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Mathematical modeling of starting modes and static characteristics of a wound-rotor induction motor in phase coordinates

***Abstract**. The algorithms for calculating steady-state modes, static characteristics, and transients in an asynchronous drive based on a wound-rotor induction motor and a starter in a rotor circuit are presented. The calculation is based on a mathematical model of an induction motor that takes into account the saturation of the magnetic circuit. The electromagnetic processes are described by a nonlinear system of equations of electrical equilibrium composed in a fixed three-phase coordinate system, the coefficients of which are the differential inductances of the circuits. The steady-state modes and static characteristics are calculated by solving the boundary value problem using the continuation method in the parameter.*

***Streszczenie.** Przedstawiono algorytmy obliczania trybów pracy w stanie ustalonym, charakterystyk statycznych i stanów nieustalonych w napędzie opartym na silniku indukcyjnym z wirnikiem fazowym i rozrusznikiem obwodu wirnikowego. Obliczenia oparte są na modelu matematycznym silnika indukcyjnego uwzględniającym nasycenie obwodu magnetycznego. Procesy elektromagnetyczne opisane są nieliniowym układem równań równowagi elektrycznej w ustalonym trójfazowym układzie współrzędnych, z indukcyjnościami różnicowymi obwodów jako współczynnikami. Tryby pracy w stanie ustalonym i charakterystyki statyczne oblicza się rozwiązując problem wartości granicznych metodą kontynuacji w parametrze. (**Modelowanie matematyczne trybów rozruchu i charakterystyk statycznych silnika indukcyjnego z uzwojonym wirnikiem we współrzędnych fazowych**)*

Keywords: mathematical modeling, induction motor, wound-rotor, phase coordinate system, transients, static characteristics.
Słowa kluczowe: modelowanie matematyczne, silnik indukcyjny, wirnik fazowy, układ współrzędnych fazowych, stany przejściowe, charakterystyki statyczne.

Introduction

Today, many electric drives are in operation, including mechanical handling equipment based on wound-rotor induction motors (WRIM) with a relay-contactor controller [1]. Since the modernization of crane equipment by switching to variable frequency drives requires significant material and time costs, it should be expected that they will operate until the full use of their operational life. In addition, in some industrial installations in recent decades, synchronized asynchronous electric drives have been used [2 - 6], based on WRIM, the rotor winding of which is connected to the starting rheostat through contact rings during startup, and after reaching a rotor speed close to synchronous speed, it is switched to power from a DC source, and the motor operates as a synchronous motor. Increasing the efficiency of such electric drives can be achieved by improving their operation control system, which has not yet been exhausted. For this purpose, it is necessary to have software tools that would allow analyzing any operating mode with high accuracy and speed [7-10].

The switching on of additional resistive resistances in the rotor circuit makes it possible to obtain the necessary starting characteristics, reduce starting currents, and also to carry out rheostat control of the rotor speed. Starting currents are mainly inductive in nature, but losses from them are significant [7], especially with frequent starts that occur in mechanical handling equipment [1]. The use of rheostats allows the heat generated by inrush currents to be removed from the motor and thus prevent overheating of the windings, but it is important to choose its parameters and the law of regulation so that the power loss is minimized in specific operating conditions.

In some electric drives, when starting the WRIM, an inductive reactance (reactor) is switched on in series or in parallel in addition to the resistance [8, 9]. In addition, special induction rheostats are also used [12]. The inductive element acts as an automatic rotor current regulator. At the initial moment of a startup, when the rotor current frequency is equal to the supply voltage frequency, the reactor inductive reactance is maximum and limits the value of the starting current, and as the rotor speed increases, the emf induced in the rotor winding decreases in proportion to the slip s and the resulting inductive reactance of the rotor circuit decreases simultaneously. As a result, the rotor current decreases more slowly than when the starting rheostat is switched on without a reactor. Reducing the inductive reactance of the reactor leads to an increase in the power factor, and, as a result, the electromagnetic torque under these conditions changes more slowly than at the beginning of acceleration [7].

By switching on inductive reactances in addition to resistances [2], the starting characteristics can be improved, but the choice of their parameters requires preliminary research on a mathematical model. Optimization of the starting and control characteristics will allow existing electric drives designed for rheostat control to use their resource before they are replaced by frequency-controlled ones.

The choice of the parameters of the starting device, the performance and energy efficiency of the electric drive, as well as the control of its operation, require research by mathematical modeling methods [2, 10, 12, 14]. Based on the above, the purpose of this paper is to develop a mathematical model of an electric drive system based on WRIM, which would make it possible to perform a wide range of studies of operating modes that may arise during the operation of a technological installation. Obviously, such a model cannot be based on calculation methods and algorithms developed using a mathematical model of a motor based on a known single-phase T-shaped substitute circuit [3, 8, 9] or a system of linear differential equations (DE). Mathematical models using electromagnetic field equations [19] require a significant amount of computer memory and computational time. The mathematical model of WRIM in the form of the DE of electromechanical equilibrium adequately reflects the processes of electromechanical energy conversion if the flux coupling and inductive parameters are determined taking into account the actual magnetization characteristics calculated on the basis of the geometry of the magnetic circuit, which makes it possible to adequately take into account saturation [6, 13, 16].

The electromagnetic processes in the WRIM transformed to the axes *x*, *y* taking into account the saturation of the magnetic circuit, were considered in [6], but the use of orthogonal coordinates is limited to symmetric modes, while the three-phase coordinate axis system allows us to consider asymmetric modes that may occur during the operation of the electric drive.

The complete study of electromechanical processes can be made by using mathematical models in the phase coordinate system [15]. Obviously, such a model should not only take into account all the main factors [15] that affect the motor characteristics but also have high performance.

By choosing the parameters of the rheostat elements and the optimal algorithm for their switching, it is possible to improve the technical and economic performance of the electric drive. An increase in the resistance of the rotor circuit due to an external rheostat leads to a decrease in the starting current of the motor and, at the same time, an increase in electromagnetic torque. As the rotor speed increases, the rheostat resistance is reduced (mostly discretely), and when the motor reaches the required speed, the rheostat resistance is completely removed, and the rheostat is shorted.

Transition processes

The object of study is a WRIM-based electric drive with three-phase windings on the stator and rotor. A three-phase voltage system is applied to the stator winding, and the rotor winding is connected to the starting device by means of slip rings, the phases of which can have an active or active-inductive load. Let us consider the problem of developing a mathematical model of a rheostat-controlled electric drive that allows us to calculate static and dynamic characteristics in order to select the parameters of the starter and analyze its operation in various operating modes. It should be noted that resistive and inductive elements connected in series or in parallel can be included in the rotor circuit [2]. In addition, asymmetric modes may occur due to different values of the phase parameters of the starter. Therefore, for a comprehensive study of the electric drive, it is necessary to develop mathematical models that allow us to adequately describe electromagnetic processes in the motor, taking into account the saturation of the magnetic circuit, as well as algorithms for calculating modes and characteristics, taking into account the law of change in load torque.

The system of equations of electrical equilibrium of the stator circuits (phases A, B, C) and rotor circuits (phases a, b, c), compiled according to Kirchhoff's laws for instantaneous values of currents and voltages of the WRIM, has the following form [19]

$$\begin{gathered}
\frac{d\psi_A}{dt}-\frac{d\psi_B}{dt}=-r_A i_A+r_B i_B+u_{ABm}\sin\left(\omega_0 t+\frac{\pi}{6}\right);\\
\frac{d\psi_B}{dt}-\frac{d\psi_C}{dt}=-r_B i_B+r_C i_C+u_{ABm}\sin\left(\omega_0 t-\frac{\pi}{2}\right);\\
i_A+i_B+i_C=0;\\
\frac{d\psi_a}{dt}-\frac{d\psi_b}{dt}=-r_a i_a+r_b i_b-\frac{\omega}{\sqrt{3}}\left(\psi_b-2\psi_c+\psi_a\right)-u_{ab};\\
i_a+i_b+i_c=0;
\end{gathered} \tag{1}$$

де U_{ABm}, U_{BCm} – amplitude values of the corresponding phase-to-phase supply voltages of the stator winding; $\psi_k, i_k,\ r_k\ (k=A,\ B,\ C,\ a,\ b,\ c)$ – flux linkage, currents, and resistances of the stator winding and transformed to the three-phase fixed coordinate system of the rotor winding; $\omega=\omega_0(1-s)$ – angular speed of rotor rotation; u_{ab}, u_{bc} – phase to phase voltages at the output terminals of the phase rotor winding, which in the case of the active-inductive nature of the starter are determined by the following formulas

$$u_{ab}=r_{pa}i_a-r_{pb}i_b+L_{pa}\frac{di_a}{dt}-L_{pb}\frac{di_b}{dt};$$

$$u_{bc}=r_{pb}i_b-r_{pc}i_c+L_{pb}\frac{di_b}{dt}-L_{pc}\frac{di_c}{dt}$$

r_p, L_p is the resistance and inductance of the starter device.

To calculate the transient process by numerical integration of the nonlinear system (2), it is advisable to reduce the finite equations written according to Kirchhoff's first law to differential equations. This makes it possible to reduce it to the DE system in the form

$$P\frac{d\vec{\psi}}{dt}=\vec{f}\left(\vec{\psi},\vec{i},\vec{u},t\right), \tag{2}$$

where

$$\vec{\psi}=\begin{bmatrix}\psi_A\\ \psi_B\\ \psi_C\\ \psi_a\\ \psi_b\\ \psi_c\end{bmatrix};\quad \vec{i}=\begin{bmatrix}i_A\\ i_B\\ i_C\\ i_a\\ i_b\\ i_c\end{bmatrix};\quad \vec{u}=\begin{bmatrix}u_{AB}\\ u_{BC}\\ 0\\ 0\\ 0\\ 0\end{bmatrix};\quad \vec{f}=\begin{bmatrix}f_1\\ f_2\\ f_3\\ f_4\\ f_5\\ f_6\end{bmatrix}$$

$$P=\begin{bmatrix}1&-1&0&0&0&0\\ 0&1&-1&0&0&0\\ 1&1&1&0&0&0\\ 0&0&0&1&-1&0\\ 0&0&0&0&1&-1\\ 0&0&0&1&1&1\end{bmatrix}$$

$\vec{f}$ is a vector whose elements are the functions of the right-hand sides of the equations (1).

Since the flux linkages are determined by the set of currents of all WRIM circuits, which in the dynamic mode depend on time, then $\frac{d\vec{\psi}}{dt}=\frac{\partial\vec{\psi}}{\partial\vec{i}}\frac{d\vec{i}}{dt}$;

$$\frac{\partial\vec{\psi}}{\partial\vec{i}}=\begin{bmatrix}L_{AA}&L_{AB}&L_{AC}&L_{Aa}&L_{Ab}&L_{Ac}\\ L_{BA}&L_{BB}&L_{BC}&L_{Ba}&L_{Bb}&L_{Bc}\\ L_{CA}&L_{CB}&L_{CC}&L_{Ca}&L_{Cb}&L_{Cc}\\ L_{aA}&L_{aB}&L_{aC}&L_{aa}&L_{ab}&L_{ac}\\ L_{bA}&L_{bB}&L_{bC}&L_{ba}&L_{bb}&L_{bc}\\ L_{cA}&L_{cB}&L_{cC}&L_{ca}&L_{cb}&L_{cc}\end{bmatrix}$$

– is a matrix, the elements of which are the differential inductances $L_{j,k}$ of the WRIM circuits (own at $j=k$ and mutual at $j\neq k$), for the determination of which the characteristics of magnetization of the magnetic circuit by the main magnetic flux $\psi_\mu = \psi_\mu(i_\mu)$, stator $\psi_{\sigma 1} = \psi_{\sigma 1}(i_1)$, and rotor leakage fluxes $\psi_{\sigma 2} = \psi_{\sigma 2}(i_2)$ [4] are used in accordance with the description in [19],

where $i_1 = \sqrt{\frac{2}{3}\left(i_A^2 + i_B^2 + i_C^2\right)}$; $i_2 = \sqrt{\frac{2}{3}\left(i_a^2 + i_b^2 + i_c^2\right)}$;

$i_\mu = \sqrt{\frac{2}{3}\left(i_{\mu A}^2 + i_{\mu B}^2 + i_{\mu C}^2\right)}$ – modules of spatial vectors of current.

To calculate the transient, it is necessary to supplement the DE system (1) with the rotor dynamics equation

$$\frac{d\omega}{dt} = \frac{p_0}{J}\left(M_e - M_c(t)\right), \tag{3}$$

where the electromagnetic torque of the motor is determined by the formula [17]

$$M_e = \frac{p_0}{\sqrt{3}}\left(\psi_{\mu A}\left(i_B - i_C\right) + \psi_{\mu B}\left(i_C - i_A\right) + \psi_{\mu C}\left(i_A - i_B\right)\right);$$

p_0 – a number of pole pairs; J – a reduced moment of inertia of the electric drive system; $\psi_{\mu\eta}$ – projections of the working flux linkage vector on the phase axis $(\eta = A, B, C)$.

The DE system (1)-(3) makes it possible to calculate any dynamic mode for a given load torque law $M_C(t)$. The flux linkage of each circuit is considered as the sum of the working and leakage fluxes linkages, so the corresponding differential inductances of the matrix also consist of the sum of the two inductances. If the starting rheostat is connected in series with an inductive reactance, its parameters are added according to the rotor winding resistances and inductive leaking reactances.

An example of the results of calculating the process of starting up the ID (P=250 kW, U = 380 V, I = 263 A, p = 3) with the rated load is shown in Fig. 1

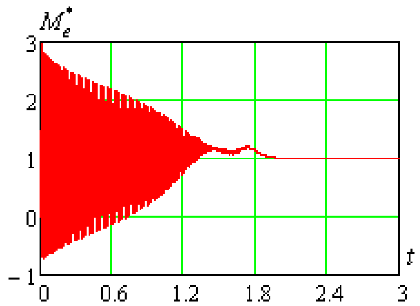
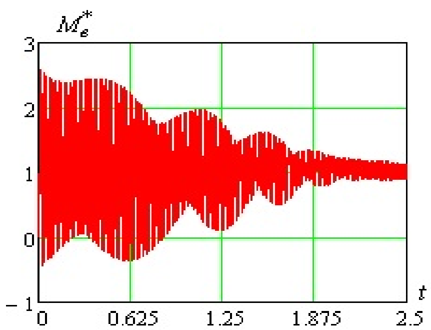
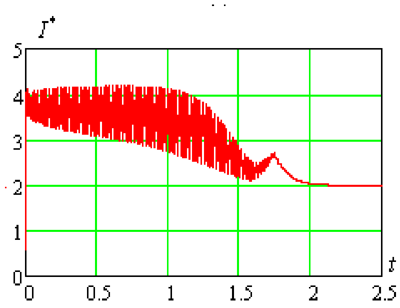


Fig. 1. Time dependence of the electromagnetic torque of the WRIM during starting with symmetry of the starting rheostat resistances in the phase rotor circuit



a)



b)

Fig. 2. Time dependence of the electromagnetic torque of the WRIM (a) and the stator current (b) during starting in the case of asymmetry of the resistances of the starting rheostat in the rotor circuit.

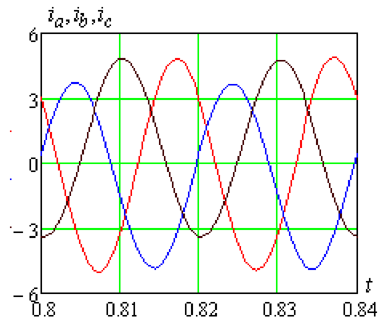


Fig. 3. Fragment of the transient process during startup in case of asymmetry of the starting rheostat resistances in the phase rotor circuit

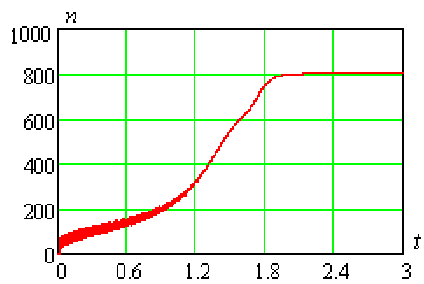


Fig. 4. Time dependence of the rotor rotation speed of the WRIM during startup in the case of asymmetry of the starting rheostat resistances in the rotor circuit

As can be seen from Fig. 4, in the case of asymmetry of the starting rheostat phase resistances, the motor did not reach the nominal speed of rotation of the rotor.

Boundary value problem and static characteristics

To select the value of any parameter of the starting device, it is necessary to determine the dependence of the rest of the coordinates, and above all, the electromagnetic torque, on this parameter for each slip, i.e., to obtain a family of static characteristics [16]. To calculate them, it is necessary to remove the time coordinate from equations (1) by moving from a continuous change in coordinates to their discrete values.

Under the condition of a constant speed ω of the rotor rotation, the variable coordinates included in the DE system (1) are periodic functions with a period T=2π. This makes it possible to consider the problem of calculating the steady-state operation of the WRIM with constant slip as a boundary problem with periodic boundary conditions. The application of the specified method of obtaining a periodic solution makes it possible to obtain periodic dependences of a set of coordinates on one, assumed to be independent, without resorting to the calculation of the transient process, and is the most effective in terms of the amount of calculations. Note that the result of the calculation of the steady state is not a set of numerical values of the coordinates, which corresponds to the independent variable, but the functional dependencies of the coordinates during the period. A set of dependencies that correspond to a change in an independent coordinate determine a multidimensional static characteristic.

In order to find the periodic solution of the DR system (1) by the method of solving the boundary value problem, it is necessary to carry out their algebraization. Many methods of solving boundary-value problems are known, starting from the evolutionary method, the essence of which is the numerical integration of the DR system (1) to the actual stabilization, as well as difference and approximation methods. The article proposes the calculation algorithm of the projection method of algebraization developed by the authors [18], based on the theory of splines, the essence of which in relation to the solution of this problem is as follows.

Using the values of the coordinates in n nodes in the period T, we approximate each component of the vector $\vec{\psi}$ by a cubic spline. As a result, we will get a continuous function, which is described by the equation at each j-th time section $h_j = t_j - t_{j-1}$

(4) $$\psi(t) = a_j + b_j(t_j - t) + c_j(t_j - t)^2 + d_j(t_j - t)^3,$$

where a_j, b_j, c_j, d_j – spline coefficients; $j = (\overline{1, n})$ – number area.

The ratio between spline coefficients is determined by the properties of spline functions of the third order. In particular, as follows from (4), at $t = t_j$

$$\psi(t_j) = a_j\,, \qquad \left.\frac{d\psi}{dt}\right|_{t=t_j} = f_j = -b_j\,.$$

As a result of the approximation, taking into account periodic boundary conditions ($a_{n+j} = a_j$, $b_{n+j} = b_j$, $c_{n+j} = c_j$, $d_{n+j} = d_j$), we obtain the algebraic analogue of the system (2) in the form of a vector equation

$$H\vec{\Psi} - \vec{F} = \vec{0}\,, \tag{5}$$

where H – a square matrix of size 6×n of the transition from a continuous change of coordinates over a period to their discrete values, the elements of which are determined exclusively by a grid of nodes [16];

$$\vec{\Psi} = \left(\overline{\vec{\psi}_1, \vec{\psi}_n}\right),\ \vec{F} = \left(\overline{\vec{f}_1, \vec{f}_n}\right),$$

– column vectors composed of vector values $\vec{\psi}$, $\vec{f}$ at period nodes.

Vector equation (5) is a discrete analog of the system (1), and its solution is a vector $\vec{I} = \left(\overline{\vec{i}_1, \vec{i}_n}\right)^*$, which is a grid representation of the vector $\vec{i}\left(t\right) = \vec{i}\left(t+T\right)$ on the period. To obtain it, we will use the parameter continuation method in combination with Newton's method. To do this, we will select the disturbance vector $\vec{U} = \left(\overline{\vec{u}_1, \vec{u}_n}\right)$ and present the vector $\vec{F}$ in the form of a sum

$$\vec{F} = \vec{Z} + \vec{U}\,.$$

We introduce the scalar parameter ε into the system (5) by multiplying by ε the vector $\vec{U}$ of nodal values of the applied voltages, that is, assuming $\vec{U} = \varepsilon\vec{U}$.

Considering that

$$\vec{\Psi} = \vec{\Psi}\left(\vec{I}\right)\,; \quad \vec{F} = \vec{F}\left(\vec{\Psi}, \vec{I}, \varepsilon\right),$$

we differentiate the equation obtained by entering the parameter with respect to ε. As a result, we will get DE of the form

(6) $$W\frac{d\vec{I}}{d\varepsilon} = \vec{U}\ ,$$

where $W = \left(H - \dfrac{\partial\vec{F}}{\partial\vec{\Psi}}\right)\dfrac{\partial\vec{\Psi}}{\partial\vec{I}} - \dfrac{\partial\vec{F}}{\partial\vec{I}}$ – the Jacobian matrix in which the corresponding derivatives are block-diagonal matrices in which

$$\frac{\partial\vec{\psi}}{\partial\vec{i}} = \left.\frac{\partial\vec{\psi}}{\partial\vec{i}}\right|_{t=t_j}$$

– matrix of differential inductances of WRIM circuits in nodes of the period grid;

$$\left.\frac{\partial f}{\partial i}\right|_{t=t_j} =$$

$-r_A$					
	$-r_B$				
		$-r_C$			
$\alpha(L_{bA}-L_{cA})$	$\alpha(L_{bB}-L_{cB})$	$\alpha(L_{bC}-L_{cC})$	$\alpha(L_{ba}-L_{ca})-$ $-r_a$	$\alpha(L_{bb}-L_{cb})$	$\alpha(L_{bc}-L_{cc})$
$\alpha(L_{cA}-L_{aA})$	$\alpha(L_{cB}-L_{aB})$	$\alpha(L_{cC}-L_{aC})$	$\alpha(L_{ca}-L_{aa})$	$\alpha(L_{cb}-L_{ab})-$ $-r_b$	$\alpha(L_{cc}-L_{ac})$
$\alpha(L_{aA}-L_{bA})$	$\alpha(L_{aB}-L_{bB})$	$\alpha(L_{aC}-L_{bC})$	$\alpha(L_{aa}-L_{ba})$	$\alpha(L_{ab}-L_{bb})$	$\alpha(L_{ac}-L_{bc})-$ $-r_c$

$$\alpha = \omega/\sqrt{3}\,.$$

The calculation of the static characteristic as a function of coordinates ξ taken as independent is carried out by the differential method. Its essence is that the nonlinear system of finite equations (5) is differentiated by the argument ξ (for example, the resistance $\xi = r_p$ in the rheostat circuit), and then integrated numerically. As a result of differentiation ξ, we get the DE system

(7) $$W\frac{d\vec{I}}{d\xi} = \frac{\partial\vec{F}}{\partial\xi}$$

Therefore, the following algorithm is proposed for calculating static characteristics.

Given the value of the rotor speed of rotation ω=0 (s=1), we perform several steps of integration of system (6) over ε (from ε = 0 to ε = 1) by Euler's method, which makes it possible to obtain periodic dependences of currents (and, therefore, flux linkages) at s=1 with some approximation caused by the errors of the Euler method, but they are sufficient to ensure the convergence of the iterative process. We refine the obtained nodal coordinate values according to the iterative scheme of the Newton method, according to which the increments $\Delta\vec{I}$ of the nodal coordinates at the *k*-th step of the iteration are calculated according to the formula

(8) $$W\Delta\vec{I}^{(k)} = \vec{Q}\left(\vec{I}^{(k)}\right),$$

where $\vec{Q}(\vec{I}^{(k)})$ – the residual vector of the system (6) at given values of the vector $\vec{U}$ of nodal values of the applied voltages. Note that the Jacobi matrix in equation (5) is the same as by integration, which makes it possible to combine these two calculation stages in a single algorithm. Changing the slip value from *s*=1.0 to zero, we find the coordinate values that correspond to them only by the iterative method since the coordinate values obtained from the previous step are usually in the vicinity of the convergence of the iterative process. Based on the calculated nodal values of flux linkages and currents for each node, we calculate the electromagnetic moment.

The described algorithm for calculating static characteristics in phase coordinates can be used both for the analysis of symmetric and asymmetric modes. In the case of a symmetric mode, the mathematical model in orthogonal *x*, *y* coordinates described in [6] is simpler, but it has a number of limitations. In particular, it is impossible to analyze an asymmetric mode, which may occur in the case of non-simultaneous or incomplete switching on of the stages of the starting device. Thanks to the use of an effective method of finding a periodic solution, the developed algorithm and mathematical model can be applied to symmetric modes. In this case, the dependence of the coordinates on the period will be in the form of straight lines, and therefore you can limit yourself to a small number of nodal points on the period. The program has a high speed of operation, which gives reason to consider it universal in terms of application for both symmetric and asymmetric modes.

The algorithm for calculating the static characteristics necessary for selecting the parameters of the WRIM starting device and controlling the electric drive consists of two stages: at the first stage, the dependence of the starting moment on the resistances of the rheostat during slip *s*=1,0 is calculated, and then, by calculating the starting mechanical characteristics, the necessary values of the resistances of the sections of the starting rheostat are selected.

To obtain the dependence of coordinates on slip *s* by the differential method, system (7) has the form

(9) $$W\frac{d\vec{I}}{ds} = \frac{\partial\vec{F}}{\partial s}.$$

The algorithm for switching the elements of the starting device created on the basis of the results of the calculations of the static characteristics is finally checked by calculating the transient electromechanical process according to the algorithm described at the beginning of the article.

An example of the results of the calculation of the periodic dependences of the stator, rotor, and electromagnetic torque currents with asymmetry of the resistances of the starting device is shown in Fig. 5.

Average values of the electromagnetic torque and effective current values are used to construct static characteristics.

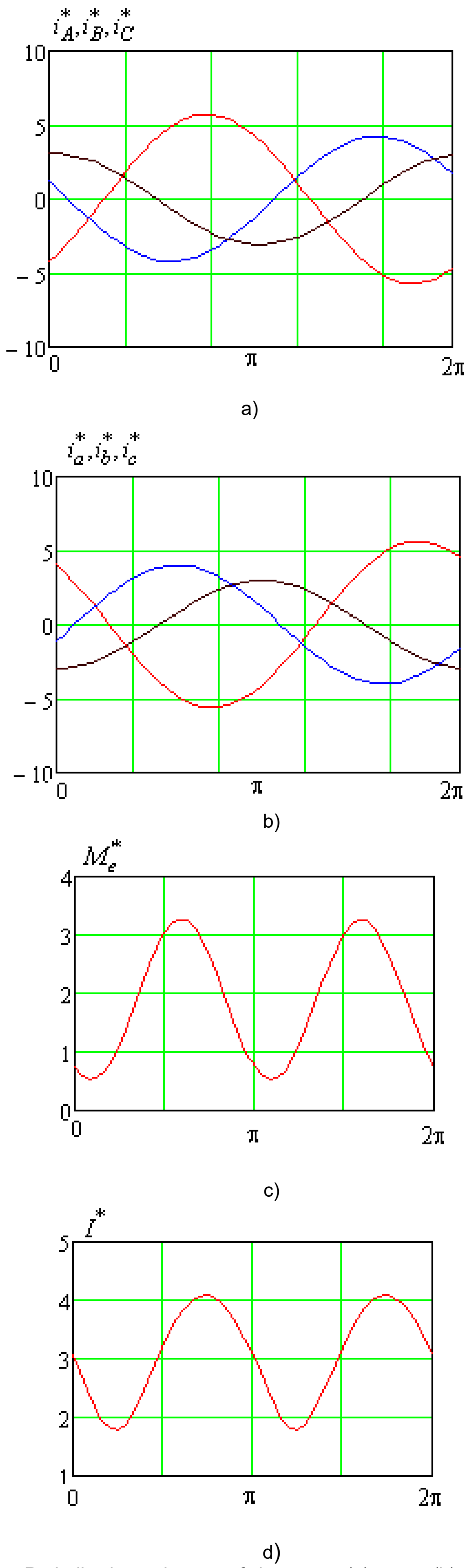


Fig. 5. Periodic dependences of the stator (a), rotor (b) currents, electromagnetic moment (c), and space vector of current (d) with *s*=1,0 and asymmetry of the starting rheostat phase resistances

Conclusions

Algorithms for calculating the transient processes and static characteristics of the WRIM have been developed, which make it possible to perform a set of design and operational studies necessary to ensure the reliable and efficient operation of the electric drive system with a relay-contactor control system.

The basis of the developed algorithms is a non-linear mathematical model of the WRIM based on equations of state that describe electromagnetic and electromechanical processes in a three-phase system of coordinate axes, which makes it possible to calculate not only symmetric but also asymmetric modes of operation of the electric drive, taking into account the saturation of the magnetic field.

The calculation programs created based on the developed algorithms ensure the adequacy of the analysis results and have a high speed, thanks to which they can be used not only for the design of regulated electric drives but also for their control systems in order to form the necessary characteristics.

The task of calculating the steady state is performed by solving the boundary value problem for the DE system of the electromagnetic equilibrium of the motor circuits. Algebraization of DR is carried out by approximating periodic dependences of coordinates by splines of the third order, taking into account periodic boundary conditions. The continuation method by parameter is used to calculate static characteristics. The elements of the Jacobi matrix include active leakage flux calculated based on the characteristics of magnetization by the main magnetic flux and leakage fluxes.

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