m_3 + m_4) g}{\sin(\omega_2 \cdot t) \cdot m_1}. \quad (27)$$

The natural frequency of oscillations of the system relative to the axis OX and OZ [24]:

$$k_x^2 = \frac{\omega_2 (m_2 + m_3 + m_4) + c_x}{m_1}; \quad (28)$$

$$k_z^2 = \frac{\omega_2 (m_2 + m_3 + m_4) + c_z}{m_1}. \quad (29)$$

By solving the obtained equations as linear differential equations of the second order with constant coefficients, the dependences of the motion of the executive body of the studied machine are obtained. Due to the scattering of energy in the system under study, the free oscillations are damped, as a result of which the obtained equations for the steady-state will take the form [17, 23]:

$$x = \frac{F_{mx} \alpha_x \omega_2}{(k_x^2 - \omega_2^2)^2 + \alpha_x^2 \omega_2^2} \sin(\omega_2 t) + \frac{F_{mx} (\omega_2^2 - k_x^2)}{(k_x^2 - \omega_2^2)^2 + \alpha_x^2 \omega_2^2} \cos(\omega_2 t); \quad (30)$$

$$z = \frac{F_{mz} (k_z^2 - \omega_2^2) \sin \omega_2 t}{(k_z^2 - \omega_2^2)^2 + \alpha_z^2 \omega_2^2} + \frac{F_{mz} \alpha_z \omega_2 \cos \omega_2 t}{(k_z^2 - \omega_2^2)^2 + \alpha_z^2 \omega_2^2}. \quad (31)$$

The amplitude of oscillations about the axis OX and OZ has the form:

$$A_x = \frac{F_{mx}}{\sqrt{(k_x^2 - \omega_2^2)^2 + \alpha_x^2 \omega_2^2}}; \quad (32)$$

$$A_z = \frac{F_{mz}}{\sqrt{(k_z^2 - \omega_2^2)^2 + \alpha_z^2 \omega_2^2}}. \quad (33)$$

The absolute amplitude of oscillations:

$$A = \sqrt{A_x^2 + A_z^2}. \quad (34)$$

Taking into account the equation (32) and (33):

$$A = \sqrt{ \left(\frac{ \left(\frac{(m_2 + m_3)\omega_2{}^2 e - m_4 \omega_2{}^2 l}{m_1} \right) + \frac{m_2 \cdot \omega_2^2 \cdot e + m_3 \cdot r \cdot \omega_2^2 - m_3 \cdot 2ku \cdot r_d \cdot \omega_3^2 + m_4 \cdot \omega_2^2 \cdot l}{\cos(\omega_2 \cdot t) m_1} }{ \sqrt{ \left(\frac{\omega_2 (m_2 + m_3 + m_4) + c_x}{m_1} - \omega_2^2 \right)^2 + \left(2\sqrt{\frac{\omega_2 (m_2 + m_3 + m_4) + c_x}{m_1} - \omega_2^2} \right)^2 \cdot \omega_2^2 } } \right)^2 + \left(\frac{ \frac{(m_2 + m_3)\omega_2{}^2 e - m_4 \omega_2{}^2 l}{m_1} - \frac{(m_1 + m_2 + m_3 + m_4) g}{\sin(\omega_2 \cdot t) \cdot m_1} }{ \sqrt{ \left(\frac{\omega_2 (m_2 + m_3 + m_4) + c_z}{m_1} - \omega_2^2 \right)^2 + \left(2\sqrt{\frac{\omega_2 (m_2 + m_3 + m_4) + c_z}{m_1} - \omega_2^2} \right)^2 \cdot \omega_2^2 } } \right)^2 } \quad (35)$$

Also, using equation (35), the parameters of the vibration field for the steady-state, which are proportional to the amplitude and frequency of oscillations, namely: vibration velocity *υ = A · ω*; vibration acceleration ***a*** *= A · ω*^{2}; oscillation intensity *I* = ***a*** *· υ = A*^{2} *· ω*^{3} [15].

The power of the drive of the studied machine [19]:

$$N_{pd} = (N_{Fmax} + N_{fr}) / \gamma_{dr}, \quad (36)$$

where: N_{Fmax} – maximum power developed by the forcing force to generate the required oscillations; N_{fr} – power consumption for friction in the support nodes; γ_{dr} – efficiency of the drive.

Power developed by the forcing force:

(37) $$N_F = F_m \cdot \upsilon,$$

Power consumption for friction [20]:

(38) $$N_{fr} = 0{,}5 \cdot F \cdot \mu \cdot d_{sh} \cdot \omega_2{}^2,$$

where: $\mu = 0.05\ldots0.08$ – coefficient of friction; d_{sh} – the diameter of the drive shaft ($d_{sh} = 0.04\,m$).

For further analysis, we used the numerical values of constants and other experimental data obtained as a result of exploratory research on the basis in laboratory Vinnytsia National Agrarian University. The software algorithm that was created to analyze the amplitude-frequency ranges of the crusher and estimate the energy consumption of the drive is based on developed mathematical model, which is presented in the form of differential equations (17-38), which were entered into the working field of the software environment MathCad 15.0 [6]. Visualization of the obtained results is carried out in the form of an array of numerical values, three-dimensional graphs of amplitude-frequency and energy characteristics. In fig. 3 shows a fragment of the listing for automated determination of influence of design and technological parameters on the values amplitude of vibrations, vibration speed, vibration acceleration, vibration intensity and power consumption of the drive.

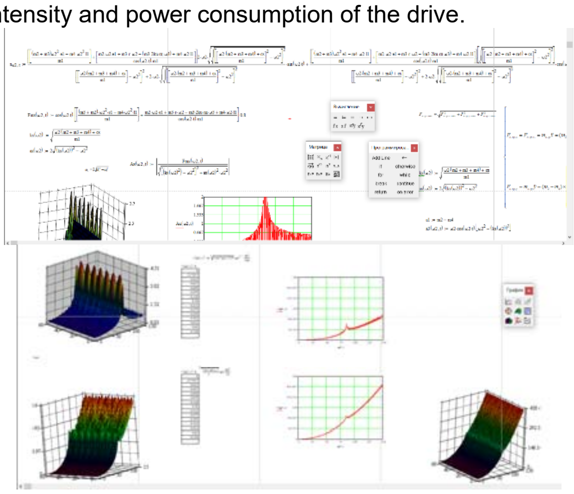

Fig. 3. Fragment of software algorithm for analyze the amplitude-frequency ranges of the vibratory crusher and visualization of the obtained results

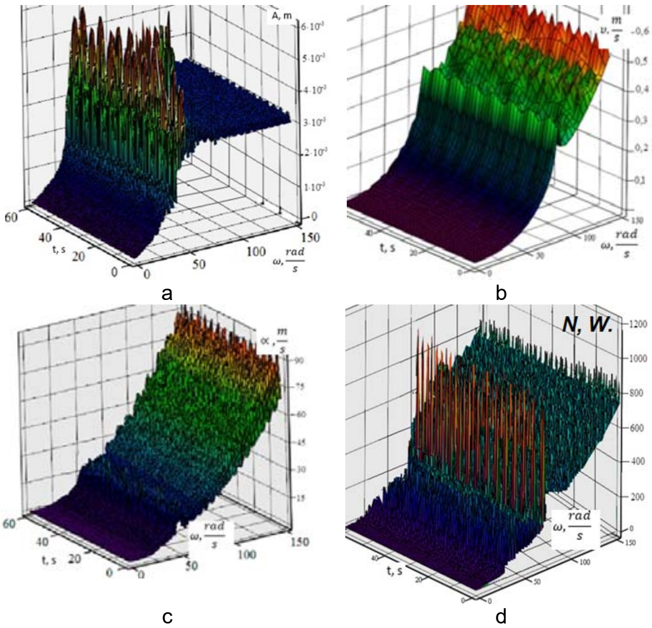

Fig. 4. Amplitude-frequency and energy characteristics of vibrating crusher: a) amplitude of oscillations; b) vibration speed; c) vibration acceleration; d) power consumption

As a result of mathematical analysis of compound equations of motion in the software environment MathCad 15.0, graphical dependencies for the main kinematic characteristics of the studied equipment are obtained (Fig. 4).

Theoretical analysis of the presented differential equations of motion of the executive equipment's of the developed vibrating disk crusher and graphical dependences (Fig. 4) are showed that during its operation without material supply, the resonant mode on the axis OZ to Az = 3.9 mm, on the axis OX-Ax = 2.25 mm at an angular velocity of the drive shaft 70 rad s^{-1}. As a result, the peak values of the total amplitude of oscillations are observed at 70 rad s^{-1} and are A = 4.5 mm (Fig. 4a).

Analyzing the graphical dependence of the vibration speed on the angular velocity of the drive shaft operating modes are observed at values up to 0.3 m s^{-1} at 71 rad s^{-1}, peak values are observed in resonant mode at 150 s^{-1} (Fig. 4b), and are 0.5 m s^{-1}. Considering the graphical dependence of vibration acceleration (Fig. 4c) on angular velocity, the maximum value of 82 m $s^{2\ -1}$ is observed at 150 s^{-1}.

Further analysis of the mathematical model in the use a wide range of analytics tools in the MathCad 15.0 software allowed to theoretically determine the operating frequency range of the vibrating disc crusher, in which the value of power consumption is close to the most rational values [6] and is *N* = 650… 750 W: *ω* = 100… 115 rad / s, *A* = 3 … 3.1 mm, *v* = 0.28 ... 0.31 m / s, *a* = 40 ... 43 m / s^2.

To verify the mathematical model, a series of experimental studies were performed and the real values of the amplitude-frequency characteristics of the developed equipment were established [20]. Experimental and theoretical graphs of the distribution of the main parameters of the studied system are presented (Fig. 5).

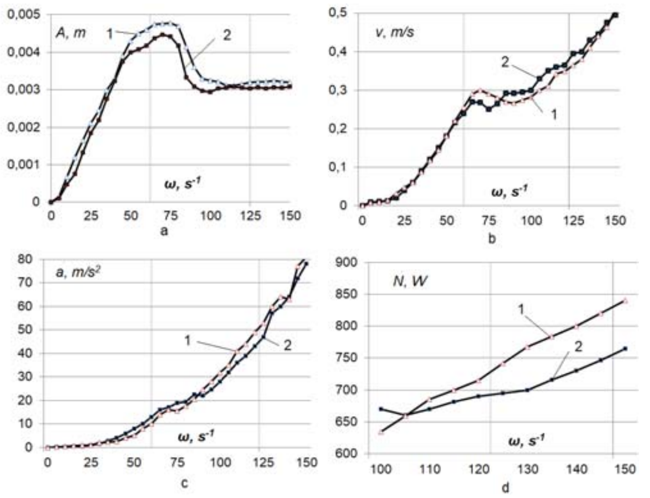

Fig. 5. Comparison of theoretical and experimental research results: a) amplitude of oscillations; b) vibration speed; c) vibration acceleration; d) power consumption; 1 – theoretical research; 2 – experimental research

Thus, it was found that the discrepancy between theoretical and experimental results is 7.2–12.1% and does not exceed the recommended value for vibratory crushers (up to 15%).

Conclusions

The result of the analysis of the proposed mathematical model of the vibrating crusher is the analytical and graphical dependences of the main kinematic parameters of oscillations. Recommended crusher operating parameters: ω=100…115 s^{-1}, *A*=3.1… 3.2 mm, *v* = 0.28 ... 0.31 m / s, a=40...43 m/s^2. The energy consumption for the crusher drive is N=650…750 W. The peak values of the total amplitude in the resonant mode are A_{max}=4.5 mm at ω=70 s^{-1}.

The comparative analysis of deviations of theoretical and experimental research on power and amplitude-frequency parameters of the developed equipment revealed a discrepancy of the received values within 7.2–12.1% that confirms the adequacy of the developed mathematical models. So, the proposed mathematical model with sufficient accuracy and reliability reflects the modes of oscillation of the machine and can be used when substantiating the parameters of the vibrating disc crusher.

Funding

This research was supported and funded by the Ministry of Education and Science of Ukraine under grant No 0121U108589.

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