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Identification of inclusions with LDA in the EIT

***Abstract**. The article presents a method using probes placed on one side, which were used to collect measurements in electrical tomography on the presence of inclusions in the object. Linear discriminant analysis was used for this purpose. The results of the linear discriminant analysis method are presented. The presented algorithm was used in the process of converting the electrical input values into conductance, which are represented by the pixels of the output image.*

***Streszczenie.** W artykule została zaprezentowana metoda wykorzystująca sondy umieszczone po jednej stronie, które posłużyły do zbierania pomiarów w tomografii elektrycznej na temat występowania wtrąceń w obiekcie. W tym celu została wykorzystana Liniowa analiza dyskryminacyjna. Przedstawiono wyniki badań nad metodą liniową analizą dyskryminacyjną. Przedstawiony algorytm został wykorzystany w procesie konwersji wejściowych wartości elektrycznych na konduktancję, które są reprezentowane poprzez piksele obrazu wyjściowego (**Identyfikacja inkluzji za pomocą LDA w EIT**).*

Keywords: EIT. LDA, identification of inclusions.
Słowa kluczowe: EIT, LDA, wykrywanie wtrąceń.

Introduction

The main purpose of discriminant analysis (pattern classification) is to find a classification rule [1], [2]. Mainly, the task is to determine membership in a certain class on the basis of observations of the independent variable. The decision on class membership is made based on knowledge of the distribution of the independent variable and the distribution of class apriori.

There are many methods for solving optimization tasks in inverse problems [3-11].To create the classification rule regarding whether the finite element belongs to inclusion or background for each finite element, we define the following objects: learning data set as $D=\{(x_i,y_i): x_i \in R^m, y_i \in \{0,1\}, 1 \le i \le n\}$ and probabilistic space as $(\Omega,\mathcal{F},P)$. Each component (x_i,y_i) of learning set D is created as follows: $x_i \in R^m$ denotes measurements obtained from sensors, but $y_i \in \{0,1\}$ denotes membership of finite element to background when $y_i=0$ or to inclusion when $y_i=1$.

The main aim of reconstruction is creating the classifier $f: R^m \to \{0,1\}$, where based on measurement from sensors, we can designate membership of finite element to inclusion or background. In the presented case, the LDA was used to define the classifier. Let Y be a random variable with discrete distribution Y: $\Omega \to \{0,1\}$ We assume that the conditional distribution $f_k(x)=P(X=x|Y=k)$ of the random variable X is the normal distribution $N(\mu_k,\Sigma_k)$ for $k \in \{0,1\}$. We define the apiori distribution of the random variable Y as

(1) $$\pi_k=\frac{n_k}{n}$$

where $n_k=\#\{i: y_i=k\}$ oraz $\pi_0+\pi_1=1$.

We construct the decision rule based on Bayes' theorem and

(2) $$P(Y=k|X=x)=\frac{f_k(x)\pi_k}{\sum_{j=0}^{1} f_j(x)\pi_j}$$

for $k \in \{0,1\}$.

When determining class membership based on equation (1), we compare the values of $f_k(x)\pi_k$, a larger value of the product for k means a higher probability that the random variable Y will take the value of k (i.e., observation x belongs to class k) [2], [12], [1].

For linear discriminant analysis, we assume that the covariance matrix of the random variable X for each group is identical, i.e. $\Sigma_0=\Sigma_k=\Sigma$.

The conditional distribution of the random variable X belonging for class k, $k \in \{0,1\}$ is given by the formula

(3) $$f_k(x)=P(X=x|Y=k)=\frac{1}{(2\pi)^{m/2}\sqrt{|\Sigma|}}exp\left(-\frac{1}{2}(x-\mu_k)^T\Sigma^{-1}(x-\mu_k)\right)$$

From equation (2), poring the probabilities of belonging to two different classes sufficiently analyze the logarithm of the quotient of these probabilities, ie.

(4)
$$\begin{aligned}\log\frac{P(Y=1|X=x)}{P(Y=0|X=x)} &= \log\frac{f_1(x)}{f_0(x)}+\log\frac{\pi_1}{\pi_0}\\ &= \log\frac{\pi_1}{\pi_0}-\frac{1}{2}(\mu_1+\mu_0)^T\Sigma^{-1}(\mu_1-\mu_0)+x^T\Sigma^{-1}(\mu_1-\mu_0)\\ &= \log\pi_1-\log\pi_0-\frac{1}{2}\mu_1^T\Sigma^{-1}\mu_1+\frac{1}{2}\mu_0^T\Sigma^{-1}\mu_0+x^T\Sigma^{-1}\mu_1-x^T\Sigma^{-1}\mu_0\end{aligned}$$

The function

(5) $$\delta_i(x)=\log\pi_i+x^T\Sigma^{-1}\mu_i-\frac{1}{2}\mu_i^T\Sigma^{-1}\mu_i$$

is called a linear discriminant function for a class $k \in \{0,1\}$.

We define the plane separating the two classes as $H=\{x \in \mathbb{R}^m: P(Y=1|X=x)=P(Y=0|X=x)\}$ and using that formula (4) and (5) we have

(6) $$H=\{x \in \mathbb{R}^m: \delta_1(x)=\delta_0(x)\}.$$

The plane H splits the entire transpose R^m into two separable sets, where the membership of the observed signal to a set is equivalent to the membership of the corresponding class.

In view of the above, we can present the decision rule in the form of

(7) $$\hat{Y}=\begin{cases}1, & \delta_1(x)>=\delta_0(x),\\ 0, & \delta_1(x)<\delta_0(x)\end{cases}=\underset{k\in\{0,1\}}{argmax}\,\delta_k(x).$$

As estimators of the unknown parameters of the distributions of observations for each class, we determine as follows:

expected values

(8) $$\hat{\mu}_k=\frac{1}{n_k}\sum_{i:y_i=k} x_i$$

for $k \in \{0,1\}$;

covariance matrix

(9) $$\hat{\Sigma} = \frac{1}{n-2}\sum_{k=0}^{1}\sum_{i:y_i=k}(x_i-\widehat{\mu_k})(x_i-\widehat{\mu_k})^T$$

In EIT, the predictors are highly correlated, so to overcome the problem of the singularity of Σ matrix, it is necessary to use regularization techniques [13-15].

2 ROC

We determine the probability of inclusion for each element based on the readout X. The probability of inclusion will occur when P(Y=1|X) according to equations (1)-(2) and assume

(10)
$$\text{Area membership} = \begin{cases} \text{inclusion}, & P(Y=1|X) >= l \\ \text{Background}, & P(Y=0|X) < l \end{cases}$$

for level l∈(0,1).

Basic terminology and coefficients describing the recognition of inclusions in the field of view. In the following discussion, we take the absence of a trumpet in the finite element location as the negative case (N), while the occurrence of inclusion is the positive case (P). For our considerations in the confusion matrix, we determine the values: TP (True Positive) - the number of finite elements for which inclusions were correctly recognized, TN (True Negative) - the number of finite elements for which the absence of inclusions was correctly recognized, FP (False Positive) - the number of finite elements without inclusions for which they were recognized as having inclusions (false alarm), FN (False Negative) - the number of finite elements with inclusions for which they were recognized as having no inclusions.

Table 1. Matrix of confusions

	Positive case	Negative case
Positive Prediction	TP	FP
Negative Prediction	FN	TN

The basic coefficients are determined as follows

$$Accuracy = \frac{TP+TN}{TP+TN+FP+FN},$$
$$TruePositiveRate = Sensivity = \frac{TP}{TP+FN},$$
$$Specificity = 1 - FalsePositiveRate = \frac{TN}{TN+FP},$$
$$PositivePredictiveValue = \frac{TP}{TP+FP},$$
$$NegativePredictiveValue = \frac{TN}{TN+FN},$$
$$Prevalence = \frac{TP+FN}{TP+TN+FP+FN},$$
$$DetectionRate = \frac{TP}{TP+TN+FP+FN},$$
$$DetectionPrevalence = \frac{TP+FP}{TP+TN+FP+FN},$$
$$BalancedAccuracy = \frac{Sensivity + Specificity}{2},$$
$$FalseAlarmRate = \frac{FP}{TP+FP}.$$

Analyzing the above, the Accuracy value is the portion of the viewing area that has been correctly recognized by the model. It is also one of the measures that directly shows the correctness of recognition.

In tomography for image reconstruction, it is necessary to describe the ability to find inclusions in the visual area. To determine the ability of a classifier based on the use of logistic regression ([16,17]), a curve describing the operational characteristics of the receiver (ROC curve) is determined. This curve shows the relationship between sensitivity and specificity during reconstruction. The diagonal line in the ROC plot describes the strategy based on guessing inclusions during reconstruction. If the curve is above the diagonal, the identification technique is clearly superior to guessing. The area under the curve in the literature is called the AUC (Area under ROC curve) and denotes a measure of predictability.

3 Examples

3.1 Field of view

Basic properties of the field of view:
- Number of electrodes: 8;
- Type of electrodes: linear;
- Number of nodes: 848;
- Number of finite elements: 1555.

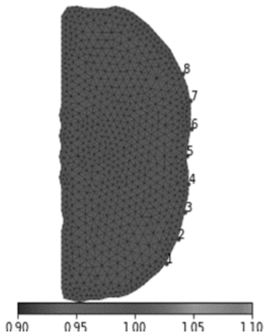

Fig.1. Area of reconstruction

3.2 Example reconstructions

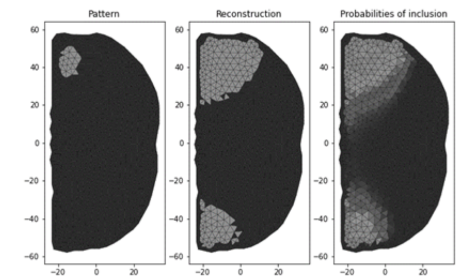

Fig.2. Example 1

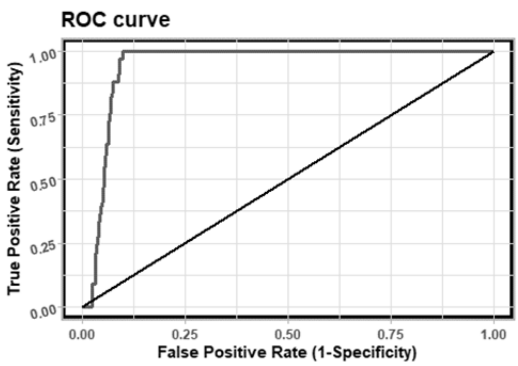

Fig.3. Analysis ROC for example 1

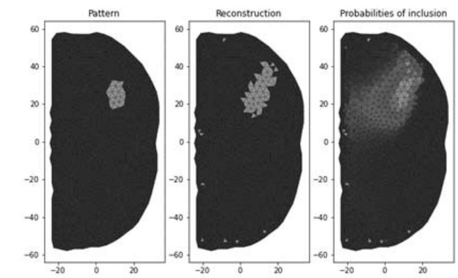

Fig.4. Example 2

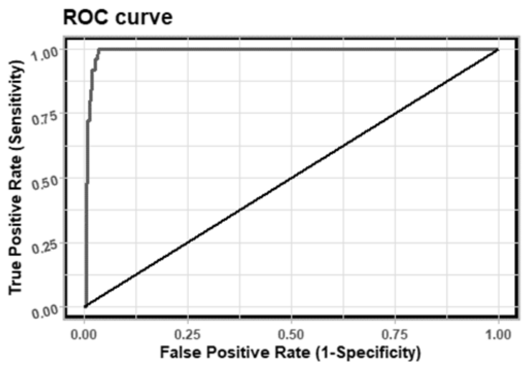

Fig.5. Analysis ROC for example 2

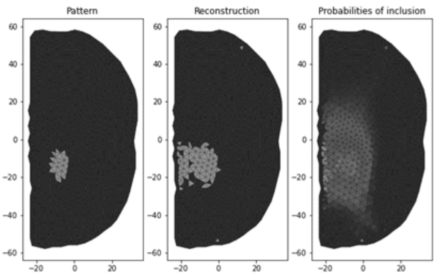

Fig.6. Example 3

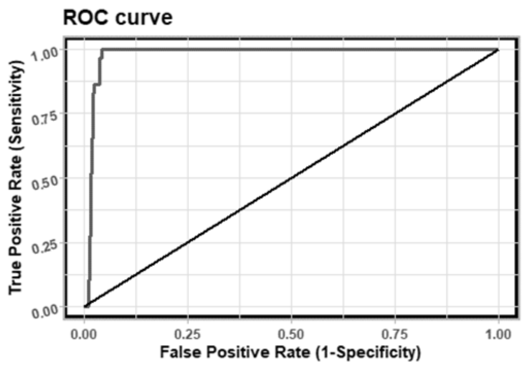

Fig.7. Analysis ROC for example 3

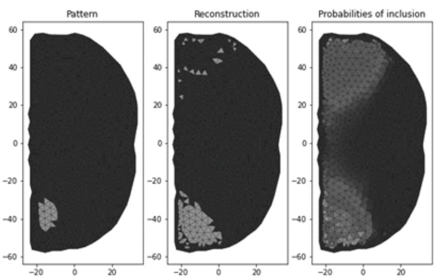

Fig.8. Example 4

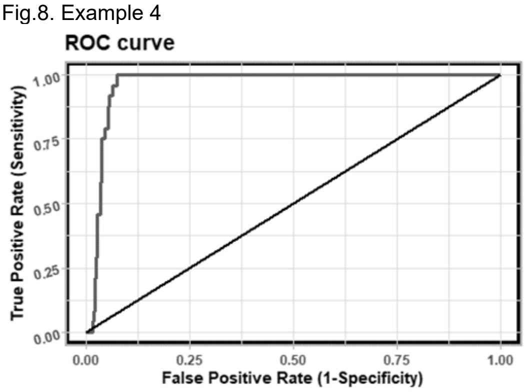

Fig.9. Analysis ROC for example 4

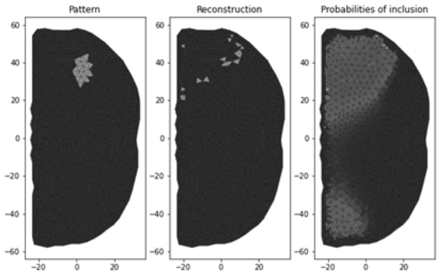

Fig.10. Example 5

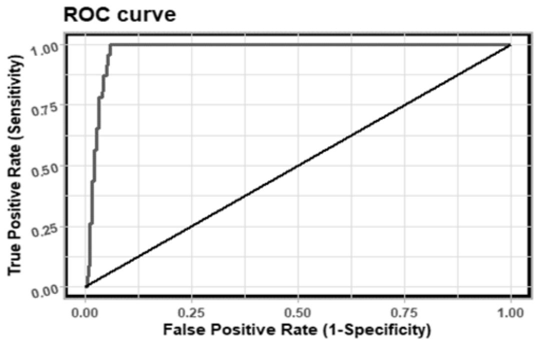

Fig.11. Analysis ROC for example 5

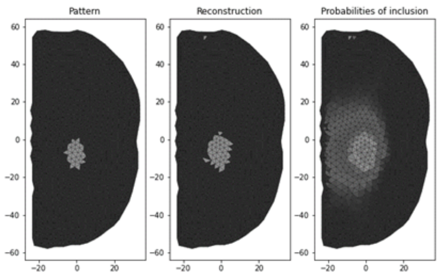

Fig.12. Example 6

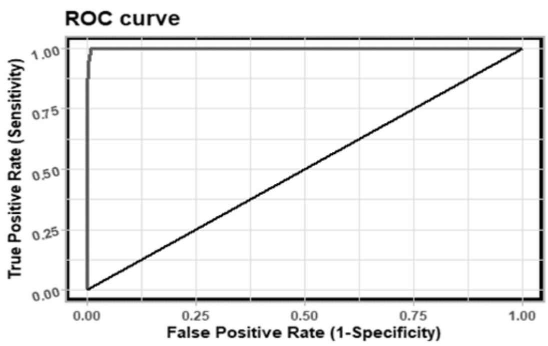

Fig.13. Analysis ROC for example 6

Table 2: Basic characteristics of the reconstruction of 3 first examples

	Example 1	Example 2	Example 3
Accuracy	0.832	0.970	0.950
Sensitivity	1.000	0.960	1.000
Specificity	0.829	0.971	0.950
Pos Pred Value	0.112	0.348	0.274
Neg Pred Value	1.000	0.999	1.000
Detection Rate	0.021	0.015	0.019
AUC	0.946	0.990	0.981
Kappa	0.170	0.499	0.412
chi-squared	259.004	40.196	75.013
p.val	0.000	0.000	0.000

Table 3: Basic characteristics of the reconstruction of 3 last examples

	Example 4	Example 5	Example 6
Accuracy	0.943	0.978	0.987
Sensitivity	0.875	0.130	1.000
Specificity	0.944	0.991	0.987
Pos Pred Value	0.198	0.176	0.667
Neg Pred Value	0.998	0.987	1.000
Detection Rate	0.014	0.002	0.026
AUC	0.965	0.975	0.999
Kappa	0.306	0.139	0.794
chi-squared	74.557	0.735	18.050
p.val	0.000	0.391	0.000

Summary

Analyzing the probability plot of the finite element membership of the inclusion area, it is impossible to determine the boundaries of the inclusion accurately. Electrodes are located on one side of the viewing area. If the inclusion is located in the lower part below electrode 1 and the upper part above electrode 8, then or from Figures 1 and 4 it is clear that the position of the inclusion cannot be accurately determined. -The signals from the electrodes are highly correlated. This causes problems in estimating the apiori distributions of the signals belonging to the corresponding classes.

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