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doi:10.15199/48.2023.01.22

A dead-beat internal model control for trajectory tracking in discrete- time linear system

***Abstract**. In this paper, a robust control law is applied. This command is called dead-beat internal model control. The application of this command on a discrete-time linear system presents good performance in precision, low overshoots and tracking of reference trajectories, which shows the effectiveness of the proposed command. The effect of the controller by dead-beat internal model that the error vanishes in a finite and minimal number of sampling periods and remains zero thereafter.*

***Streszczenie.** W niniejszym artykule zastosowano solidne prawo kontrolne. To polecenie nazywa się kontrolą martwego modelu wewnętrznego. Zastosowanie tego polecenia na liniowym systemie dyskretnym daje dobre wyniki w precyzji, małych przeregulowaniach i śledzeniu trajektorii odniesienia, co świadczy o skuteczności proponowanego polecenia. Efekt działania regulatora przez martwy model wewnętrzny polegający na tym, że błąd zanika w skończonej i minimalnej liczbie okresów próbkowania, a następnie pozostaje zerowy. (**Wewnętrzna kontrola modelu martwego uderzenia do śledzenia trajektorii w dyskretnym systemie liniowym**)*

Keywords: dead-beat response, internal model control, discrete-time linear systems, zero static error.
Słowa kluczowe: odpowiedź impulsu martwego, sterowanie modelem wewnętrznym, układy liniowe z czasem dyskretnym, y.

Introduction

When the classic PI and IP controllers do not make it possible to obtain the desired performance and when there is not enough computing power to implement a standard predictive regulation, the Internal Model Control or IMC (Internal Model Control), turns out to be an interesting approach [1, 2]. Indeed, this type of controller is robust, easy to adjust online and easy to maintain, i.e., to estimate with the process, because it contains an explicit model of the process. In addition, several system methodologies of this controller have been developed, which makes its design simple. The interest of the IMC approach is to show how a performance-robustness compromise can be made depending on the uncertainty of the model parameters [4, 5].

For complex systems, for which an equivalent classic controller does not exist, IMC synthesis provides a simple controller, offering good performance, and for which, again, the performance-robustness trade-off can be easily chosen. We can even imagine adjusting it online thanks to the design parameters [6].

The most interesting design step of this internal model control strategy is based on the choice of the controller. In general, the presence of a compensator is necessary when designing the controller which must, however, be physically feasible, stable and ensure certain performance [7, 8].

However, if the system is subject to disturbances, the difference between the output of the system and that of the model is non-zero, which makes it necessary to add a filter, called robustness, to ensure the stability of the system and improve its closed-loop robustness with respect to noise and modeling errors. To respect the closed-loop dynamic behavior of the system, a reference model at the setpoint level can be implemented according to the characteristics of a perfect controller [9].

The objective of internal model control for discrete systems is to design a digital controller such that the position error of the controlled system, controlled in discrete time, is strictly zero at any instant kT. Suppose the process contains an integrator (its continuous-time transfer function has a zero pole). Note first that if the position error and all its derivatives are zero at time k_0, it suffices that the command be zero at any time $k \geq k_0$: indeed, if the command is canceled at instant k_0, the integrator ensures that the output retains the constant value it has at instant k_0. We are therefore looking for a control sequence of maximum length k_0, such that the error in response to a setpoint step is zero for all $k \geq k_0$ [10].

To cancel the static error in finite time a combination between the dead-beat response method and the control by internal model will be realized in this paper. In fact, the dead-beat response method is part of the class of so-called "model" methods. The idea is to design a controller that responds in finite absolute time, while ensuring zero static error with respect to a particular input.

The adjustment of this system adds an additional specification, since such a system is said to be dead-beat response when the output stabilizes at the desired value without oscillation between the sampling instants, for a given input-type, in a finite number of samples, without overshoot.

This paper is organized as follows. Section II presents a generality on the internal model control strategy. Section III presents dead-beat internal model control. In section IV, an example is employed to illustrate the effectiveness of the proposed controller. Some conclusions are drawn in section V.

Internal Model Control

In the late 1970s and early 1980s, a control algorithm known as the internal model was developed. The development of this algorithm was intended to take advantage of open loop regulators. These advantages are the ease of the synthesis of the controller, the possibility of systematically considering the robustness, and the advantages in closed loop which are the possibility of obtaining zero error in steady state at setpoint steps or at disturbances of non-zero mean [1, 11].

Internal model control is based on knowledge of an assumed model of the process. The uncertainty of the model is directly considered. It is possible to compensate the performance of the control system by its robustness to process modifications or modeling errors.
Indeed, this type of controller is robust, easy to adjust and easy to maintain that's to say to evolve with the process [12]. The struc

Wewnętrzna kontrola modelu martwego uderzenia do śledzenia trajektorii w dyskretnym systemie liniowym ture of the internal model controller is given by Fig. 1.

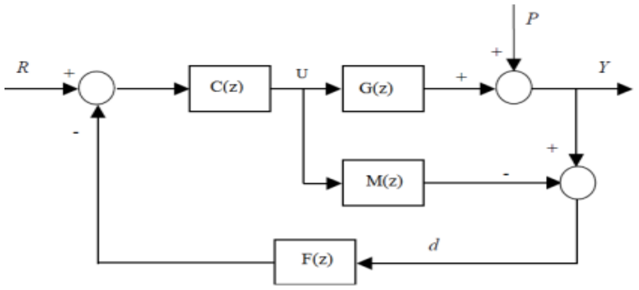

Fig.1. Structure of internal model control

In the internal model structure, the regulation part is composed of two parts: the controller C(z) and the model M(z).

In the internal model structure, the effect of the manipulated variables is subtracted from the output of the process. If we assume that the model is perfect and that the system is not subject to any disturbance, then the feedback signal, the difference between the output of the process and that of the model, is identically zero [6, 7].

The command *U* is defined as a function of the setpoint *R* and the disturbance by the following equation, knowing that $F(z)=1$

(1) $$U(z)=\frac{C(z)}{1+C(z)\big(G(z)-M(z)\big)}R(z)-\frac{C(z)}{1+C(z)\big(G(z)-M(z)\big)}P$$

The expression of the response *Y(z)* is described by the following equation:

(2) $$Y(z)=\frac{C(z)G(z)}{1+C(z)\big(G(z)-M(z)\big)}R(z)+\frac{1-C(z)M(z)}{1+C(z)\big(G(z)-M(z)\big)}P$$

In the case of the perfect model, this difference is equal to the disturbances. Under this perfect model assumption, the system is open-loop and therefore [11], [12]:

- The stability problems encountered in classic loops disappear. The looped system is stable if and only if the model M(z) and the internal model controller C(z) are stable.
- The role of the controller is therefore in a way to reverse the model. In other words, the internal model controller can be seen as an open loop feedforward controller. However, it does not have the disadvantages of a pure open loop because the feedback signal, which is equal to the process model deviation, that's to say to the disturbances, makes it possible to modify the setpoint in an adequate manner.

DEAD-BEAT internal model op control

Dead-beat internal model control consists in determining a controller C(z) such that the error vanishes in a finite and minimal number of sampling periods and remains zero thereafter. Such that, for a step setpoint, the output signal becomes equal to the setpoint as quickly as possible and without oscillations of the output between the sampling periods, Fig.2 [13].

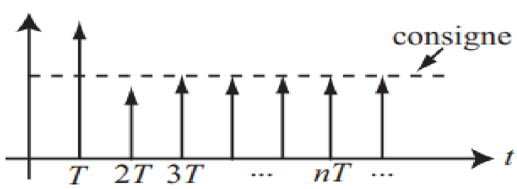

Fig.2. Evolution of system output in discrete time

To have a dead-beat response output, the control signal must be constant or zero after a certain number of sampling periods, Therefore, the control signal U(z) must be a finite polynomial in z [1], without integrator if G(z) contains one or with integrator if G(z) does not contain one [14, 15].

A discrete- time linear system controlled by a dead-beat internal model controller represented by Fig. 3:

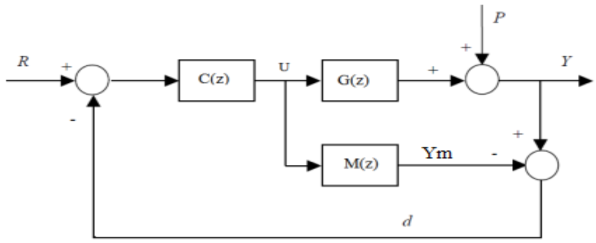

Fig.3. Control system by dead-beat response with IMC

Conditions for obtaining a dead-beat response

Condition 1:

The steady state error is zero $\varepsilon(\infty)=0$

This condition results in:

(3) $$\varepsilon(z)=R(z)-d(z)=R(z)-\big[Y(z)-Ym(z)\big]$$
$$=\big[1-H(z)\big]R(z)+Ym(z)$$

(4) $$\varepsilon(\infty)=\lim_{z\to1}\left(1-z^{-1}\right)\varepsilon(z)$$
$$=\lim_{z\to1}\left(1-z^{-1}\right)\left(\big[1-H(z)\big]R(z)+Ym(z)\right)$$

where H(z) is the closed loop transfer function without disturbance P(z)=0, is describe by the following equation [16].

(5) $$H(z)=\frac{Y(z)}{R(z)}=\frac{C(z)G(z)}{1+C(z)\big(G(z)-M(z)\big)}$$

We consider that the model is perfect, which gives us that M(z)=G(z), the transfer function in the closed loop becomes [16]:

(6) $$H(z)=\frac{S(z)}{R(z)}=C(z)G(z)$$

R(z) is the input is defined as follows:

(7) $$r(t)=t^m\rightarrow R(z)=\frac{T^mL(z)}{(z-1)^{m+1}}=\frac{T^mz^{-m-1}L(z)}{(1-z^{-1})^{m+1}}$$

(8) with L(z) : a polynomial such that $L(1)\neq0$

To get $\varepsilon(\infty)=0$ it's necessary:

(9) $$\big[1-H(z)\big]=\left(\left(1-z^{-1}\right)^{m+1}\right)Q(z)$$

Condition 2:

The transient error is limited to a finite number of error samples is of finite duration, it is necessary that Q(z) must be a polynomial, and for this number to be minimal, it is necessary that the degree of Q (z) is minimal, hence [17]:

(10) Q(z)=1

Thus the closed loop transfer function must have this form:

(11) $$H(z)=1-(1-z^{-1})^{m+1}$$

H(z) are called polynomials are called minimal prototypes. The inputs of the real systems in the industry are inputs of the types: position, velocity and acceleration. For these inputs the polynomials H(z) are given respectively by equations (12), (13) and (14).

(12) Position error is zero (m = 0) : $H(z) = z^{-1}$

(13) Velocity error is zero (m = 1) : $H(z) = 2z^{-1} - z^{-2}$

(14) Acceleration error is zero (m=2) : $H(z) = 3z^{-1} - 3z^{-2} + z^{-3}$

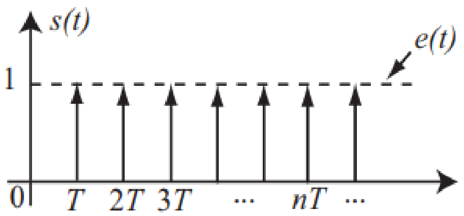

Fig.4. Evolution of system output in dead-beat response

The internal model controller is described by two equations equation (15) and equation (16). Equation (15) describes the first controller expression for perfect modeling if the model $M(z) = G(z)$.

The expression of the first controller is described by the following equation:

(15) $$C(z) = \frac{H(z)}{G(z)}$$

In most cases, we do not know exactly the mathematical modeling of the physical system that we are going to control. In this case, we will consider in the second expression of the controller by internal model that the modeling is imperfect $M(z) \neq G(z)$.

The controller equation is defined as follows:

(16) $$C(z) = \frac{F(z)}{G(z) - F(z)\left(G(z) - M(z)\right)}$$

In the case of an imperfect modeling, we will use the identification techniques of the systems. We propose a structure between its input and its output and to determine from the input-output couple, the values of the parameters of the model. The model thus found must, in its domain of validity, behave like (physical) reality or at least approach it as closely as possible.

Among the identification techniques, we can cite the methods of Strejc-Davoust, Broida and closed-loop identification.

Simulations Results

In this section, we will show the robustness of our proposed control law. We consider a linear discrete system and perform the necessary simulations to test the feasibility, efficiency and robustness of the dead-beat internal model control structure.

The study system is described by the following equation:

(17) $$G(z) = \frac{y(z)}{r(z)} = \frac{1}{2z + 0.5}$$

This system is stable in open loop, it admitted a pole $p_1 = -0.25$. The modulus of p_1 is less than 1 [18].

1st scenario: Perfect modeling

A perfect modeling of process dynamic behavior is considered in this simulation section, such that the model is identical to that of process (M(z)=G(z)). Simulations were made for this scenario knowing that the input is a unit step.

Fig. 5 shows the evolution of the output in a dotted line and its unit step reference in a solid line. We note that the output y(t) perfectly and quickly converges its setpoint. The steady-state static error is zero.

The control by internal model with dead-beat response made it possible to have a steady state without error in finite time for a step type input.

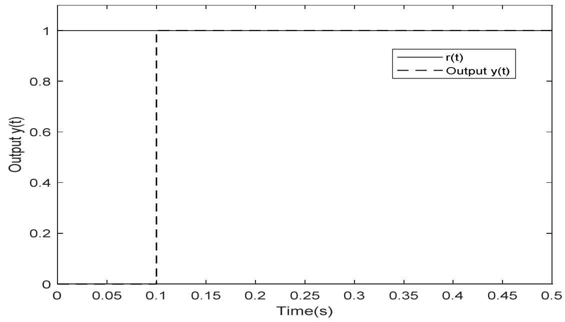

Fig.5. The step response of output $y_1(t)$

Fig. 6 presents the temporal evolution of the dead-beat internal model control u(t).

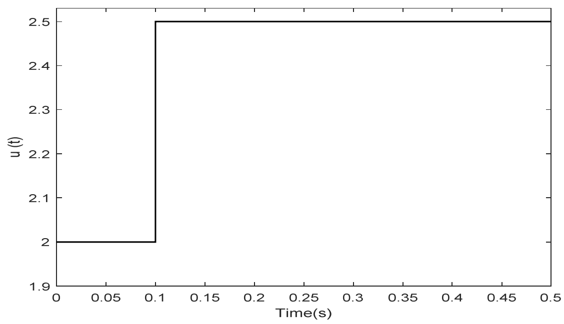

Fig.6. The control input u(t)

2nd scenario: Imperfect modeling

Modeling is imperfect if the model is unable to perfectly describe the dynamic behavior of the process such that $M(z) \neq G(z)$ [19].

The input of the system is a position step of amplitude 0.5.

The model of the internal model command structure is defined as follows:

(18) $$M(z) = \frac{2}{z}$$

Fig.7 presents the time evolution of the system response y(t) which is stabilized in zero period, which was predictable from the behavior model. There is no hidden oscillation since the control, too, stabilizes in zero period, Fig.8.

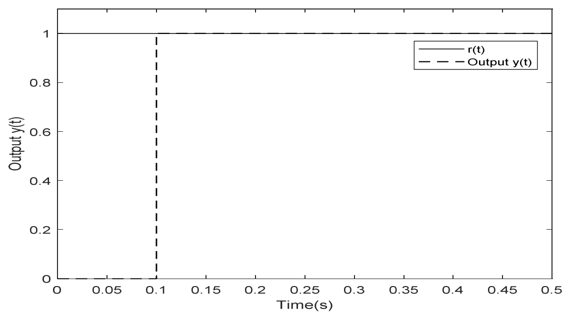

Fig.7: The step response of output $y_1(t)$

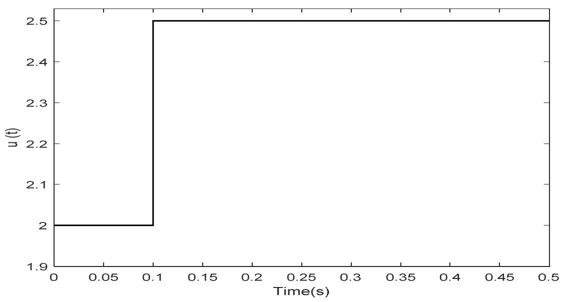

Fig.8: The control input u(t)

Fig. 9 presents the temporal evolution of the output y(t) in dotted line and its reference in solid line. The input considered in this simulation is a unit slope ramp type input. We notice that the output evolves without overshoot with a steady-state error is zero when input is unit ramp signal.

This applied control law makes it possible to follow the setpoint perfectly without permanent position or velocity errors.

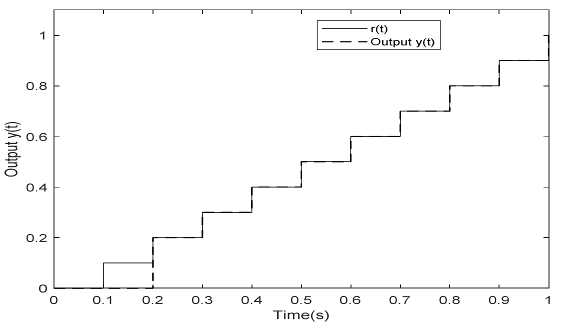

Fig.9: The control input u(t)

Conclusion

In this work, we applied the dead-beat internal model control on a discrete linear system. Two structures of controllers are proposed for our study system.

The simulation results show that the proposed control structure is robust. The output of the discrete system is stable and reaches its steady state for different types of canonical inputs in a finite number of samples without overshoot.

The results are satisfactory have proved the effectiveness and reliability of the proposed method.

Generally, this new method is simple, has robust performance and easy to implement in engineering processes.

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